DEPARTMENT OF ENGINEERING MANAGEMENT

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RESEARCH PAPER 2022-007
SEPTEMBER 2022

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D/2022/1169/007

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Storing inventory in both a reserve and forward area is a way to improve the efficiency of a B2C warehouse. Due to the small size of the forward area order picking distances can be reduced, although internal replenishments from the reserve to the forward area are necessary to keep the warehouse up and running. In the warehouse literature, the organisation of a forward area is mostly studied from a tactical point of view, discussing the size of the forward area, the selection of products to store there and the number of locations to allocate to each product, also known as storage capacity. The operational implications of these decisions are often ignored or simplified, possibly resulting in (replenishment) cost underestimations. In this paper we study how the forward area could be replenished such to avoid stockouts during order picking whilst integrating practical issues that rise during the operation, e.g., limited replenishment force and time. We present the internal warehouse replenishment problem (IWRP), that determines which products to replenish, in which quantity and by which replenisher. The IWRP is solved for different sizes of a rectangular forward area and differing storage and replenishment capacities whilst considering various inventory policies and planning horizons. We consider an out-of-rack forward reserve system where picking and replenishing is performed in alternating waves. The IWRP is optimally solved using a standard mixed integer program solver. Results show how all parameters influence the objective, and reveal practical issues that should be considered when configuring the forward area. For certain parameter combinations instances were not solved within a realistic time, hence the call for a heuristic solution method.

Keywords: Inventory; Warehousing; Forward reserve; Replenishment

1 Introduction

A forward reserve storage system divides the storage area of a warehouse in a *reserve area* or *bulk area*, and a *forward area* (Wu et al., 2020), also referred to as the *fast-pick area* (Bartholdi and Hackman, 2019). In the reserve area, products are stored in bulk quantities (e.g., pallets), while in the typically smaller forward area a selection of products is stored in a format more convenient for item picking (e.g., bins). Due to the restricted size of the forward area, the picking time, still the primary cost in a B2C warehouse, can be reduced (De Koster et al., 2007). As such, the implementation of a forward area complements the list of storage allocation, batching and routing policies studied and used to improve order picking speed and efficiency (Boysen et al., 2019). For an overview and detail of such policies, we refer to Gu et al. (2007). It should be noted that although of all warehouse activities order picking is considered to have the most direct impact on customer order fulfilment (Roodbergen et al., 2015), all efforts to improve this efficiency are lost if pick locations are not restocked as they should be (Carrasco-Gallego and Ponce-Cueto, 2009). Therefore, *restocking* or *replenishing* is a crucial operation to support a smooth order picking process, although it is often neglected.

To ensure (sufficient) stock to be available in the forward area, replenishments from the reserve to the forward area need to be planned and executed, referred to as *internal replenishments*. Bartholdi III and Hackman (2008) visualise the situation as a ”warehouse in a warehouse”, and point out that picking and replenishment efforts must be balanced to organise matters efficiently. Indeed, a large forward area will reduce the frequency of internal replenishments, but will increase the order picking travel instance, and vice versa. This trade-off is the primary objective in what is known as the *forward reserve problem* which focusses on the organisation of a forward reserve system by determining the size of the forward area, which stock keeping units (SKUs) to store there, which capacity to allocate to these SKUs and where in the forward area to store the SKUs (Jiang et al., 2020). Studies found in the warehouse literature, address one or multiple of these decisions while reducing material handling costs of picking and replenishing (e.g., Hackman et al. (1990)) or minimizing the replenishment cost (e.g., Walter et al. (2013)). Acknowledging the tactical focus of these studies, it is noteworthy that often strict assumptions are taken regarding the practical execution of internal replenishments. It is, for example, common to assume that replenishment quantities can be replenished in one trip (e.g., Bartholdi III and Hackman (2008)) and that replenishment effort is independent of the location in the forward area that needs to be replenished. The actual time required to successfully perform the replenishment is neglected, costs are possible underestimated and operational problems can be encountered.

An example of such an operational problem is that a replenishment is scheduled but no replenisher is available to execute the task. The outcome is that replenishments possibly happen too late and a stockout is experienced. Congestion among pickers and replenishers contributes to this delay, especially if both picking and replenishing take place simultaneously. To reduce congestion,Van den Berg et al. (1998) mainly perform (unit-load) replenishments before picking, referred to as *advance replenishments*. Replenishing in advance allows to replenish multiple loads (Wu et al., 2020), and to coordinate the replenishment of loads in one trip if less than pallet-load replenishments are considered. In this paper we extend the concept of advance replenishment and assume that replenishment and picking are organised in alternating *waves*, with a wave defined as a group of orders released simultaneously to the pick area (Ardjmand et al., 2018), often characterised by the same delivery time (Rasmi et al., 2022). That way, the forward area is replenished in advance of picking, knowing the demand of the upcoming pick wave(s). We subject the replenishment waves to practical constraints, e.g., restricted work force and time available for replenishment. Because of these restrictions it is possible that not all required replenishments, those necessary to avoid a stockout, can be performed. Any stockout experienced during the pick wave is handled by means of an *emergency pick*: the missing items to complete order fulfilment are directly brought from the reserve area to the picking depot. Emergency picks greatly disrupt the picking process as efficiently planned pick tours cannot be carried out. Therefore, the minimization of emergency picks (elaborated in section 3.2.3), realised through the minimization of the number of SKUs experiencing a stockout, is a relevant objective that we pursue.

Our contribution to the research on the forward reserve problem is two-fold. (1) We present the *internal warehouse replenishment problem* (IWRP), a model that organises advance replenishments by determining which SKUs to restock in which quantity by which replenisher, while considering relevant practical constraints. The IWRP is solved whilst minimizing the number of SKUs experiencing a stockout during the upcoming pick wave(s), with a stockout occurring if the inventory level post replenishment is exceeded by the product's demand in the upcoming pick wave. The IWRP is solved for a number of consecutive replenishment and pick waves, highlighting the dynamic aspect of the problem. Baita et al. (1998) define *dynamicity* as a situation in which the same set of questions has to be answered repetitively over a fixed horizon, and where future decisions are influenced by decisions made earlier. Dynamicity is originally introduced in the context of the inventory routing problem (IRP), which, due to resemblances with the IWRP, has inspired the model proposed in this paper. (2) We solve the mixed integer model for the IWRP with a standard MIP-solver for different parameters, decisions made on the tactical level, such as the inventory policy, storage capacity, replenishment resource availability and demand data. The results of the experimental study allow us to answer some research questions, centred around the influence each parameter has on the number of stockouts. We emphasize that the aim of this research is not to provide the most efficient operational replenishment plan with (near) optimal replenishment routes, but to study and conclude on the impact of tactical decisions on the number of stockouts whilst delivering a replenishment plan that meets all operational constraints.

The remainder of this paper is organised as follows. In section 2 we present a literature review. On the one hand, we focus on the forward reserve storage policy studied in the warehouse literature. On the other hand, we review the literature on the inventory routing problem given the resemblances with the IWRP proposed in this paper. In section 3 we describe in detail the IWRP model and assumptions taken. The mathematical model is presented in section 4. The experimental setup and results are discussed in section 5. We conclude this study and elaborate on future research in section 6.

2 Literature review

2.1 The forward reserve storage policy

In the following sections, we review the literature on the forward reserve storage policy on the strategical, tactical and operational level.

2.1.1 Implementation and design of the forward area

Storing inventory in both a reserve and forward area is highlighted by many studies as an opportunity to improve picking performance. Thomas and Meller (2015) compared 476 design possibilities for a manual, rectangular, case-picking warehouse and found that the worst performing designs, i.e., designs with the highest amount of labour hours necessary for picking, put-away and replenishment (analytical models presented by Thomas and Meller (2014)), are the ones with no forward area. A clear guideline as of when it is beneficial to store a product in the forward and reserve area, is provided by Wu et al. (2020). Their analytical study is based on response travel time models which the authors developed for an AS/R system housing both the forward and reserve area. For such a system, the authors indicate that the forward storage policy pays off, i.e., outperforms an ABC class-based storage policy, as long as the number of picks per replenishment is strictly larger than one. This implicates that at least two picking orders should be picked from one replenishment load. For an out-of-rack forward reserve system, with the forward and reserve area situated apart, the authors recommend a ratio of at least 3 because replenishments take up more time.

Once the call for a forward reserve system is made, various implementations are possible differing in design and equipment. The forward and reserve area can be located in different racks, an out-of-rack system, or both can be situated in the same rack. In the latter set-up it is common to designate the lower levels as forward area, while the upper levels are dedicated to reserve storage. Equipment-wise, forward reserve systems can differ in level of automation, with replenishment and/or picking performed automatically or manually. An overview of the different implementations and relevant literature on each system is presented by Wu et al. (2020).

2.1.2 The forward reserve problem

The forward reserve problem entails several decision problems (Wu et al., 2020):

1. What is the size of the forward area?
2. Which products to store in the forward area?
3. How much storage locations to allocate to each product in the forward area?
4. Jiang et al. (2020) add to this list the assignment of storage locations to products in the forward area.

Hackman et al. (1990) are the first to present a model to tackle problems 2 and 3. By means of a heuristic procedure, they reduce the picking and replenishing material handling costs of a forward area organised with an AS/R system. In fact, they balance the cost of picking from the forward area (cost a) and the cost of picking from the reserve area (cost b) on the one hand, for which holds $a < b$, and the replenish cost on the other hand. Bartholdi III and Hackman (2008) continue the work of Hackman et al. (1990) with the selection of SKUs stored in the forward area to be known. They determine the optimal storage capacity of small-sized products in the forward area while minimizing the total number of restocks annually. They compare their optimal storage policy to the Equal Space strategy (EQS), that partitions space equally, and the Equal Time strategy (EQT), that divides space such that each product is restocked the same number of times. Bartholdi III and Hackman (2008) prove the outperformance of their optimal storage policy, which also disproves the common belief in warehouse industry that the EQT strategy minimizes the number of restocks. An optimal solution approach to the problem outlined by Hackman et al. (1990) is proposed by Gu et al. (2010) with a branch-and-bound algorithm.

Van den Berg et al. (1998) continue the work on the forward reserve problem with a two-fold contribution. First, they relax the assumption that one trip suffices to replenish a product in the forward area. The authors argue that product storage is often organised in pallets and that only one replenishment can be executed per trip. Secondly, the authors study a set-up where picking and replenishing are not solely executed simultaneously. Instead, most unit-load replenishments are performed during idle periods (e.g., morning shift or weekend), referred to as *advance replenishments*. *Concurrent replenishments* during picking take place in case a product experiences a stockout, which one aims to avoid given the time-intensive character of the operation. The authors present a mathematical model to determine which quantities of which products to store in the forward and which products to replenish in advance while minimizing the expected amount of labour time (picking and concurrent replenishing). A greedy heuristic is proposed, inspired by the well-known Knapsack problem. Gagliardi et al. (2008) continue the work on palletized forward areas, assuming a set-up where replenisher and picker(s) work simultaneously, apart from weekend shifts dedicated to replenishments. The authors propose an iterative improvement heuristic to solve the allocation problem (the selection of SKUs is predetermined) while optimizing the total number of stockouts. The authors test different allocation ratios that all have in common to allocate at least two locations to each SKU, allowing replenishments before the inventory level falls to zero.

A considerable flaw in the studies discussed so far, except studies on unit-load replenishments, is that assignment and/or allocation decisions are made assuming the forward area space can be continuously partitioned among SKUs (Bartholdi and Hackman, 2019). Walter et al. (2013) tackle the drawback of this *continuous* or *fluid* model, and present the discrete counterpart of the forward reserve problem that allocates discrete units of space. The authors propose and discuss on several repair heuristics to convert the continuous output of the fluid model to an integer solution.

Frazelle (1994) extends the forward reserve problem by including the forward area's size as a decision variable. Heragu et al. (2005) determine how available space should be divided among a cross-dock, reserve and forward area, as well as to which area products will be allocated. The authors present a heuristic approach to solve the problem jointly while minimizing material handling and storage costs. Thomas and Meller (2014) present a model to configure the forward area where the upper aisles are dedicated as reserve area. Specifically, the model determines the optimal warehouse shape whilst considering travel distances for put-away, order picking and internal replenishments. For an overall optimal warehouse shape, the frequency of operations and distances associated with each operation need to be considered.

Wu et al. (2020) introduce response travel time models for a forward reserve system where both the forward and reserve area are located in an AS/R system. The model is used to size both areas and solve the assignment problem whilst minimizing the response time. In these models the authors consider the probability of picking an item from the forward or reserve area, the expected retrieval time, and the average inventory restocked by external suppliers. The authors additionally take into account the occurrence of single and dual-command cycles, and test both a random and ABC class-based storage policy for the forward area. Because of these additions, the work of Wu et al. (2020) touches upon many aspects related to the organisation of a forward reserve policy that go beyond most forward reserve studies. The authors eventually conclude on the implementation guidelines of such a forward reserve system, as discussed in section 2.1.1.

In the forward reserve problem the underlying idea is to maximise the value of the restricted space available in the forward area, hence the comparison with the Knapsack problem. Yu and de Koster (2010) and Bahrami et al. (2019) stress this challenge and propose innovative ways to deal with it. The former study a dynamic storage system that swaps products autonomously such that only the products required for the current batch of picking orders are provided in the forward area. Bahrami et al. (2019), on the other hand, study an alternative storage assignment policy for the forward area based on the sharing concept. Storage locations are either shared or dedicated. Shared locations are used to restock any product in the forward area for which upcoming demand exceeds the predetermined storage capacity stored at the dedicated locations.

2.1.3 Planning of internal replenishments

Once the tactical decisions are made, the internal replenishments need to be planned for execution. We find a limited number of operational studies on the internal replenishment. A summary is given below, making a distinction between studies on picker-to-parts systems and studies on parts-to-picker systems.

Picker-to-parts systems
To the best of our knowledge, the operational issues of the internal replenishment activity are mainly studied for picker-to-parts systems in which picking and replenishing take place simultaneously. Gagliardi et al. (2008) assume the performance of weekend replenishment shifts to stock up the forward area when no picking is performed, but their actual optimization only relates to the replenishments happening during week shifts. The authors propose different replenishment heuristics to determine the next replenishment order in a pick-to-belt pick area. The dominant replenishment heuristic, i.e., the one resulting in the least number of stockouts, requires upcoming demand as well as pick list sequences to be known in advance, and replenishes the first SKU that is about to experience a stockout. The performance of the replenishment heuristic, however, depends on the space allocation heuristic, discussed in section 2.1.2. De Vries et al. (2014) study an environment in which picking and replenishing solely take place simultaneously, in the same wave. The wave policy allows to know the demand of the upcoming wave which the authors use to optimize the sequence of replenishment orders. De Vries et al. (2014) propose several replenishment policies to determine the priority for products replenished in the next wave. Batching of replenishment orders is allowed as long as priorities are respected. Both studies have in common to assume a fixed time for each replenishment order, ignoring the replenishment location and the time it takes to get there.

The question of which location to replenish becomes even more important when a *scattered storage policy* is adopted. The policy scatters a single product over different storage locations in the pick area and aims to reduce picker travel distances based on the idea that the probability increases to find requested items in locations close to each other (Gámez Albán et al., 2020). Although the policy is primarily studied in terms of the picking activity (Weidinger et al., 2019), Weidinger and Boysen (2018) emphasize the role of replenishments in the implementation of this storage policy. Frequent replenishments ensure a high overall inventory level of products with many locations for pickers to pick from. Replenishments triggered at a low inventory level allow more open storage locations to chose from when scattering items over the warehouse. Weidinger and Boysen (2018) present the scattered storage assignment (SSA) problem that assigns empty storage locations to products to maximize the scateredness of products in the pick area. Their conclusions support our belief that the replenishment component cannot be neglected: the authors re-assessed the advantages of an optimized scattered storage assignment over a randomized one when not only the picking but also the replenishment travel component were included. Also Bahrami et al. (2019) take into account both picking and replenishment travel distances for completeness.

Parts-to-picker systems
Boywitz et al. (2019) study the replenishment in an A-frame system. An A-frame system is a very efficient system which pushes items from a channel in an autonomous way towards a central conveyer as the order requesting this product passes by. The picking process itself is fully automated, the replenishment process is organised manually. Passing the orders on the conveyor in an optimized sequence helps to scatter the replenishments and give replenishers the time they need to move from channel to channel and perform the required replenishment. Jiang et al. (2020), on the other hand, tackle the replenishment activity in a robotic mobile fulfilment system (RMFS). The authors study the synchronization between order picking and replenishment. The authors argue that the high number of shelf visits and resulting high picking costs can be reduced if replenishments were performed more thoughtfully, by for example storing products requested by the same order on the same shelve. Replenishment time, on the other hand, increases, such that the challenge is to find the optimal balance between picking and replenishment effort. The authors present the picking-replenishment synchronization mechanism (PRSM) and propose a variable neighborhood search with a divide-and-conquer paradigm to solve it.

Research opportunities
Summarising the literature on the forward reserve problem, brings us to a practical remark. Solving the forward reserve problem happens often without considering the practical, operational roll-out of the tactical choices that were made, or by taking strict assumptions. It is, for example, unclear to what extent the storage location of SKUs in the forward area is included in the replenishment cost. Bartholdi III and Hackman (2008) state that because the forward pick area is such a small part of the entire warehouse, travel within this area is considered only a fraction of replenishment cost. Other studies acknowledge replenishment time not to be negligible, but assume all replenishments take the same amount of time. We find this to be in contradiction to the vast research on the warehouse order picking activity, that focusses on the minimization of pickers' travel distance or time in the pick area (De Koster et al., 2007); (Van Gils et al., 2018). Solving the forward reserve problem with these practical simplifications can result in more stockouts than anticipated, more concurrent replenishments, and cost underestimations and inefficiencies in general.

Research on systems where replenishments mainly take place between two pick waves remains scarce despite its opportunities to avoid concurrent replenishments (Wu et al., 2020). Advance replenishments allow to replenish multiple loads rather than performing replenishments when a stockout is imminent or already has occurred. Replenishments can also be performed in a more coordinated way, whereas in a simultaneous system replenishment orders happen more scattered in time with less interesting consolidation opportunities. Advance replenishments also allow to reduce congestion as pickers and replenishers coincide to a minor extent (Van den Berg et al., 1998). Time available for advance replenishments in such a system, however, is not infinite: time spent on replenishing is considered time not spent on picking. It is therefore in the warehouse's interest to restrict the replenishment time, as well as the number of available replenishers to keep congestion to a minimum and avoid further fulfilment delays.

To achieve more efficiency, whether expressed in a minimisation of stockouts or a minimal cost, the practical organisation of internal replenishments should be, to some extent, considered in the tactical decision making progress. We are, however, aware that the forward reserve problem already qualifies as an NP-hard problem (Walter et al., 2013). Integrating practical issues would make matters even more complex. Nonetheless, it would be interesting to study the influence of tactical choices on the stockout behaviour when practical limitations are imposed. To the best of our knowledge, such research is not available for a manual picker-to-parts system where picking and replenishing happens alternately. Although research

Multi-vehicle IRP	
A supplier is responsible for the deliveries of a product to a set of customers or retailers, which he assigns to one of the available vehicles.	
Reference	**Remarks**
Coelho et al. (2012)	Matheuristic solution method
Guemri et al. (2016)	Exact solution method
Raa and Aghezzaf (2009)	Integration of fleet sizing and restricted driving time per period
Coelho et al. (2012)	Integration of service-related inconveniences.
	E.g., driver consistency: serve customer by one vehicle (↔ split delivery)
Multi-product IRP	
A supplier is responsible for the distribution of multiple products to a set of customers or retailers.	
Reference	**Remarks**
Coelho et al. (2014)	Literature review
Moin et al. (2011)	Genetic algorithm
Mjirda et al. (2014)	Two-phase variable neighborhood search
	(1) Solve vehicle routing problem with replenishment quantity equal to demand,
	(2) improve solution through neighbourhood exploration
Mjirda et al. (2016)	State of the art
	General variable neighbourhood search, approaching replenishment quantity as integer problem, rather than continuous
Multi-product and multi-vehicle IRP (MMIRP)	
The demand of multiple customers has to be met through the delivery of multiple products, organised over a limited fleet of vehicles.	
Reference	**Remarks**
Coelho and Laporte (2013)	Presenting the mixed-integer lineair programming formulations of the MMIRP assuming a shared storage policy
	Solved with a branch-and-cut algorithm
Guemri et al. (2016)	Grasp-based heuristic
	Improves results of Coelho and Laporte (2013) for large instances
Hasni et al. (2017)	Variable neighborhood search algorithm
	Dominating Coelho and Laporte (2013) in terms of computation time
Cordeau et al. (2015)	Decomposed solution method approaching inventory and routing subproblem separately,
	followed by feedback model to improve overall solution

Table 1: Overview of literature on the multi-vehicle, multi-product, and multi-product and multi-vehicle IRP.

in the warehouse literature is limited, we find many similarities between the planning of internal replenishments and the inventory routing problem (IRP), a problem extensively studied within the context of vehicle routing. The decisions typically handled are more of an operational kind, tackling some of the issues that often remain untouched in the forward reserve problem. A brief overview of the IRP is given in the next section.

2.2 The inventory routing problem

The inventory routing problem (IRP) studies the combination of inventory management and vehicle routing decisions (Moin and Salhi, 2007). The problem arises typically in a vendor-managed inventory (VMI) context, where the supplier takes control over the inventory decisions faced by the customers, and coordinates the deliveries to all customers. Because aggregated information is available at the supplier, serving customers can be done more efficiently, resulting in significant cost reductions. A basic version of the IRP is described by Coelho et al. (2014), although the authors remark that a standard version of the problem not really exists. In this so-called basic IRP, a single distributor or supplier delivers a product to multiple customers, referring to a *one-to-many IRP*. The IRP comprises three subproblems:

1. When to serve a customer.
2. The quantity delivered at a customer.
3. How to combine customers in a route, as well as the determination of the route.

The IRP aims to answer these questions while minimizing the total inventory and distribution cost. The inventory cost is incurred by both supplier and customers, although some papers assume only one of both parties to incur this cost (e.g., Mjirda et al. (2014)). The distribution cost is determined by the route, that is the distance travelled when moving from the supplier to the customer(s) and back. Often, studies decompose the distribution cost into a variable and a fixed component to penalise the use of vehicles (e.g., Raa and Aghezzaf (2009), Moin et al. (2011)). Among the constraints, we find capacity-related restrictions: each customer has limited storage space to store deliveries. The vehicle capacity, on the other hand, restricts the number of units that can be replenished in one trip.

The IRP has been studied for different variants. Coelho et al. (2014) distinguish variants based on (1) the structure of the IRP, and (2) the demand information that is available to solve the IRP, differing static from dynamic IRPs. With regards to the structure of the IRP, IRPs are distinguished based on: the time horizon, the structure (one-to-one, one-to-many or many-to-many), routing, inventory policy, inventory decision, fleet composition and fleet size. The authors give an extensive overview of studies on the IRP, which they classify according to the categorisation just described. Similarities with the IWRP we envision, are most noticeable with IRPs studied for a multi-product and/or multi-vehicle environment. An overview of research on these problems is summarised in table 1.

3 The internal warehouse replenishment problem

3.1 Problem description

The aim of the IWRP is to organise internal replenishments for an out-of-rack forward reserve system with alternating replenishment and pick waves. We assume the replenishment operation to be organised in bins, although only minor adaptations would be required for alternative handling units (e.g., pallet). We subject the system to practical limitations and aim for a minimum number of SKUs experiencing a stockout in the upcoming pick wave. We define a stockout as follows:

Stockout: An SKU experiences a stockout if no stock is available in the forward area (inventory level = 0 items), while there are still unfulfilled customer orders requesting the SKU. As a consequence, an emergency pick will be required.

The replenishment operation is exposed to two capacity constraints.

- **Storage capacity constraint:** A maximum number of items can be stored in the forward area of each SKU, determined by the number of storage locations allocated to the SKU. Determining this quantity for each SKU is part of the forward reserve problem (discussed in section 2.1.2) and solved a priori.
- **Replenishment capacity constraint:** A restricted capacity is available to perform replenishments. The capacity is determined by the number of available replenishers, the maximum number of bins that can be replenished in a single tour (restricted by the cart's capacity), and the time available for each replenisher between two pick waves.

Both constraints force to make deliberate choices as probably not all desired replenishments can be executed when requested. The IWRP is decomposed into five questions:

1. Which SKUs will be advance replenished?
2. For each SKU: how many bins will be replenished of each SKU, referred to as replenishment bins?
3. For each replenishment bin: with which other bin(s) will the replenishment bin be batched?
4. For each replenishment batch: by which replenisher will the replenishment batch be executed?
5. For each replenishment batch: can a feasible tour be determined, taking into account the predetermined dedicated storage assignment?

Note that the aim of the IWRP is not to determine optimal replenishment routes, but to minimize the number of stockouts. Improving replenishment tours, i.e., visiting locations in a shorter way, is only pursued if the time saved can be employed to perform more replenishments and avoid additional stockouts.

These five questions are to be solved for each replenishment wave, given the following information:

- **Initial inventory information:** all relevant inventory information related to the previous pick wave. This includes the stock level of each SKU, and whether or not a less-than-full bin, also called *broken bin*, of the SKU is available in the reserve area at the end of the previous pick wave. The organisation of broken bins will be discussed in section 3.2.3.

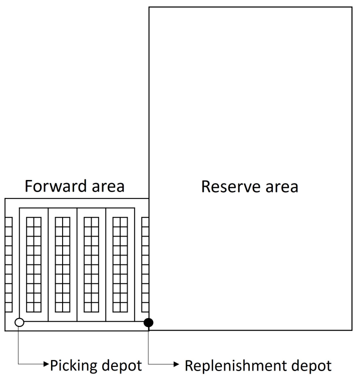


Figure 1: Visualisation of the forward and reserve area in an out-of-rack implementation.

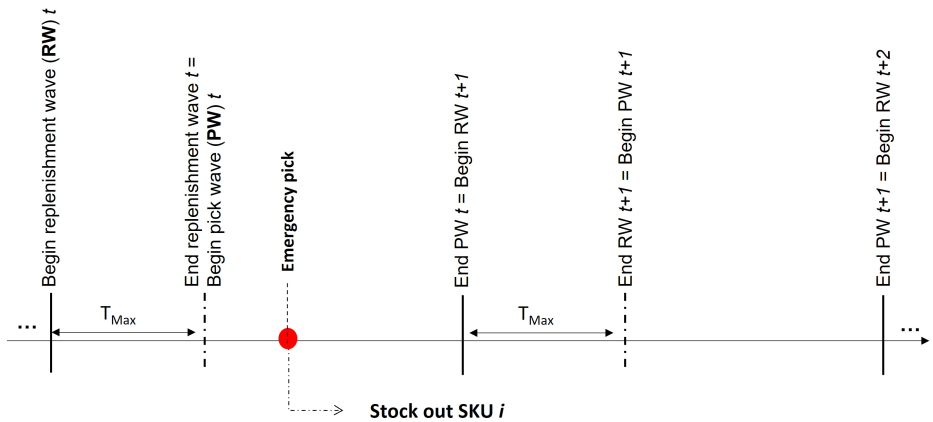


Figure 2: Organisation of replenishment waves (RW) and pick waves (PW). Replenishments are always performed in advance of picking; taking into account the demand of the upcoming pick wave(s).

- **Capacity-related data:** data related to the storage capacity and replenishment capacity.
- **Demand:** the demand of the upcoming T pick wave(s) is known for all SKUs.
- **Layout and storage allocation:** the layout of the forward area is given and the assignment of SKUs to storage locations is predetermined. The shortest Manhattan distance to move between SKUs is known.

3.2 Problem context and assumptions

In this section, we give more details on the replenishment set-up and list the assumptions necessary to model the internal replenishment operation we assume in this paper.

3.2.1 Organisation forward and reserve area

In the remainder of this paper, we consider the organisation of the reserve area (the layout characteristics, the storage policy, the planning of retrievals from the reserve area,...) out of scope. The following assumptions hold:

A 1 *An out-of-rack forward/reserve area is assumed (see fig. 1).*

The replenishment depot, where all replenishment routes start and end, is located at the intersection of the forward and reserve area. The picking depot is situated elsewhere.

A 2 *There is sufficient stock in the reserve area to execute all planned internal replenishments.*

The planning of external replenishments is not part of the optimization problem.

A 3 *The reserve area is efficiently organised such that all bins required for an internal replenishment, are retrieved in time.*

The bins are then temporarily stored on a cart, depending on the replenishment batch to which the bin is assigned. The cart is picked up by a replenisher to store the bins from the cart in the forward area at the right storage location.

A 4 *Replenishers are allowed to execute multiple replenishment batches consecutively, as long as the available replenishment time allows.*

Delays due to congestion between replenishers are neglected, nor do we consider any set-up time between two batches executed by the same replenisher.

3.2.2 Organisation replenishment and pick waves in the forward area

Replenishments and picking take place in the forward area in alternating waves, illustrated in fig. 2. The time available to perform replenishments is fixed and the same for each replenishment wave, represented by T_{max}. In each replenishment wave, each replenisher has T_{max} [time units] available. Any time remaining is not transferable to other replenishers nor future replenishment waves.

In the remainder of this paper, the organisation of the pick wave is not considered, despite the influence picking can have on the replenishment operation. Indeed, the sequence in which customer orders are picked may impact the timing of a stockout, but it is not relevant for our objective. We consider the planning of the picking operation to be given and assume:

A 5 *The customer orders to be picked in the upcoming pick wave(s) are known at the beginning of the planning horizon.*

We know which products are requested as well as the total quantity requested of each product by the customer pick orders in the upcoming pick wave(s). The number of pick wave(s) for which this information is known, depends on the length of the planning horizon.

A 6 *Customer order fulfilment in the reserve area is not considered, unless stockouts occur and emergency picks need to be performed in the reserve area.*

Details on the emergency pick operation are elaborated in section 3.2.3.

A 7 *All customer orders of pick wave t are fulfilled by the end of pick wave t.*

Incomplete orders cannot be transferred to the next pick wave, but are fulfilled by an emergency pick.

A 8 *Replenishments are organised in bins. Each storage location in the forward area is able to fit one bin.*

3.2.3 Emergency picks

In case a stockout occurs, emergency picks take place during the pick wave to meet **assumption 7**. We deliberately choose the term emergency pick over emergency replenishment. With an emergency replenishment, the product that experienced a stockout is restocked in the forward area and the replenishment quantity can take any value of the interval [the quantity necessary to complete order fulfilment; the quantity to reach the inventory level associated with the storage capacity of the product]. An emergency pick only restocks the items actually missing for order fulfilment, although the term 're-stock' might be confusing as the items are not stored in the forward area but brought directly to the picking depot. This way, pickers do not have to make a detour but go to the picking depot, which they have to do anyway to close their pick tour.

Knowing the number of items that are missing of a particular SKU, an emergency order is sent to the reserve area, where the right amount of bins is retrieved from the storage system. It is likely that the number of missing items is not equivalent to a full bin. Imagine an SKU of which 10 items fit into one bin. The SKU experiences a stockout during a pick wave of size 22. An emergency request is sent, and three bins (30 items) of the respective SKU are retrieved from the reserve area. With 22 items sent to the picking depot for order completion, 8 items remain in the reserve area in a broken bin. A broken bin can be handled in two ways:

1. Schedule the bin as a replenishment and store it in the forward area in the next replenishment wave. If so, we make the following assumption:

   A 9 *A broken bin is treated as a full replenishment bin.*

   Broken or full, each bin takes the same amount of space and requires the same amount of time for storage.

2. Keep the broken bin in the reserve area, and use the items in the bin to execute emergency picks (if necessary) in the next pick wave. If this is the case, we do not allow the respective SKU to be replenished in the next replenishment wave. I.e.,

   A 10 *if there is a broken bin of an SKU after a previous emergency pick, replenishments for this particular SKU can be planned only if the broken bin is scheduled for replenishment in the next replenishment wave as well.*

	Inventory level begin RW, in FA	**Items in broken bin** begin RW, in RA	**Advance replenishment**	**Demand**	**Stockout** Yes\no (nb items missing)	**Inventory level** end PW, in FA	**Items in broken bin** end PW, in RA
1. Feasible	0	0	1 bin: 10 items	9	No	1	0
2. Feasible	0	8	1 bin: 8 items	9	Yes (1)	0	1 bin: 9 items
3. Infeasible	0	8	1 bin: 10 items $\leftrightarrow$ assumption 10	9	No	1	1 bin: 8 items
4. Feasible	0	8	0 bins	9	Yes (9)	0	1 bin: 9 items
5. Infeasible	0	8	0 bins	9	Yes (9)	0	2 bins: (1) 8 items (2) 1 item $\leftrightarrow$ assumption 11

Table 2: Illustration of 5 scenarios with a feasible or infeasible solution with respect to the treatment of a broken bin. The example is worked out for an SKU of which 10 items fit into one bin. The inventory level in the forward area at the end of the pick wave (PW) (column 7) is determined by the inventory level at the beginning of the replenishment wave (RW), advance replenishments (column 4) and demand (column 5). The replenishment quantity depends on the existence of a broken bin in the reserve area (RA) (column 3). Quantities are expressed in items unless mentioned otherwise.

This assumption is imposed such that the amount of less-than-full bins circulating in the reserve area is minimized. For the same reason, we assume

A 11 *in case of an emergency pick, the broken bin is used first.*

Different scenarios arise, which we illustrate by means of an example in table 2. We distinguish 5 situations, all with an inventory of 0 items in the forward area to start with and a demand of 9 items in the upcoming pick wave. A stockout is unavoidable unless a replenishment is performed (one full replenishment bin = 10 items). It is the model that decides which quantity to replenish (column 4). The third and fifth scenario lead to infeasible solutions as they strike with **assumption 10** and **assumption 11**.

3.2.4 Layout and storage assignment in the forward area

Notwithstanding the findings of Thomas and Meller (2014), we keep the determination of an optimal forward area shape and storage assignment out of scope, and consider these decisions as predetermined. The only assumption we take:

A 12 *A non-scattered, dedicated storage policy is adopted to assign SKUs to storage locations.*

As such, the model is not supposed to deal with the question 'which location to replenish?', but rather 'which SKU to replenish and in which quantity?'. As such, the index i, is used to refer to SKU i, as well as to its storage location. When multiple locations are allocated to SKU i, locations are chosen such that no or minimal distance is travelled when moving between them (*non-scattered*). The location center of SKU i is then determined by taking the average of x- and y-coordinates of all locations where the SKU is stored, and is used to determine the travel distance to other SKUs.

4 Mathematical model

In this section, we describe the mathematical model that defines the IWRP as set out in section 3. The indices, sets and decision variables are defined in table 3. All parameters are detailed in table 4. In the following sections, we will guide the reader through all constraints. Note that the model is set out in an if-then formulation, and with non-linear expressions (e.g., absolute value and ceiling operator). The linear mathematical model implemented in a standard MIP-solver, can be found in Appendix A.

4.1 Objective function

The objective function minimizes the number of SKUs that experience a stockout in the pick wave(s) $t \in T$:

$$Min \sum_{i \in V} \sum_{t \in T} Z_{it} \tag{1}$$

with the binary variable Z_{it} defined:

$$Z_{it} = \begin{cases} 1, & \text{SKU } i \text{ faces a stockout in pick wave } t \\ 0, & \text{otherwise} \end{cases} \qquad \forall i \in \mathcal{V}, t \in T$$

The objective function does not take into account the size of the stockout, nor does it make a distinction among products; all stockouts contribute equally to the objective function.

Indices		
i		SKUs, location centers
r		Replenishers
k		Batches
t		Waves
		Depending on the variable, t refers to replenishment wave t or pick wave t
Sets		
$\mathcal{V}$		Set of SKUs i ($\#\mathcal{V} = N$)
V_0		Set containing the replenishment depot v_0
$\mathcal{R}$		Set of replenishers r ($\#\mathcal{R} = R$)
$\mathcal{K}$		Set of batches k ($\#\mathcal{K} = K$)
$\mathcal{T}$		Set of waves t ($\#\mathcal{T} = T$)
E		Set of edges connecting two nodes i and j (= storage locations to visit)
Decision variables - Inventory problem		
Z_{it}	$\in \{0,1\}$	= 1 if SKU i faces a stockout in pick wave t
I_{it}	$\in \mathbb{N}$	Inventory level of SKU i at the end of pick wave t, expressed in items
$Emerg_{it}$	$\in \mathbb{N}$	The size of an emergency pick for SKU i in pick wave t, expressed in items
$AdvItem_{ikrt}$	$\in \mathbb{N}$	Number of items replenished of SKU i in batch k, executed by replenisher r, in replenishment wave t
$AdvBin_{ikrt}$	$\in \mathbb{N}$	Number of full bins replenished of SKU i in batch k, executed by replenisher r, in replenishment wave t
$BrokenBin_{ikrt}$	$\in \{0,1\}$	= 1 if the broken bin of SKU i available at the end of pick wave t-1, is stored in the forward area by batch k, executed by replenisher r in replenishment wave t
$BrokenItem_{ikrt}$	$\in \mathbb{N}$	Number of items of SKU i in the broken bin available at the end of pick wave t-1, stored in the forward area by batch k, executed by replenisher r in replenishment wave t
$Broken_{it}$	$\in \{0,1\}$	= 1 if there is a broken bin of SKU i at the end of pick wave t-1
$BrokenQ_{it}$	$\in \mathbb{N}$	Number of items of SKU i in the broken bin at the end of pick wave t-1
$ReserveBin_{it}$	$\in \mathbb{N}$	Number of full bins of SKU i retrieved from the reserve area to perform an emergency pick in pick wave t
$Repl_{it}$	$\in \{0,1\}$	= 0 if SKU i cannot be replenished in replenishment wave t
Decision variables - Routing problem		
x_{ijkrt}	$\in \{0,1\}$	= 1 if storage location center i is visited before storage location center j by batch k, executed by replenisher r, in replenishment wave t
$Route_{ikrt}$	$\in \{0,1\}$	= 1 if storage location center i is visited by batch k, executed by replenisher r in replenishment wave t

Table 3: Decision variables for the IWRP.

Parameters	
General	
N	Number of SKUs stored in the forward area
T	The number of waves for which the IWRP is solved simultaneously
Demand	
D_{it}	Total number of items requested of SKU i in pick wave t
Inventory	
I_{i0}	The initial stock level of SKU i, available at the beginning of replenishment wave 1
$Broken_{i0}$	= 1 if a broken bin of SKU i exists in the reserve area, at the beginning of replenishment wave 1
$BrokenQ_{i0}$	The number of items of SKU i available in the broken bin, at the beginning of replenishment wave 1
SKU specifics	
b_i	Number of items of SKU i that fits into one bin
b_{max}	The largest number of items carried in one bin, over all SKUs
s_i	The replenishment level for SKU i associated with the chosen inventory policy
C_i	Storage capacity of SKU i associated with the chosen forward area configuration
Batch	
Q_k	The capacity of batch k, expressed by number of bins
Distances - routing	
c_{ij}	Shortest Manhattan distance between storage location center of SKU i and storage location center of SKU j
T_{travel}	Time required to travel 1 meter (seconds/meter)
T_{store}	Time required to replenish one bin in a storage location (seconds/bin)
T_{max}	Maximum time to replenish per replenishment wave, equal to the time between two pick waves
M	A large positive number

Table 4: Parameters for the IWRP.

4.2 Stock evolution and stockouts

The stock evolution of an SKU is presented in eq. (2), where the final inventory of pick wave t is determined by the inventory level at the end of the previous pick wave ($I_{i,t-1}$), replenishments performed ($AdvItem_{ikrt}$) over all batches and replenishers in replenishment wave t, and the demand of pick wave t. The variable $Emerg_{it}$, representing the size of an emergency pick of SKU i in pick wave t, is added to eq. (2) to avoid a negative stock level in case of a stock out.

$$I_{it} = I_{i,t-1} + \sum_{k \in \mathcal{K}} \sum_{r \in \mathcal{R}} AdvItem_{ikrt} - D_{it} + Emerg_{it} \qquad \forall i \in \mathcal{V}, t \in T \quad (2)$$

No stock out is recorded if the final inventory of the previous pick wave and the performed replenishments suffice to meet the upcoming demand. Z_{it} will automatically take the value zero, given the objective, and $Emerg_{it}$ is set to zero as no emergency picks have to be performed eq. (3).

$$Z_{it} = 0 \implies Emerg_{it} = 0 \qquad \forall i \in \mathcal{V}, t \in T \quad (3)$$

If the pick wave's demand exceeds the inventory level post replenishments, a stock out is recorded (eq. (4)). $Emerg_{it}$ is forced to equal the number of items that is missing of SKU i (eq. (5)) to ensure all customer orders of pick wave t are fulfilled by the end of the pick wave (a 7). Setting $Emerg_{it}$ equal to the absolute difference of the inventory level post replenishment and the pick wave's demand is crucial as otherwise $Emerg_{it}$ might take a larger value and as such (a) result in a larger inventory level for the next replenishment wave without the performance of an actual replenishment (the items are brought to the replenishment depot, not to the storage location of the product), and/or (b) give the opportunity to optimize the content of broken bins which is not a matter to be optimized. Due to eq. (5), I_{it} equals zero in case of a stockout.

$$I_{i,t-1} + \sum_{k \in \mathcal{K}} \sum_{r \in \mathcal{R}} AdvItem_{ikrt} - D_{it} < 0 \implies Z_{it} = 1 \qquad \forall i \in \mathcal{V}, t \in T \quad (4)$$

$$Z_{it} = 1 \implies Emerg_{it} = |I_{i,t-1} + \sum_{k \in \mathcal{K}} \sum_{r \in \mathcal{R}} AdvItem_{ikrt} - D_{it}| \qquad \forall i \in \mathcal{V}, t \in T \quad (5)$$

4.3 Broken bin

In section 3.2.3, we highlighted the possible circulation of broken bins in the reserve area, indicated by the binary variable $Broken_{i,t-1}$. To correctly determine the number of items in such a broken bin at the end of a pick wave, the following questions need to be answered:

1. What happens with the broken bin if one is available in the reserve area at the beginning of the replenishment wave? Is the bin stored in the forward area in the current replenishment wave, or is it kept in the reserve area and used for potential emergency picks? If such a broken bin is available, both options can be considered (eq. (6)).

   $$Broken_{i,t-1} = 1 \implies \sum_{k \in \mathcal{K}} \sum_{r \in \mathcal{R}} BrokenBin_{ikrt} = 0, or = 1 \qquad \forall i \in \mathcal{V}, t \in T \quad (6)$$

   The storage of a broken bin in the forward area cannot be distributed over multiple batches:

   $$BrokenBin_{ikrt} = 1 \implies BrokenItem_{ikrt} = BrokenQ_{i,t-1} \qquad \forall i \in \mathcal{V}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R}, t \in T \quad (7)$$

2. Do we need to perform emergency picks in the next pick wave?
   In case of an emergency pick, we use the items of SKU i in the broken bin, $BrokenQ_{i,t-1}$, first (a 11). If the broken bin not suffices to complete all pick orders containing SKU i, additional full bins will be retrieved from the storage system in the reserve area. The number of additional bins, $ReserveBin_{it}$, is determined by eq. (8). The ceiling function ensures an integer result.

   $$ReserveBin_{it} = \lceil \frac{Emerg_{it} - (BrokenQ_{i,t-1} - \sum_{k \in \mathcal{K}} \sum_{r \in \mathcal{R}} BrokenItem_{ikrt})}{b_i} \rceil \qquad \forall i \in \mathcal{V}, t \in T \quad (8)$$

Previous equations allow us to determine the number of items in a broken bin at the end of the pick wave t:

$$BrokenQ_{it} = BrokenQ_{i,t-1} - \sum_{k\in\mathcal{K}}\sum_{r\in\mathcal{R}} BrokenItem_{ikrt} - Emerg_{it} + b_i \cdot ReserveBin_{it} \qquad \forall i \in \mathcal{V}, t \in T \quad (9)$$

For a non-zero value of $BrokenQ_{it}$, the variable $Broken_{it}$ is set to 1 (eq. (10)); otherwise, $Broken_{it}$ equals 0 (eq. (11)).

$$BrokenQ_{it} > 0 \implies Broken_{it} = 1 \qquad \forall i \in \mathcal{V}, t \in T \quad (10)$$

$$BrokenQ_{it} = 0 \implies Broken_{it} = 0 \qquad \forall i \in \mathcal{V}, t \in T \quad (11)$$

4.4 Inventory replenishment level & replenishment quantity

SKUs are considered for replenishment only if their inventory level is strictly less than a predetermined value, s_i, dependent on the chosen inventory policy. Otherwise, the SKU is automatically ignored for replenishment in the current replenishment wave, as stated by eq. (12).

$$I_{i,t-1} - s_i \geq 0 \implies Repl_{it} = 0 \qquad \forall i \in \mathcal{V}, t \in T \quad (12)$$

By adopting different inventory policies, we study the impact of the SKU replenishment level on the number of SKU stock-outs.

The total number of items replenished in batch k by replenisher r ($AdvItem_{ikrt}$), is given by eq. (13) where component one refers to the replenishment of the broken bin, and component two refers to the replenishment of full bins.

$$AdvItem_{ikrt} = BrokenItem_{ikrt} + AdvBin_{ikrt} \cdot b_i \qquad \forall i \in \mathcal{V}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R}, t \in T \quad (13)$$

Full bins of a particular SKU can only be replenished to the forward area if no broken bin circulates in the reserve area, that is if (1) no broken bin exists after the previous pick wave (eq. (14)), or (2) if the broken bin is replenished in the forward area in the current replenishment wave (eq. (15)).

$$Broken_{i,t-1} = 0 \implies AdvBin_{ikrt} \geq 0 \qquad \forall i \in \mathcal{V}, \forall r \in \mathcal{R}, \forall k \in \mathcal{K}, t \in T \quad (14)$$

$$\sum_{k\in\mathcal{K}}\sum_{r\in\mathcal{R}} BrokenBin_{ikrt} = 1 \implies AdvBin_{ikrt} \geq 0 \qquad \forall i \in \mathcal{V}, \forall r \in \mathcal{R}, \forall k \in \mathcal{K}, t \in T \quad (15)$$

At no point in time it is allowed to store more items of SKU i in the forward area than the predetermined storage capacity. The maximum number of bins (full or broken) that can be replenished is determined by eq. (16). The flooring function ensures a correct transformation from available storage capacity originally expressed in items, to an expression in bins.

$$\sum_{k\in\mathcal{K}}\sum_{r\in\mathcal{R}} AdvBin_{ikrt} + BrokenBin_{ikrt} \leq \left\lfloor \frac{C_i - I_{i,t-1}}{b_i} \right\rfloor \qquad \forall i \in \mathcal{V}, \forall k \in K, \forall r \in \mathcal{R}, t \in T \quad (16)$$

For each replenishment of SKU i performed in replenishment wave t by replenisher r in batch k, a visit to SKU i needs to be scheduled, represented by the binary variable $Route_{ikrt}$ (eq. (17)). Visiting locations empty handed, i.e., without performing a replenishment, is not allowed (eq. (18)). We argument that such travel will not impact the objective function, but it is considered an unnecessary activity and it skews the replenishment effort actually required.

$$AdvBin_{ikrt} + BrokenBin_{ikrt} > 0 \implies Route_{ikrt} = 1 \qquad \forall i \in \mathcal{V}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R}, t \in T \quad (17)$$

$$AdvBin_{ikrt} + BrokenBin_{ikrt} = 0 \implies Route_{ikrt} = 0 \qquad \forall i \in \mathcal{V}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R}, t \in T \quad (18)$$

Equation (19) sets $Route_{ikrt}$ to zero if an SKU is not allowed to be replenished, i.e., the predetermined inventory criterion has not been met (see eq. (12)).

$$Repl_{it} = 0 \implies \sum_{k\in\mathcal{K}}\sum_{r\in\mathcal{R}} Route_{ikrt} = 0 \qquad \forall i \in \mathcal{V}, t \in T \quad (19)$$

The number of (less-than-)full replenishment bins assigned to a batch cannot exceed the capacity of that batch. We assume all batches to be homogeneous with a capacity Q_k, expressed in bins:

$$\sum_{i \in V} (AdvBin_{ikrt} + BrokenBin_{ikrt}) \leq Q_k \qquad \forall k \in \mathcal{K}, \forall r \in \mathcal{R}, t \in T \quad (20)$$

4.5 Routing

Each batch visiting at least one SKU for replenishment, starts its tour at the replenishment depot:

$$\sum_{i \in V} Route_{ikrt} > 0 \implies Route_{0krt} = 1 \qquad \forall k \in \mathcal{K}, \forall r \in \mathcal{R}, t \in T \quad (21)$$

Equation (22) forces that the visit of SKU i replenished in batch k, executed by replenisher r in replenishment wave t, is preceded and succeeded by the visit of another SKU j replenished in the same batch k. The equation also holds for the replenishment depot. As such, batches visiting at least one SKU, end the tour at the replenishment depot.

$$Route_{ikrt} = \sum_{j \in V \cup V_0, j \neq i} x_{ijkrt} = \sum_{j \in V \cup V_0, j \neq i} x_{jikrt} \qquad \forall i \in \mathcal{V} \bigcup V_0, \forall k \in \mathcal{K}, \forall r \in \mathcal{R}, t \in T \quad (22)$$

Each replenisher has T_{max} [time unit] to perform all replenishment batches assigned to him. The execution of a replenishment batch, includes

- Travelling between storage location centres, at a cost of T_{travel} [time unit] per [length unit].
- Storing the replenishment bin in the dedicated storage location, at a cost of T_{store} [time unit] per bin.

The total time required for each replenisher to perform replenishments is expressed by the left-hand side of eq. (23) and may not exceed T_{max}.

$$\sum_{k \in \mathcal{K}} (T_{travel} \cdot (\sum_{i \in V \bigcup V_0} \sum_{j \in V \bigcup V_0} c_{ij} \cdot x_{ijkrt}) + T_{store} \cdot \sum_{i \in V} (AdvBin_{ikrt} + BrokenBin_{ikrt})) \leq T_{max}$$
$$\forall r \in \mathcal{R}, t \in T \quad (23)$$

Subtour elimination constraints are added to the model by means of lazy constraints, i.e., constraints added to the model only when they are violated by the current solution (Aguayo et al., 2018). The lazy constraint callback function is called to check for any subtours in the incumbent solution. A subtour is identified when a tour is associated with multiple lengths, the lengths of the various subtours. In that case, the smallest subtour which does not contain the depot, is selected. S is defined as a subset of V that contains the locations visited by this particular subtour. The following constraints are added to the model (based on the example available at https://www.gurobi.com/documentation/9.5/examples/tsp_cpp_cpp.html):

$$\sum_{i \in S} \sum_{j \notin S} x_{ijkrt} \geq Route_{hkrt}, \qquad \forall h \in S, \forall k \in \mathcal{K}, \forall r \in \mathcal{R} \quad (24)$$

with h being an SKU/storage location center included in subset S, and t to be known as the subtour was identified for this particular replenishment wave. Note that subtour elimination constraints are added each time a new subtour has been identified, and are complementary to lazy constraints already added [1].

4.6 Boundaries

Equations (25) to (29) pose an upper bound on the variables.

$$I_{it} \leq C_i \qquad \forall i \in \mathcal{V}, t \in T \quad (25)$$
$$AdvItem_{ikrt}, AdvBin_{ikrt} \leq C_i \qquad \forall i \in \mathcal{V}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R}, t \in T \quad (26)$$
$$BrokenQ_{it} \leq b_{max} \qquad \forall i \in \mathcal{V}, t \in T \quad (27)$$
$$BrokenItem_{ikrt} \leq b_{max} \qquad \forall i \in \mathcal{V}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R}, t \in T \quad (28)$$
$$Emerg_{it}, ReserveBin_{it} \leq D_{it} \qquad \forall i \in \mathcal{V}, t \in T \quad (29)$$

[1] The website of Gurobi highlights that node solutions should comply with lazy constraints added previously, although this is not always the case. No further information was provided (Gurobi, 2021).

In theory, the number of batches per replenisher is infinite. In reality, only a limited number of batches can be performed due to the restricted replenishment time. Knowing the time to store one replenishment bin and the distance from the replenishment depot (i=0) to the storage location closest to the depot (j=close), K takes the value:

$$K = \lfloor \frac{T_{max}}{c_{0,close} T_{travel} + T_{store}} \rfloor \tag{30}$$

We highlight that for the model just described, a feasible solution always exists as long as the initial inventory does not exceed the storage capacity.

5 Computational study

5.1 Experimental setup

In this paper, the proposed IWRP-model is solved to optimality using the Gurobi software (version 7.0.2) with an optimality gap set to 0%. For each IWRP-instance, the replenishments for a number of consecutive replenishment waves are planned. The instances, detailed in section 5.2.2, are solved 12 consecutive times with the final inventory of pick wave t used as initial inventory of replenishment wave $t + 1$. We also say that one instance contains 12 sub-problems (illustrated in fig. 3). Of these 12 replenishment waves, the first 6 are used to reach a steady state and to create a realistic inventory situation for replenishment wave 7 to start with. For this reason, the results of the first 6 replenishment waves are excluded from the analysis.

In the experiment, different values for the planning horizon, T, are tested. The planning horizon indicates the number of replenishment waves T that are to be solved simultaneously, and requires the demand of the upcoming T pick waves to be known. Replenishment waves 1-6 are scheduled one replenishment wave at a time, T=1 (fig. 3a). Replenishment waves 7 to 12 are either solved assuming $T = 1$ (fig. 3b and fig. 3d) or $T = 3$ (fig. 3c). After planning T replenishment waves, the planning horizon moves forward one pick wave at a time. To illustrate, if T=1, we consecutively solve replenishment wave 7, 8, ..., and 12. When T=3, we consecutively solve replenishments waves 7-8-9, 8-9-10,..., and 12-13-14 (illustrated in fig. 3c).

For each instance, the demand between pick waves differs ($D_{it} \neq D_{i,t+1}$), although not significantly as the demand of each pick wave follows the same distribution. Additionally, replenishment waves 7 to 12 are solved three times, each time with a different demand to add variety to the experiment. We refer to the different demand sets as D (fig. 3a, 3b, 3c), D' (fig. 3d) and so on.

The maximum computation time for each sub-problem, independent of the planning horizon or any other parameter value, is set to 3600 seconds. If no solution is found within this time, no input is generated for the next sub-problem, and the model moves on to the next instance. Only if all 12 sub-problems are solved, the instance is indicated as "solved".

All instances were solved on an Intel(R) Core(TM) i7-4790 CPU, 3.60GHz and 16 GB of RAM.

5.2 Instance generation

The mathematical model is solved for a random dataset. To create the instances, characteristics such as the storage capacity of SKUs, layout dimensions and product assignment in the forward area are considered to be known. Therefore, precomputation is required, discussed in section 5.2.1. The characteristics of the IWRP-instances are discussed in section 5.2.2.

5.2.1 Precomputation

To obtain the average demand per pick wave, crucial to configure the forward area, random picking orders are generated. We do this consecutively for 50 and 100 SKUs. Picking orders are generated assuming a B2C environment, where typically orders contain 1.6 items on average (Weidinger et al., 2019). We follow this guideline for 75% of orders. The remaining orders contain on average 3 items per order to take into account slightly larger orders. The demand distribution follows the Pareto principle, where 20% of products account for 80% of total demand (Weidinger and Boysen, 2018). The detailed distribution can be found in fig. 4a, as well as other data considered known before precomputation. According to these guidelines, 150

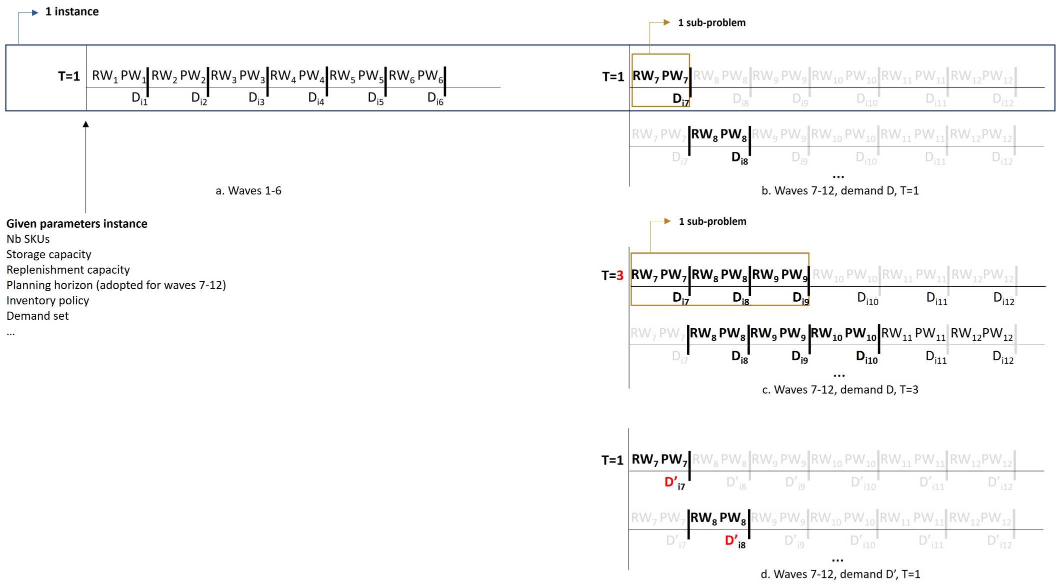


Figure 3: Visualisation of the experimental setup.

orders per pick wave are generated, 10.000 pick waves in total. The average demand per pick wave and its standard deviation are then used to configure the forward area, with the primary question '*how much locations are allocated to each SKU?*'.

In this paper, we do not solve the above question to optimality, but we are aware that the available storage capacity can influence the objective significantly. To determine the storage capacity for each SKU, we return to the basics of inventory management and manage the inventory in the forward area by means of a *periodic review policy*. With the periodic review policy, the inventory level of products is checked after a fixed interval of duration *r*. After each interval, an internal replenishment order is submitted such that the inventory position increases to the inventory level S, also known as the *base-stock level* (BSL). In some settings, as will be the case for the IWRP, an order is submitted only if the current inventory level falls below a predetermined inventory replenishment level *s*. The periodic review policy is therefore also known as the *(s,S)-policy* (Arrow et al., 1951). We note that due to capacity restrictions, replenishing up to the base-stock level (S) will not always be feasible in our model.

The base-stock level is determined using the average demand during period r and replenishment lead time L, as well as a safety stock component to cover demand uncertainty during time $r + L$, with a service level α in mind. The base-stock level (S) formula is defined as follows:

$$S = (r + L) \cdot \mu_D + SS, \tag{31}$$

with μ_D representing the average demand of an SKU during a time period r, in our case, pick wave. The safety stock level, SS, is determined as follows:

$$SS = z \cdot \sqrt{\mu_{r+L}\sigma_D^2 + \mu_D^2\sigma_{r+L}^2}, \tag{32}$$

with σ_D representing the standard deviation of the demand during a pick wave, and μ_{r+L} and σ_{r+L} representing the average period + lead time, standard deviation of the period + lead time respectively. The safety factor, z, is associated with the *cycle service level* or *P1-service level*, α, indicating '*the probability of not having a stockout in a replenishment cycle*' (Cardós et al., 2006). The desired service level is specified for each SKU.

In what we define as the initial forward area configuration, abbreviated *LI*, the storage capacity allocated to SKU i is determined based on the above theory. The number of locations allocated to SKU i, is determined as follows:

$$NbLocations_i^{LI} = \max(2, \lceil \frac{r \cdot \mu_{D_i} + z_i \cdot \sigma_{D_i} \cdot \sqrt{r}}{b_i} \rceil) \tag{33}$$

			Storage allocation policy		
			Initial	W	E
FA size		*Number of storage locations*			
	Large	$NbLocations^{LI}$	**LI**	**LW**	**LE**
	Medium	80% of $NbLocations^{LI}$	/	**MW**	**ME**
	Small	60% of $NbLocations^{LI}$	/	**SW**	**SE**

Table 5: Possible forward area configurations included in the experimental study. Each configuration is characterised by a forward area size (expressed in a total number of storage locations), and an allocation policy to determine how many storage locations are allocated to each SKU.

In eq. (33) the base-stock level formula can be recognized although some adaptations were enforced:

- As picking and replenishing do not take place simultaneously, the lead time, L, is set to 0, and therefore excluded from the formula. r is set to the duration of 1 pick wave.
- The formulas eq. (31) and eq. (32) are expressed in items, while the forward area in our experiments is organised in bins. The original base-stock level formula is divided by the number of items in a bin b_i, and ceiled to the nearest integer to move from an item- to a bin-expression. Note that b_i differs among SKUs (see fig. 4a). Sizing is pre-determined and follows a uniform distribution.
- Similar to Gagliardi et al. (2008), we impose that each SKU stored in the forward area receives at least two locations. With only one location, which would be the case for low demanded products, replenishment is only feasible when the stock level of the product equals zero.

The experiments in this paper are carried out assuming a service level of 95%.

Summing the number of locations obtained by eq. (33) over all SKUs N, results in a total number of locations, $NbLocations^{LI}$, which we associate with a *large* forward area; the largest forward area size we consider in our experiments. To study the impact of a smaller forward area size on the number of SKU stockouts, we define a medium and small forward area, where the total number of storage locations is equal to 80%, respectively 60% of the total number of locations allocated in a large forward area. With a reduction in the forward area size, an alternative allocation technique is required to distribute the available storage locations over the N SKUs; the selection of SKUs to be stored in the forward area is not re-assessed. We test two allocation policies for the medium and small forward area sizes. For completeness, we also test both policies for the large forward area size and compare results to the results obtained by the initial policy:

1. **Policy W**: allocates locations according to the algorithm of Walter et al. (2013), which takes into account the demand of the product. We set the replenishment cost c_i to one for all SKUs and use the R_4 repair heuristic, proposed by Walter et al. (2013), to obtain an integer solution.
2. **Policy E**: based on the EQS policy presented in Bartholdi III and Hackman (2008). Distributes locations evenly among the SKUs. Leftover locations are assigned randomly.

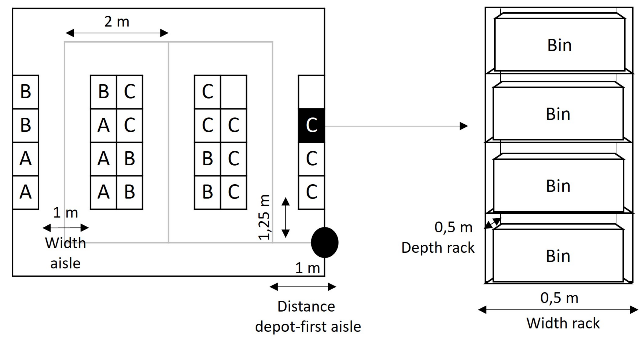
The combination of forward area sizes and allocation policies leads to seven forward area configurations, including configuration LI (summarized in table 5). Each forward area configuration results in a different storage capacity for SKU i. C_i is determined by multiplying the number of locations allocated to SKU i by the number of items in one bin b_i.

The layout of the forward area in our experiments is fixed to a parallel, one block construction, as illustrated in fig. 4b. With regards to the dimensions of the forward area layout, we consider part of the data to be fixed, such as the width and depth of the racks and the (cross-)aisles (summarized and visualised in fig. 4). The number of racks per aisle and the number of aisles, on the other hand, are determined in precomputation to fit the number of storage locations calculated earlier. As all racks have the same length, it is possible that the final number of storage locations exceeds the number of storage locations required for a particular forward area configuration. Any remaining locations will be kept empty to keep the storage allocation rules as pure as possible.

Knowing how many locations are allocated to each product, the products can be assigned to the locations. We adopt a class-based storage policy: high demand products (A products) are stored closest to the depot, while low demand products (C products) are stored in the back of the forward area. Within a class, products are randomly assigned to locations (Yu et al., 2015). Acknowledging that order picking still remains the most costly operation in warehousing (De Koster et al.,

Pick order information	
Average number of items per order	75% of orders: 1.6 items, 25% of orders: 3 items
Number of items per bin (b_i)	5, 10 or 20
Demand distribution	A products: 5% of SKUs, 65% of total demand
	B products: 15% of SKUs, 15% of total demand
	C products: 80% of SKUs, 20% of total demand
	Based on the 20%/80% demand curve presented by Guo et al. (2016)
Forward area layout information	
Width rack	0.5 m
Depth rack	0.5 m
Height rack	Not integrated in calculations
Storage locations per rack	4
Width aisle	1 m
Width cross aisle	2 m
Distance depot to first aisle	1 m

(a) Pick order and forward area layout information.



(b) Visualisation of the forward area and rack dimensions.

Figure 4: Data considered given and fixed for precomputation.

2007), we apply the class-based storage policy from the pickers' point of view, i.e., the most interesting locations are those located closest to the picking depot. An example is worked out in fig. 4b.

Precomputation delivers the following output:

- The storage capacity, C_i, for each SKU, adapted to the chosen forward area configuration.
- A distance matrix, which includes the shortest travel distance in the forward area for each pair of SKUs, c_{ij}. We assume racks on both sides of the aisle to be accessible without additional movements (the replenisher is positioned in the center of the aisle). Vertical movements are also ignored.

A summary of the statistics related to the demand generated in precomputation, is given in table 6.

	50 SKUs			100 SKUs		
	A	**B**	**C**	**A**	**B**	**C**
Average demand per product, per pick wave	99.74	6.49	1.56	39.85	3.03	0.8
Total average demand per pick wave	199.48	45.4	63.77	199.27	45.39	63.99
Average standard deviation on the demand per product, per pick wave	8.46	2.42	1.25	5.96	1.7	0.89
Average base-stock level per product	114	11,14	5	50	7	3
Average safety stock level per product	14	4.14	3	10	3	2

Table 6: Statistics demand, expressed in items.

5.2.2 IWRP-instances

IWRP-instances differ in terms of: the number of SKUs, the demand set, the planning horizon, the number of replenishers, the replenishment time, the storage capacity and the inventory replenishment level. The latter depends on the chosen inventory policy, detailed in table 8. Each instance takes a different combination of the parameter values included in our experiment, summarised in table 7, which results in 7560 instances solved in total. For parameters, such as the travel and storage time, a single parameter value is considered. For all instances, we impose that the inventory level of SKU i at the beginning of replenishment wave 1 is set to 50% of its storage capacity, and that no broken bins are available in the reserve area.

5.3 Results

In this section we discuss the results in terms of the capability of instances to be solved within limited computation time, as well as the objective function, the total number of SKU stockouts. A detailed overview of results is shown in Appendix B.

Parameters	*Parameter values*
Nb SKUs (N)	50, 100
Demand set (D)	D, D”, D”’
Planning horizon (T)	1, 3 pick waves
Inventory related parameters	
Storage capacity (C_i)	
(depends on the forward area configuration)	LI, LW, MW, SW, LE, ME, SE
Inventory replenishment level (s)	
(depends on inventory policy, detailed in table 8)	BIN, SS, SSCeiled, BSL, BSLCeiled, CAP
Initial inventory level (I_{i0})	The initial inventory level of SKU i in replenishment wave 1 equals 50% of its storage capacity C_i
Initial available broken bin ($BrokenQ_{i0}$)	In replenishment wave 1, the reserve area holds no broken bins
Replenishment related parameters	
Number of replenishers (R)	1, 3, 5
Bins per replenishment batch (Q_k)	10 bins
Replenishment time (T_{max})	30, 60, 90, 120, 240 seconds
Travel time (T_{travel})	1 s/m
Storage time (T_{store})	5 s/bin

Table 7: Overview of parameter and parameter values.

Inventory policy	**Inventory replenishment level** Take the minimum of C_i and ...
BIN	one full bin + 1 item.
SS	the safety stock (eq. (32)).
SSCeiled	the safety stock level ceiled to the nearest full-bin equivalent + 1 item.
BSL	the base-stock level (eq. (31)).
BSLCeiled	the base-stock level ceiled to the nearest full-bin equivalent + 1 item.
CAP	the storage capacity of the SKU.

Table 8: Inventory policies tested.

50 SKUs							
T=1	*72.96% of instances solved, average computation time: 16.14 s*						
FA configuration	**LI**	**LW**	**LE**	**MW**	**ME**	**SW**	**SE**
	44.44%	52.59%	80.00%	75.93%	83.33%	84.07%	90.37%
Inventory policy	**Bin**	**SS**	**SSCeiled**	**BSL**	**BSLCeiled**	**Cap**	
	71.75%	73.33%	71.43%	73.65%	73.33%	74.29%	
Nb replenishers	**1**	**3**	**5**				
	78.25%	63.65%	76.98%				
Replenishment time	**30**	**60**	**90**	**120**	**240**		
	37.83%	75.40%	70.37%	88.62%	92.59%		
T=3	*50.21% of instances solved, average computation time: 106.15 s*						
FA Configuration	**LI**	**LW**	**LE**	**MW**	**ME**	**SW**	**SE**
	21.85%	25.56%	55.56%	52.96%	56.30%	65.93%	73.33%
Inventory policy	**Bin**	**SS**	**SSCeiled**	**BSL**	**BSLCeiled**	**Cap**	
	49.21%	55.24%	49.52%	53.97%	47.94%	45.40%	
Nb replenishers	**1**	**3**	**5**				
	23.17%	59.05%	68.41%				
Replenishment time	**30**	**60**	**90**	**120**	**240**		
	4.23%	41.80%	56.61%	69.31%	79.10%		

100 SKUs							
T=1	*55.50% of instances solved , average computation time: 33.36 s*						
FA configuration	**LI**	**LW**	**LE**	**MW**	**ME**	**SW**	**SE**
	39.63%	37.04%	64.81%	41.85%	68.15%	61.85%	75.19%
Inventory policy	**Bin**	**SS**	**SSCeiled**	**BSL**	**BSLCeiled**	**Cap**	
	54.92%	59.05%	55.24%	56.19%	54.29%	53.33%	
Nb replenishers	**1**	**3**	**5**				
	45.24%	51.43%	69.84%				
Replenishment time	**30**	**60**	**90**	**120**	**240**		
	33.33%	44.71%	50.26%	64.81%	84.39%		
T=3	*20.42% of instances solved , average computation time: 247.35 s*						
FA Configuration	**LI**	**LW**	**LE**	**MW**	**ME**	**SW**	**SE**
	3.33%	16.67%	18.89%	15.19%	22.59%	7.78%	58.52%
Inventory policy	**Bin**	**SS**	**SSCeiled**	**BSL**	**BSLCeiled**	**Cap**	
	14.92%	33.65%	16.19%	25.40%	15.24%	17.14%	
Nb replenishers	**1**	**3**	**5**				
	5.40%	25.87%	30.00%				
Replenishment time	**30**	**60**	**90**	**120**	**240**		
	0.00%	13.23%	23.02%	28.31%	37.57%		

Table 9: Percentage of instances solved within one hour, set out per number of SKUs and planning horizon. Every quadrant shows the average results over 3 demand sets, over all instances for a specific parameter: forward area configuration, inventory policy, number of replenishers and replenishment time.

5.3.1 Capability of instances to be solved

The complexity of the problem is expressed through the number of instances completely solved (all 12 replenishment waves) and the average computation time per sub-problem. In tables 14 to 25 of Appendix B, a slash indicates that the model was unable to solve the instance in time, i.e., one of its sub-problems was not solved within one hour. The percentage of instances that we were able to solve through Gurobi are listed in table 9. For each parameter value, the percentages are averaged over the 3 demand sets.

Of all parameters, a longer planning horizon seems to influence this percentage the most. With T set to 3, only half (for 50 SKUs) or 20% (for 100 SKUs) of the instances can be solved in the given time. Also the number of SKUs has an impact on whether or not an instance can be solved, proven by the percentages found on the right side of table 9 that are systematically lower than the ones on the left side. This illustrates the need for a heuristic procedure, especially when instances grow to a realistic number of SKUs.

A deeper look into the results of table 9, show that in general more instances are solved as we move from a large, to medium, to small forward area, and that the E-policy seems to reduce the problem complexity as opposed to the W-policy. More replenishment capacity, determined by more replenishers and/or more replenishment time, has a positive impact on whether instances can be solved or not. The lower the replenishment capacity, the harder it generally gets to solve the sub-problems to optimality, with even 0 instances solved when T_{max} =30 s (100 SKUs, T=3). Between the inventory policies differences are rather limited, although more instances are solved with the SS-policy adopted. A possible explanation is that the SS replenishment level is often lower than the Bin replenishment level. As such, the SS-policy will consider a smaller selection of SKUs for replenishment, resulting in less replenishment quantities and schedules to be determined.

The average computation time (over replenishment waves 7-12) per sub-problem is given in table 9, where we distinct between the number of SKUs and the planning horizon. Only sub-problems of instances completely solved are taken into account, for which in general we observe an acceptable average computation time. Aligned with prior observations, the average computation time increases as the number of SKUs grows and/or the planning horizon increases.

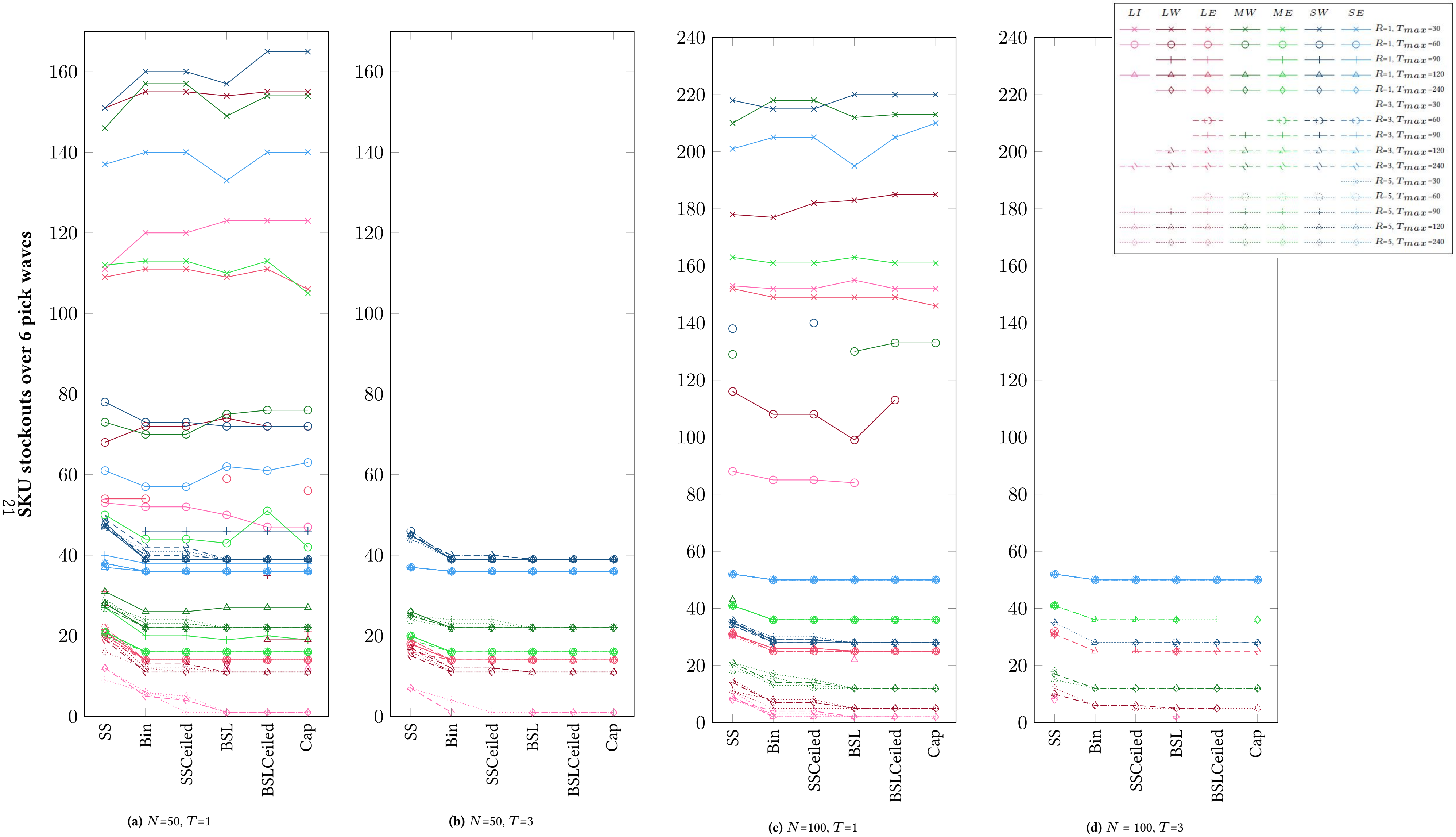
5.3.2 Stockouts

In this section, we discuss the number of SKU stockouts in terms of the different parameters. In particular, we aim to answer the following research questions:

- Does a higher replenishment level benefit the number of stockouts?
- Does a larger forward area size benefit the number of stockouts?
- Does a larger replenishment capacity benefit the number of stockouts?
- Does a longer planning horizon benefit the number of stockouts?

Inventory policy In fig. 5a and 5b (50 SKUs) and fig. 5c and 5d (100 SKUs), the total number of stockouts over 6 pick waves is set out per inventory policy (x axis), forward area configuration (color of the line graph), and replenishment capacity (pattern of the line graph). Figures a and c present the results for T=1, while figures b and d present the results for T=3. The results are shown for demand set D but can be generalised to other demand sets as well. Instances for which no outcome is registered, remained unsolved.

As fig. 5 clearly points out, an increased number of stockouts is experienced when the SS-policy is adopted. For many products, the safety stock level is exceeded by the product's average demand (see table 6). Stockouts for these SKUs are often unavoidable as they are not considered for replenishment despite the fact that their inventory level has fallen below their average demand. Other inventory policies, including the Bin-policy, generally perform better as for most products, they are associated with higher replenishment levels.



(a) N=50, T=1

(b) N=50, T=3

(c) N=100, T=1

(d) N = 100, T=3

Figure 5: Total number of stockouts over 6 pick waves, set out per inventory policy. Results are shown for 50 SKUs, demand set D.

Data	$I_{i,t-1}$	$D_{it}, D_{i,t+1}, D_{i,t+2}$	C_i	b_i				
SKU i	2	3,0,0	10 items	5 items				
SKU j	1	2,1,1	10 items	5 items				
Scenarios		**Scenario 1:** Replenish SKU i in RW t				**Scenario 2:** Replenish SKU j in RW t		
SKU i	$I_{i,t-1}$	$\sum_{k\in K}\sum_{r\in\mathcal{R}} AdvItem_{ikrt}$	D_{it}	*Stockout?*	$I_{i,t-1}$	$\sum_{k\in K}\sum_{r\in\mathcal{R}} AdvItem_{ikrt}$	D_{it}	*Stockout?*
RW/PW t	2	**5**	3	No	2	**0**	3	Yes
RW/PW $t+1$	4	0	0	No	0	0	0	No
RW/PW $t+2$	4	0	0	No	0	0	0	No
SKU j	$I_{i,t-1}$	$\sum_{k\in K}\sum_{r\in\mathcal{R}} AdvItem_{ikrt}$	D_{it}	*Stockout?*	$I_{i,t-1}$	$\sum_{k\in K}\sum_{r\in\mathcal{R}} AdvItem_{ikrt}$	D_{it}	*Stockout?*
RW/PW t	1	**0**	2	Yes	1	**5**	2	No
RW/PW $t+1$	0	0	1	Yes	4	0	1	No
RW/PW $t+2$	0	0	1	Yes	3	0	1	No
Total SKU stockouts over 3 pick waves:				3				1

Table 10: Example illustrating the impact of replenishment freedom imposed by the objective function. In both scenarios, there is only sufficient replenishment capacity for the replenishment of either SKU i or SKU j in replenishment wave (RW) t. Both scenarios have equal chances of being performed, given that the planning horizon is equal to 1 pick wave (PW).

Among the remaining inventory policies, minor to no differences arise, if we ignore the results for R=1 and T_{Max} = 30/60/90 s. Small deviations are observable for the LI, LW, MW, and SW forward area configurations. For these configurations, increased stockouts are experienced when the Bin or SSCeiled policy is adopted. Both are policies associated with lower threshold values than the BSL, BSLCeiled or Cap-policy, and consequently, consider only a limited selection of SKUs for replenishment. For forward area configurations that allocate less capacity to each product, e.g., configurations adopting the E-policy, the gap between the inventory threshold values is less pronounced, and the inventory policies tend to behave in a similar fashion.

Exceptions to the above observations arise when replenishment capacity is low (R=1, T_{max}=30/60/90 s). This behaviour could be attributed to the freedom enjoyed by the model, which is partly imposed by the objective function that treats all SKU stockouts equally. As such, the selection of SKUs to replenish is, to some extent, chosen arbitrarily. To illustrate, assume two products, SKU i and j, both requiring the same share of replenishment capacity to avoid a stockout in wave t. In table 10, we illustrate two scenarios, both with sufficient replenishment capacity to replenish only one of both SKUs in replenishment wave t. Although the choice for either SKU i or j is not decisive for pick wave t (in each scenario, the un-replenished SKU will experience a stockout), the final decision will have a major influence on the number of stockouts in future pick waves. Indeed, scenario 2 delivers a much better outcome in the end, despite the only difference being the choice of the replenished SKU in wave t. A longer planning horizon tries to tackle this problem by considering more demand information to be known up front. Arbitrariness in one specific replenishment wave, however, will always characterise the IWRP given the dynamic context of the problem. This also holds for instances with larger replenishment capacity, although with a larger capacity possibly less replenishment dilemmas arise. We argument this to be an explanation for the deviating behaviour of results obtained for instances with small replenishment capacity.

We conclude that the impact of a higher replenishment level on the number of SKU stockouts remains limited. Most significant improvements are realised as we move from the SS- to Bin-policy. Further improvements realised through a higher replenishment level are influenced by the forward area configuration, although in general, results stabilize once the BSL-policy is adopted. Because of the good solution quality and the fact that many instances are solved with the BSL-policy, we focus on the outcomes obtained with this inventory policy as we continue the discussion of the results.

Forward area configuration To illustrate the impact of the forward area configuration on the total number of stockouts, we refer to fig. 6 (50 SKUs) and fig. 7 (100 SKUs). We shift the focus to the forward area configuration represented by the pattern of the line graph, and the total replenishment capacity shown on the x-axis in increasing order. We narrowed the results to those obtained for the BSL-policy.

From fig. 6a and fig. 7a we learn that, in general, results worsen as the forward area gets smaller (LW<MW<SW, and LE<ME<SE) which is logical given that less storage capacity is allocated to the products. With regards to the allocation

policy, we find that the performance depends on the forward area size and the number of SKUs. We also note that, especially for instances of 50 SKUs, results differ between the demand sets. In general, we observe that the W-policy outperforms the E-policy in a large forward area. For a medium and small forward area, the relationship between both allocation policies is not consistent. However, all results make clear that of all forward area configurations, the LI configuration works best.

The combinatorial complexity inherent to the IWRP, makes it hard to find an explanation for each of these observations. There are, however, some findings which clarify some of the observations above:

1. Many stockouts, especially for A products, are inevitable as insufficient space is allocated. Even if locations were fully replenished, the average demand could not be met.
   The bars drawn in fig. 6c and fig. 7c show for each forward area configuration the total number of locations allocated to A, B and C products (black pattern, to be read on the left y-axis), and the capacity associated with it (grey pattern, to be read on the right y-axis). The line graphs represent the total average demand of A, B and C products. The bars and line graphs are stacked. As can be seen in fig. 6c and fig. 7c, storage capacity often fails to meet the average demand. Especially for A products, stockouts become unavoidable unless demand would deviate drastically from the average. For 50 SKUs this is the case for all configurations except LI and LW, clarifying the superior performance of the latter two. For 100 SKUs, this to true for LE, ME and SE configurations, which explains their inferior performance in fig. 7a.

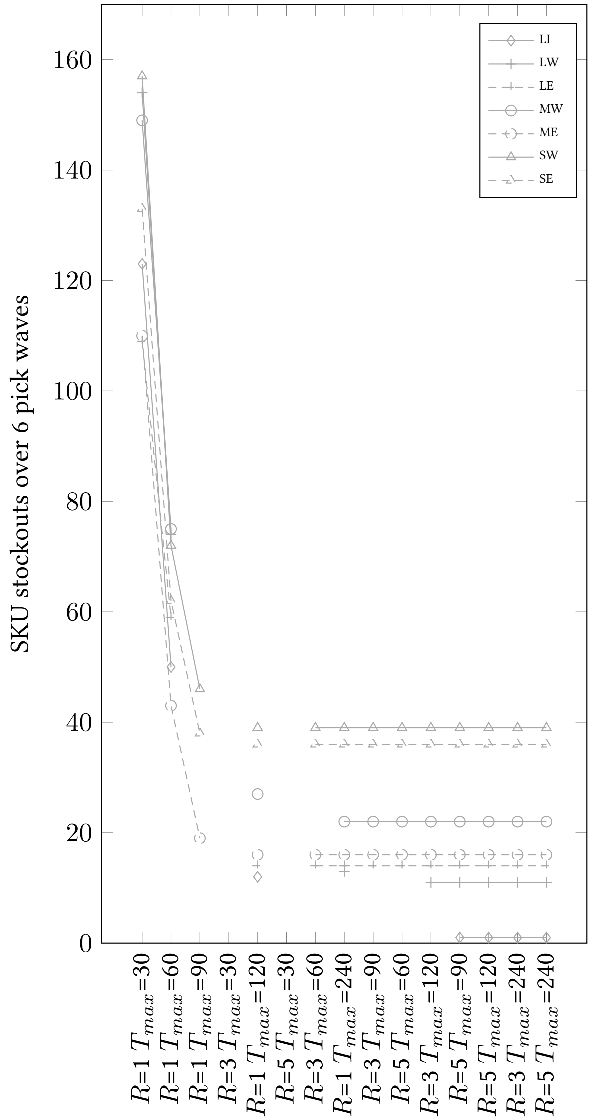
2. Having two storage locations allocated per product, is not an unnecessary luxury.
   For the initial allocation policy we stressed that each SKU should receive at least two storage locations. Due to size reductions or the adoption of other allocation policies, this guideline cannot always be met. As a result stockouts are often experienced because replenishments cannot be performed unless the SKU's inventory falls to zero. Fig. 6d and fig. 7d distinct A, B and C SKUs that received one (grey coloured) versus two or more locations (black coloured), set out over the different configurations. The figures show that the W-policy tends to allocate only one location to C-products as opposed to the initial- and E-policy, which explains the superiority of the LI configuration over the LW configuration.

3. Even with sufficient locations provided and SKUs receiving at least two locations, stockouts can happen.
   A possible explanation is that a bin always takes up one location, independent of its content (full or almost empty). For some B-products, in particular big products (b_i = 5 items), we observe that two locations suffice to meet the average demand, that is of course if both locations store full bins. If one of both locations is filled with an almost empty bin, the replenishment of the other location will not be enough to avoid a stockout.

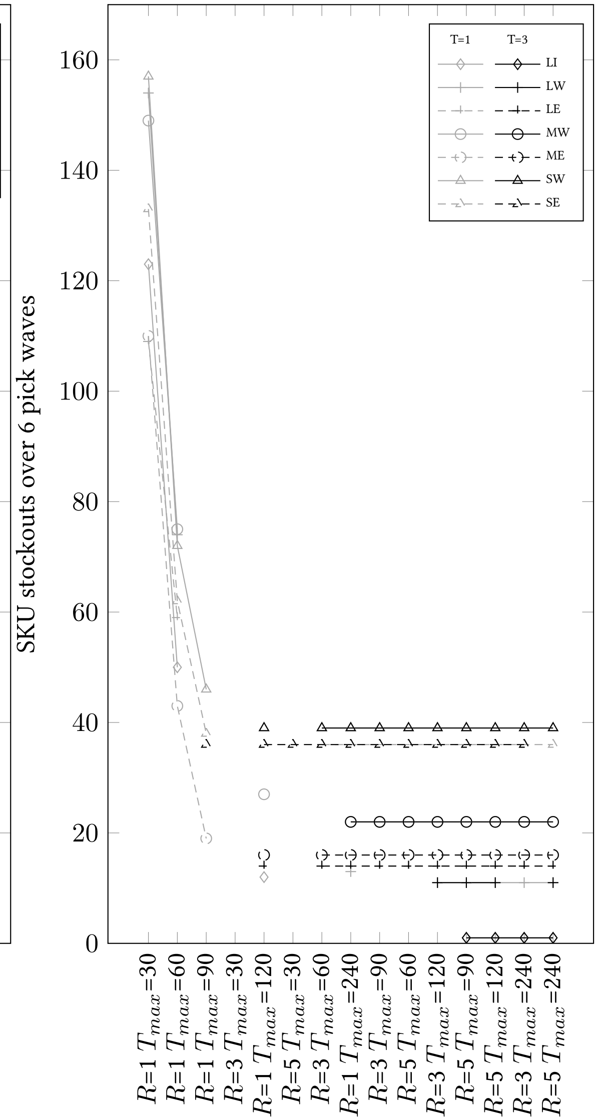
Exceptions to the observations above arise again when replenishment capacity is low (R=1, T_{max}=30/60). Part of these deviations can be attributed to the arbitrariness explained earlier. Additionally, we reason that with only little replenishment capacity available, the number of replenishment orders one can perform is limited. In these situations, having sufficient storage capacity becomes unnecessary, more so, it could be experienced as a disadvantage: larger distances possibly need to be travelled, resulting in more time dedicated to travel and less spare time for storage. (Dis)proving this hypothesis is not evident as distances between SKUs not only differ due to the forward area size, but also because of the chosen storage assignment policy that randomly assigns SKUs belonging to the same class.

We conclude that, in general, a larger forward area size positively impacts the number of SKU stockouts, although the way in which the available space is used, i.e., which allocation policy to adopt, cannot be overlooked.

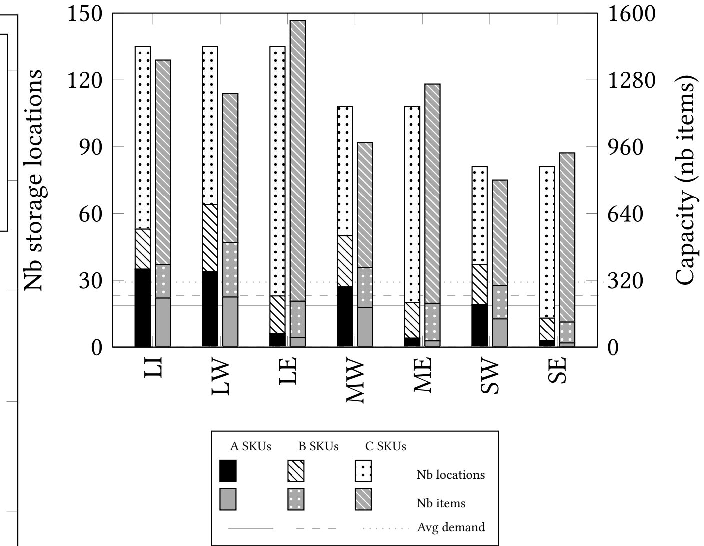
Replenishment capacity Previous observations made clear that the replenishment capacity has a significant impact on the number of stockouts. However, this only seems to be true when capacity is low, i.e., when R=1 and $T_{max} \leq 90$. From R=1 and T_{max} =120 onwards, all curves, in both fig. 6a and fig. 7a, are flat: an increase in the replenishment capacity cannot facilitate a further reduction of stockouts. The storage capacity is perceived as a bottleneck.



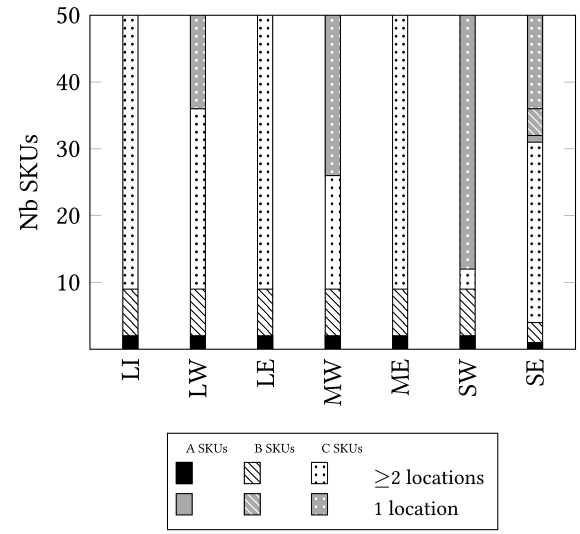
(a) Total stockouts over 6 pick waves for **T=1**



(b) Total stockouts over 6 pick waves for **T=1 and T=3**

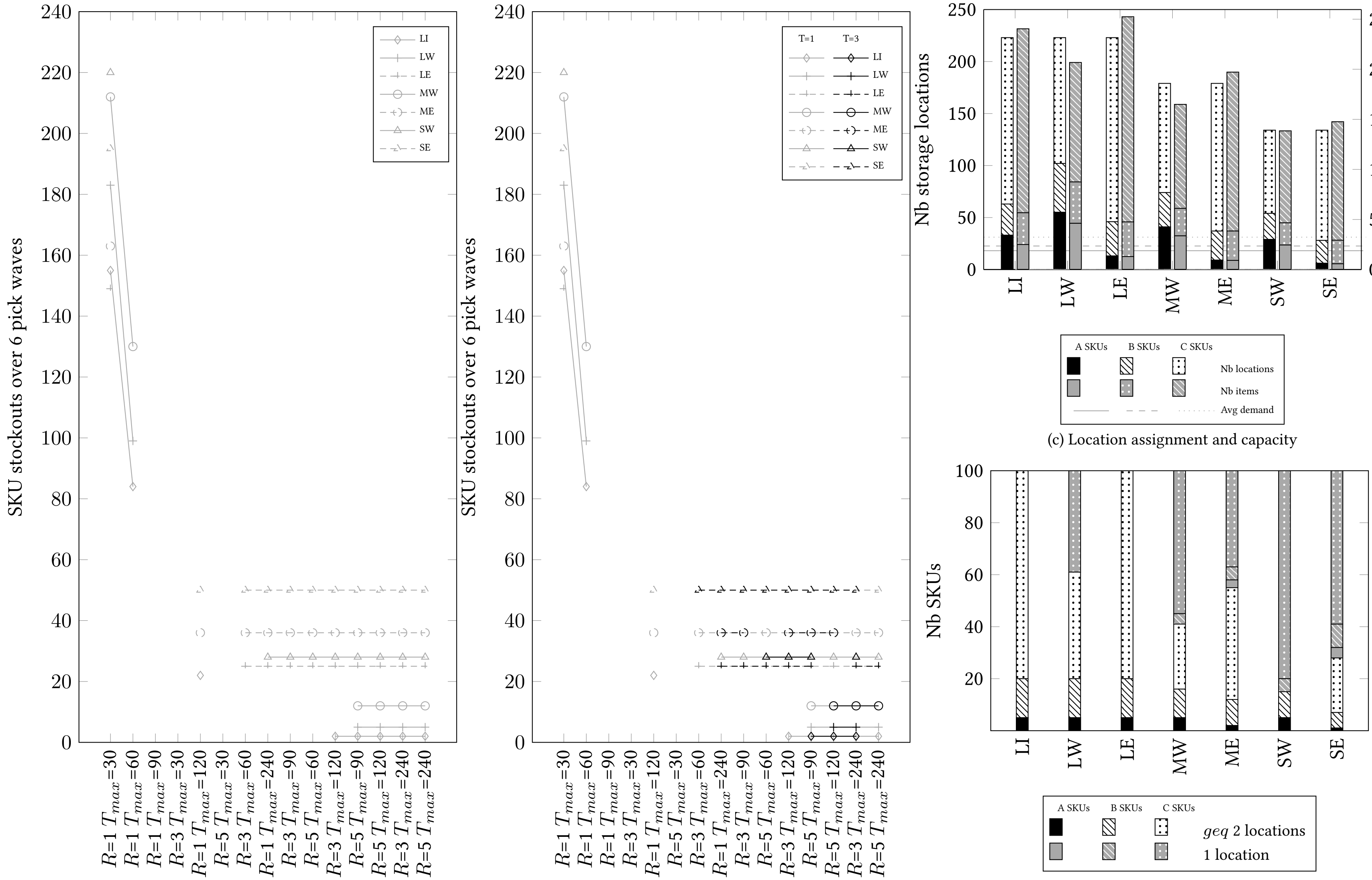


(c) Location assignment and capacity



(d) Nb SKUs with $\geq$ 2 and 1 location

Figure 6: Total number of stockouts over 6 pick waves set out over the replenishment capacity for (a) T=1 (b) T=1 & T=3. (c) Location assignment and capacity detailed per product group, for each forward area configuration. (d) Details on the location assignment detailed per product group, for each forward area configuration. Results are shown for 50 SKUs, demand set D.



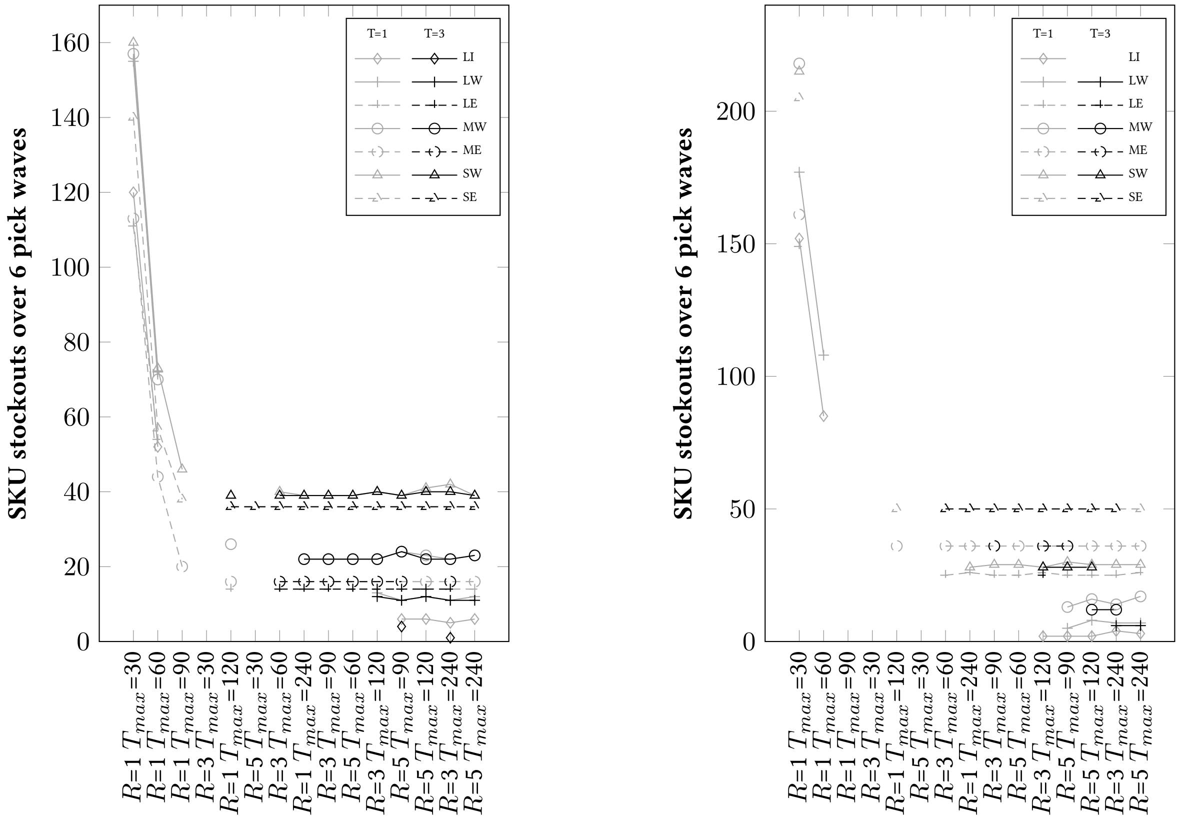
(a) Total stockouts over 6 pick waves for **T=1**

(b) Total stockouts over 6 pick waves for **T=1 and T=3**

(c) Location assignment and capacity

(d) Nb SKUs with $\geq$ 2 and 1 location

Figure 7: Total number of stockouts over 6 pick waves set out over the replenishment capacity for (a) T=1 (b) T=1 & T=3 with the BSL policy adopted. (c) Location assignment and capacity detailed per product group, for each forward area configuration. (d) Details on the location assignment detailed per product group, for each forward area configuration. Results are shown for 100 SKUs, demand set D.



(a) 50 SKUs

(b) 100 SKUs

Figure 8: Total number of stockouts over 6 pick waves set out over the replenishment capacity for (a) T=1 & (b) T=3 with the Bin-policy adopted. Results are shown for (a) 50 SKUs, (b) 100 SKUs, demand set D.

	Scenario 1: R=5 T_{max} =120				**Scenario 2:** R=3 T_{max} =240			
	$I_{i,t-1}$	$\sum_{k\in K}\sum_{r\in\mathcal{R}} AdvItem_{ikrt}$	D_{it}	*stockout?*	$I_{i,t-1}$	$\sum_{k\in K}\sum_{r\in\mathcal{R}} AdvItem_{ikrt}$	D_{it}	*Stockout?*
RW/PW t	4	10	10	No	9	0	10	Yes
RW/PW $t+1$	4	**5**	7	No	0	**14**	7	No
RW/PW $t+2$	2	10	9	No	7	0	9	Yes

Table 11: Example showing the impact of the replenishment freedom included in the model for different replenishment capacities. The example is worked out for SKU 7, b_i=5, C_i=15.

For completeness, we mention that the observation of flat curves holds when the BSL-, BSLCeiled- or Cap- policy is adopted, but not so much when the SS-, Bin- or SSCeiled-policy is applied. This can be seen in fig. 8a (50 SKUs) and fig. 8b (100 SKUs) which show the results obtained by the Bin-policy. The curves fluctuate to some extent, and, surprisingly, stockouts may rise even though replenishment capacity increases. To explain this behaviour, we emphasize that the mathematical model enjoys a certain degree of freedom: the model is allowed to choose (1) which SKUs to replenish (illustrated in table 10), (2) the replenishment quantity as long as replenishment and storage capacity are respected, allowing quantities that exceed what is deemed necessary to avoid a stockout. Because of this freedom, outcomes in future replenishment and pick waves are influenced, resulting in less or more stockouts than if this freedom were to be utilized differently. Table 11 shows an example.

In the example illustrated in table 11, replenishment wave $t+1$ starts with different inventory levels in scenarios 1 and 2. In both situations the SKU qualifies for replenishment as the inventory level has fallen below the threshold associated with the Bin-policy. In scenario 1, a replenishment of one bin is sufficient to avoid a stockout. In scenario 2, 4 items remain from a previous emergency pick. Add to this an replenishment of one bin, and a stockout in pick wave $t+1$ can be avoided. The model, however, decides to replenish not one but two full bins, whilst respecting storage and replenishment capacity. As a result, the inventory level in replenishment wave $t+2$ exceeds the threshold associated with the Bin-policy. The SKU is not considered for replenishment and a stockout is unavoidable.

The situation illustrated in table 11 arises mainly when either the Bin-, SS-, or SSCeiled-policy, not by chance the more "strict" inventory policies, is adopted. For these policies, SKUs are only considered for replenishment when inventory levels are relatively low. By replenishing additional bins, it is easy to end up with an inventory level that exceeds these threshold values, and consequently the product is not considered for replenishment in the next replenishment wave, leading to unavoidable stockouts.

We conclude that a larger replenishment capacity only benefits the number of SKU stockouts when the replenishment capacity is increased from low to medium values. Further improvements are blocked due to a more stringent constraint, the storage capacity constraint. We also find that a positive impact of a larger replenishment capacity on the number of stockouts cannot be guaranteed when more strict inventory policies than the BSL-policy (e.g., Bin-policy) are adopted.

Planning horizon To discuss the impact of a longer planning horizon on the total number of stockouts, we refer to fig. 6b and fig. 7b when the BSL-policy is adopted, and fig. 8a and fig. 8b when the Bin-policy is adopted. In all figures, the grey lines represent the results for T=1, while the black lines show the results for T=3.

In contrast to what was expected, the benefits of a longer planning horizon remain limited. This is generally the case for the BSL-policy, where T=1 and T=3 obtain the same results. For the Bin-policy, we regularly find deviations, mainly in favour of a longer planning horizon, although this is not always the case. An example is set out in table 12. For a longer planning horizon, the model decides to replenish a larger replenishment quantity. This choice can be explained by the freedom enjoyed in the model (illustrated in table 11), although the availability of more demand information could have led to a more informed decision as well. The larger replenishment quantity in the first replenishment wave allows to employ replenishment capacity in the next two replenishment waves elsewhere. Because of this decision, the SKU is not considered for replenishment until replenishment wave 11, which results in a stockout in pick wave 10. In scenario 1, such problem is not encountered.

	Scenario 1: T=1				**Scenario 2:** T=3			
	$I_{i,t-1}$	$\sum_{k\in K}\sum_{r\in\mathcal{R}} AdvItem_{ikrt}$	D_{it}	*Stockout?*	$I_{i,t-1}$	$\sum_{k\in K}\sum_{r\in\mathcal{R}} AdvItem_{ikrt}$	D_{it}	*Stockout?*
RW/PW 7	0	15	11	No	0	25	11	No
RW/PW 8	4	0	3	No	14	0	3	No
RW/PW 9	1	20	4	No	11	0	4	No
RW/PW 10	17	0	12	No	**7**	0	12	Yes
...								

Table 12: Example to illustrate the impact of a longer planning horizon. The example is worked out for SKU 6, b_i=5, C_i=25, R=5 and T_{max}=240.

We conclude that a longer planning horizon does not benefit the number of SKU stockouts. However, we recall that many instances, especially those with a longer planning horizon, were not solved. We also reason that the largest improvements of a longer planning horizon are probably expected for instances with a larger forward area. For such area sizes, more storage capacity can be allocated to SKUs and larger quantities can be replenished, which allows to anticipate on replenishments in future replenishment waves. Instances characterised by a larger forward area, however, currently often remained unsolved.

6 Conclusion

In this paper, we introduced the internal warehouse replenishment problem (IWRP) that takes place in a warehouse where inventory is stored in both a forward and reserve area. As extension to the warehouse literature which mainly focuses on the tactical organisation of a forward reserve storage system, the IWRP takes into account operational issues, e.g., limited work force and time, which are often ignored or simplified in existing research. The aim of the IWRP is to plan internal replenishments and avoid SKU stockouts in an out-of-rack forward reserve system where picking and replenishing do not take place simultaneously but in alternating waves; a forward reserve set-up that is researched to a minor extent. We discussed the details of the replenishment set-up and presented a mixed-integer linear program to plan the replenishment operation, organised in bins.

The IWRP was solved to optimality using the Gurobi software for a variety of instances. Instances differed in terms of number of SKUs, storage capacity, planning horizon, inventory policy and replenishment capacity. We found that most parameters, individually and/or in combination to other parameters, influence the number of stockouts, although some influences remain small, e.g., planning horizon. Some observations are easy to clarify, e.g., certain SKU stockouts are unavoidable when storage capacity is insufficient to meet demand (e.g., small forward area). Other findings are more complicated to explain due to the combinatorial complexity of the IWRP. Key takeaways include the recommendation to allocate minimally two locations to products stored in the forward area. Many of the stockouts experienced in the experiment can be attributed to the lack of this feature. The adoption of the base-stock level as inventory policy resulted in good and stable results. Also, increasing replenishment capacity is not a convenient way to systematically improve the number of stockouts. Indeed, at some point the increase in replenishment capacity is offset by the (limited) storage capacity.

The conclusions stated above, however, should be read with some caution as only a part of instances were solved in the available time. In particular, we believe that the power of a longer planning horizon to influence the number of stockouts is underestimated as the most promising instances in terms of this feature currently remain unsolved. We encourage the study of a heuristic approach that is able to solve realistic IWRP instances, i.e., instances with a larger number of SKUs and with sufficient storage capacity allocated, to extend and complete the observations of our experiments.

In this paper we planned the execution of internal replenishments after tactical decisions were made. Earlier, we mentioned that the integration of both levels, tactical and operational, would lead to a very complex matter. In future research, however, we encourage the consideration of practical issues when deciding on tactical matters as cost underestimations can quickly rise. Furthermore, interesting research opportunities lie in the combined optimization of the picking and replenishing operations by, for example, studying the IWRP with an objective based on pick order stockouts, rather than SKU stockouts.

Appendix A - Mathematical model

For comprehensibility, the constraints in the mathematical model given in section 4 were described in an if-then manner. These constraints can be linearised with the help of the following auxiliary variables:

Auxiliary variables	
u_{it}, v_{it}	Continuous variables to determine the absolute value of $I_{i,t-1} + \sum_{k\in\mathcal{K}}\sum_{r\in\mathcal{R}} AdvItem_{ikrt} - D_{it}$ of SKU i in wave t
	u_{it} takes a non-zero value when no stockout occurs in pick wave t
	v_{it} takes a non-zero value if a stockout occurs in pick wave t
Abs_{it}	= 1 if u_{it} >0
$MaxBin_{it}$	The maximum number of bins that can be replenished in replenishment wave t given the capacity and inventory level of SKU i at the beginning of replenishment wave t
ϵ	Very small number, e.g., ϵ = 0.001

Table 13: Auxiliary variables used in the linear mathematical model.

Objective function

$$Min \sum_{i\in V}\sum_{t\in T} Z_{it} \tag{34}$$

Inventory level & emergency replenishments

$$I_{it} = I_{i,t-1} + \sum_{k\in\mathcal{K}}\sum_{r\in\mathcal{R}} AdvItem_{ikrt} - D_{it} + Emerg_{it} \qquad \forall i \in \mathcal{V}, t \in T \tag{35}$$

$$I_{i,t-1} + \sum_{k\in\mathcal{K}}\sum_{r\in\mathcal{R}} AdvItem_{ikrt} - D_{it} \geq -M \cdot Z_{it} \qquad \forall i \in \mathcal{V}, t \in T \tag{36}$$

$$I_{i,t-1} + \sum_{k\in\mathcal{K}}\sum_{r\in\mathcal{R}} AdvItem_{ikrt} - D_{it} \leq M \cdot (1 - Z_{it}) - 1 \qquad \forall i \in \mathcal{V}, t \in T \tag{37}$$

$$I_{i,t-1} + \sum_{k\in\mathcal{K}}\sum_{r\in\mathcal{R}} AdvItem_{ikrt} - D_{it} = u_{it} - v_{it} \qquad \forall i \in \mathcal{V}, t \in T \tag{38}$$

$$u_{it} \leq M \cdot Abs_{it} \qquad \forall i \in \mathcal{V}, t \in T \tag{39}$$

$$v_{it} \leq M(1 - Abs_{it}) \qquad \forall i \in \mathcal{V}, t \in T \tag{40}$$

$$Emerg_{it} \leq u_{it} + v_{it} \qquad \forall i \in \mathcal{V}, t \in T \tag{41}$$

$$Emerg_{it} \geq u_{it} + v_{it} + M(Z_{it} - 1) \qquad \forall i \in \mathcal{V}, t \in T \tag{42}$$

$$Emerg_{it} \leq M \cdot Z_{it} \qquad \forall i \in \mathcal{V}, t \in T \tag{43}$$

Broken bin

$$BrokenQ_{it} = BrokenQ_{i,t-1} - \sum_{k\in\mathcal{K}}\sum_{r\in\mathcal{R}} BrokenItem_{ikrt} - Emerg_{it} + b_i \cdot ReserveBin_{it} \qquad \forall i \in \mathcal{V}, t \in T \tag{44}$$

$$\frac{Emerg_{it} - (BrokenQ_{i,t-1} - \sum_{k\in\mathcal{K}}\sum_{r\in\mathcal{R}} BrokenItem_{ikrt})}{b_i} \leq ReserveBin_{it} \qquad \forall i \in \mathcal{V}, t \in T \tag{45}$$

$$\frac{Emerg_{it} - (BrokenQ_{i,t-1} - \sum_{k\in\mathcal{K}}\sum_{r\in\mathcal{R}} BrokenItem_{ikrt})}{b_i} - \epsilon \geq ReserveBin_{it} - 1 \qquad \forall i \in \mathcal{V}, t \in T \tag{46}$$

$$BrokenQ_{it} \leq M \cdot Broken_{it} \qquad \forall i \in \mathcal{V}, t = 0, ..., T \tag{47}$$

$$BrokenQ_{it} \geq Broken_{it} \qquad \forall i \in \mathcal{V}, t = 0, ..., T \tag{48}$$

Internal replenishments of full & broken bins

$$\sum_{k\in\mathcal{K}}\sum_{r\in\mathcal{R}} BrokenBin_{ikrt} \leq Broken_{i,t-1} \qquad \forall i \in \mathcal{V}, t \in T \tag{49}$$

$$BrokenItem_{ikrt} \leq M \cdot BrokenBin_{ikrt} \qquad \forall i \in \mathcal{V}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R}, t \in T \tag{50}$$

$$BrokenItem_{ikrt} \leq BrokenQ_{i,t-1} \qquad \forall i \in \mathcal{V}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R}, t \in T \tag{51}$$

$$BrokenItem_{ikrt} \geq BrokenQ_{i,t-1} - M(1 - BrokenBin_{ikrt}) \qquad \forall i \in \mathcal{V}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R}, t \in T \tag{52}$$

$$I_{i,t-1} - s_i \leq M(1 - Repl_{it}) - 1 \qquad \forall i \in \mathcal{V}, t \in T \tag{53}$$

$$M \cdot (1 - (Broken_{i,t-1} - \sum_{k\in\mathcal{K}}\sum_{r\in\mathcal{R}} BrokenBin_{ikrt})) \geq AdvBin_{ikrt} \qquad \forall i \in \mathcal{V}, \forall r \in \mathcal{R}, \forall k \in \mathcal{K}, t \in T \tag{54}$$

$$\frac{C_i - I_{i,t-1}}{b_i} \geq MaxBin_{it} \qquad \forall i \in \mathcal{V}, t \in T \tag{55}$$

$$\frac{C_i - I_{i,t-1}}{b_i} + \epsilon \leq MaxBinit + 1 \qquad \forall i \in \mathcal{V}, t \in T \tag{56}$$

$$\sum_{r\in\mathcal{R}}\sum_{k\in\mathcal{K}} AdvBin_{ikrt} + BrokenBin_{ikrt} \leq MaxBin_{it} \qquad \forall i \in \mathcal{V}, t \in T \tag{57}$$

$$BrokenItem_{ikrt} + AdvBin_{ikrt} b_i = AdvItem_{ikrt} \qquad \forall i \in \mathcal{V}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R}, t \in T \tag{58}$$

$$AdvBin_{ikrt} + BrokenBin_{ikrt} \leq M \cdot Route_{ikrt} \qquad \forall i \in \mathcal{V}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R}, t \in T \tag{59}$$

$$AdvBin_{ikrt} + BrokenBin_{ikrt} \geq Route_{ikrt} \qquad \forall i \in \mathcal{V}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R}, t \in T \tag{60}$$

Routing

$$\sum_{r \in \mathcal{R}} \sum_{k \in \mathcal{K}} Route_{ikrt} \leq M \cdot Repl_{it} \qquad \forall i \in \mathcal{V}, t \in T \tag{61}$$

$$\sum_{i \in V} AdvBin_{ikrt} + BrokenBin_{ikrt} \leq Q_k \qquad \forall k \in \mathcal{K}, \forall r \in \mathcal{R}, t \in T \tag{62}$$

$$\sum_{i \in V} Route_{ikrt} \leq M \cdot Route_{0krt} \qquad \forall k \in \mathcal{K}, \forall r \in \mathcal{R}, t \in T \tag{63}$$

$$Route_{ikrt} = \sum_{j \in V \cup V_0, j \neq i} x_{ijkrt} = \sum_{j \in V \cup V_0, j \neq i} x_{jikrt} \qquad \forall i \in \mathcal{V}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R}, t \in T \tag{64}$$

$$\sum_{k \in \mathcal{K}} (T_{travel}(\sum_{i \in \mathcal{V} \cup V_0} \sum_{j \in \mathcal{V} \cup V_0} c_{ij} x_{ijkrt}) + T_{store} \sum_{i \in V} (AdvBin_{ikrt} + BrokenBin_{ikrt})) \leq T_{max}$$
$$\forall r \in \mathcal{R}, t \in T \tag{65}$$

Domain

$$Z_{it}, Repl_{it}, Broken_{it}, Abs_{it} \in \{0,1\} \qquad \forall i \in \mathcal{V}, t \in T \tag{66}$$

$$I_{it}, Emerg_{it}, BrokenQ_{it}, u_{it}, v_{it}, MaxBin_{it} \geq 0 \qquad \forall i \in \mathcal{V}, t \in T \tag{67}$$

$$BrokenBin_{ikrt}, BrokenItem_{ikrt}, AdvBin_{ikrt}, AdvItem_{ikrt} \geq 0$$
$$\forall i \in \mathcal{V}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R}, t \in T \tag{68}$$

$$x_{ijkrt}, Route_{ikrt} \in \{0,1\} \qquad \forall i \in \mathcal{V} \cup V_0, \forall j \in V \cup V_0,$$
$$\forall k \in \mathcal{K}, \forall r \in \mathcal{R}, t \in T \tag{69}$$

Added to the model by means of lazy constraint:

$$\sum_{i \in S} \sum_{j \notin S} x_{ijkrt} \geq Route_{hkrt} \qquad \forall h \in S, \forall k \in \mathcal{K}, \forall r \in \mathcal{R} \tag{70}$$

Appendix B - Detailed results

FA Config \ $R=$		1	1	1	1	1	3	3	3	3	3	5	5	5	5	5
	Inv policy \ T_{max}	**30**	**60**	**90**	**120**	**240**	**30**	**60**	**90**	**120**	**240**	**30**	**60**	**90**	**120**	**240**
	Bin	120	52	/	/	/	/	/	/	/	5	/	/	6	6	6
	SS	111	53	/	/	/	/	/	/	/	12	/	/	9	12	12
	SSCeiled	120	52	/	/	/	/	/	/	/	4	/	/	1	5	4
LI	BSL	123	50	/	12	/	/	/	/	/	1	/	/	1	1	1
	BSLCeiled	123	47	/	/	/	/	/	/	/	1	/	/	1	1	1
	Cap	123	47	/	12	/	/	/	/	/	1	/	/	1	1	1
	Bin	155	72	/	/	/	/	/	/	13	11	/	/	11	12	12
	SS	151	68	/	28	20	/	/	/	/	19	/	/	20	21	16
	SSCeiled	155	72	/	/	/	/	/	/	13	11	/	/	11	12	11
LW	BSL	154	74	/	/	13	/	/	/	11	11	/	/	11	11	11
	BSLCeiled	155	72	35	19	/	/	/	/	11	11	/	/	11	11	11
	Cap	155	72	/	19	/	/	/	/	11	11	/	/	11	11	11
	Bin	157	70	/	26	22	/	/	22	22	22	/	22	24	23	23
	SS	146	73	/	31	28	/	/	27	28	28	/	27	28	28	29
	SSCeiled	157	70	/	26	22	/	/	22	22	22	/	22	24	23	23
MW	BSL	149	75	/	27	22	/	/	22	22	22	/	22	22	22	22
	BSLCeiled	154	76	/	27	22	/	/	22	22	22	/	22	22	22	22
	Cap	154	76	/	27	22	/	/	22	22	22	/	22	22	22	22
	Bin	160	73	46	39	39	/	40	39	40	42	/	39	39	41	39
	SS	151	78	/	47	48	/	47	47	47	49	/	48	47	47	48
	SSCeiled	160	73	46	39	39	/	40	39	40	42	/	39	39	41	39
SW	BSL	157	72	46	39	39	/	39	39	39	39	/	39	39	39	39
	BSLCeiled	165	72	46	39	39	/	39	39	39	39	/	39	39	39	39
	Cap	165	72	46	39	39	/	39	39	39	39	/	39	39	39	39
	Bin	111	54	/	14	14	/	14	14	14	14	/	14	14	14	14
	SS	109	54	31	21	21	/	21	20	22	19	/	22	20	20	21
	SSCeiled	111	/	/	14	14	/	14	14	14	14	/	14	14	14	14
LE	BSL	109	59	/	14	14	/	14	14	14	14	/	14	14	14	14
	BSLCeiled	111	/	/	14	14	/	14	14	14	14	/	14	14	14	14
	Cap	106	56	21	/	14	/	14	14	14	14	/	14	14	14	14
	Bin	113	44	20	16	16	/	16	16	16	16	/	16	16	16	16
	SS	112	50	27	21	21	/	21	20	21	21	/	21	20	21	21
	SSCeiled	113	44	20	16	16	/	16	16	16	16	/	16	16	16	16
ME	BSL	110	43	19	16	16	/	16	16	16	16	/	16	16	16	16
	BSLCeiled	113	51	20	16	16	/	16	16	16	16	/	16	16	16	16
	Cap	105	42	19	16	16	/	16	16	16	16	/	16	16	16	16
	Bin	140	57	38	36	36	/	36	36	36	36	36	36	36	36	36
	SS	137	61	40	37	38	/	37	38	38	38	37	38	38	38	38
	SSCeiled	140	57	38	36	36	/	36	36	36	36	36	36	36	36	36
SE	BSL	133	62	38	36	36	/	36	36	36	36	/	36	36	36	36
	BSLCeiled	140	61	38	36	36	/	36	36	36	36	36	36	36	36	36
	Cap	140	63	38	36	36	/	36	36	36	36	/	36	36	36	36

Table 14: Total number of stockouts over 6 pick waves, listed per parameter combination. Results are shown for 50 SKUs, $T = 1$, Demand set = D.

FA Config \ $R=$		1	1	1	1	1	3	3	3	3	3	5	5	5	5	5
	Inv policy \ T_{max}	**30**	**60**	**90**	**120**	**240**	**30**	**60**	**90**	**120**	**240**	**30**	**60**	**90**	**120**	**240**
	Bin	136	54	/	/	/	/	/	/	/	1	/	/	2	4	0
	SS	128	62	32	/	/	/	/	/	/	8	/	/	7	7	8
	SSCeiled	136	54	/	/	/	/	/	/	/	1	/	/	1	2	1
LI	BSL	131	59	/	/	/	/	/	/	/	0	/	/	0	0	0
	BSLCeiled	131	55	/	/	/	/	/	/	/	0	/	/	0	0	0
	Cap	131	55	/	13	/	/	/	/	/	0	/	/	0	0	0
	Bin	166	85	/	/	12	/	/	/	8	11	/	/	10	10	10
	SS	157	80	/	/	23	/	/	/	19	19	/	/	18	19	19
	SSCeiled	166	85	/	/	12	/	/	/	8	9	/	/	10	10	10
LW	BSL	160	85	/	18	12	/	/	/	8	8	/	/	8	8	8
	BSLCeiled	166	85	36	/	12	/	/	/	8	8	/	/	8	8	8
	Cap	166	85	36	18	12	/	/	/	8	8	/	/	8	8	8
	Bin	154	70	/	23	16	/	/	16	16	17	/	16	17	16	16
	SS	155	77	41	/	21	/	/	22	23	26	/	23	22	21	22
	SSCeiled	154	70	/	23	16	/	/	16	16	16	/	16	16	16	16
MW	BSL	159	76	/	/	16	/	/	16	16	16	/	16	16	16	16
	BSLCeiled	159	74	35	23	16	/	/	16	16	16	/	16	16	16	16
	Cap	159	74	35	21	16	/	/	16	16	16	/	16	16	16	16
	Bin	166	76	/	32	31	/	31	31	32	33	/	32	34	32	32
	SS	165	75	/	33	33	/	33	33	34	34	/	33	34	33	33
	SSCeiled	166	76	/	32	31	/	31	31	32	33	/	32	34	32	32
SW	BSL	158	74	/	31	31	/	31	31	31	31	/	31	31	31	31
	BSLCeiled	165	70	/	31	31	/	31	31	31	31	/	31	31	31	31
	Cap	165	70	/	31	31	/	31	31	31	31	/	31	31	31	31
	Bin	122	/	/	15	15	/	15	15	15	15	/	15	15	15	15
	SS	117	63	/	26	26	/	25	24	25	25	/	26	27	26	25
	SSCeiled	122	/	/	15	15	/	15	15	15	15	/	15	15	15	15
LE	BSL	124	66	20	15	15	/	15	15	15	15	/	15	15	15	15
	BSLCeiled	122	62	19	15	15	/	15	15	15	15	/	15	15	15	15
	Cap	119	60	/	15	15	/	15	15	15	15	/	15	15	15	15
	Bin	119	55	/	15	15	/	15	15	15	15	/	15	15	15	15
	SS	114	59	/	/	24	/	24	23	23	24	/	24	24	25	24
	SSCeiled	119	55	/	15	15	/	15	15	15	15	/	15	15	15	15
ME	BSL	122	50	/	15	15	/	15	15	15	15	/	15	15	15	15
	BSLCeiled	119	53	/	15	15	/	15	15	15	15	/	15	15	15	15
	Cap	111	49	/	15	15	/	15	15	15	15	/	15	15	15	15
	Bin	136	58	34	32	32	/	32	32	32	32	32	32	32	32	32
	SS	135	61	39	36	36	/	36	36	36	36	36	36	36	36	36
	SSCeiled	136	58	34	32	32	/	32	32	32	32	32	32	32	32	32
SE	BSL	135	61	/	32	32	/	32	32	32	32	32	32	32	32	32
	BSLCeiled	136	64	34	32	32	/	32	32	32	32	32	32	32	32	32
	Cap	136	62	34	32	32	/	32	32	32	32	32	32	32	32	32

Table 15: Total number of stockouts over 6 pick waves, listed per parameter combination. Results are shown for 50 SKUs, $T = 1$, Demand set = D'.

FA Config \ $R=$		1	1	1	1	1	3	3	3	3	3	5	5	5	5	5
	Inv policy \ T_{max}	30	60	90	120	240	30	60	90	120	240	30	60	90	120	240
LI	Bin	135	56	/	12	/	/	/	/	/	2	/	/	2	4	2
	SS	128	66	36	24	/	/	/	/	/	13	/	/	11	12	13
	SSCeiled	135	56	/	12	/	/	/	/	/	2	/	/	1	2	1
	BSL	142	60	/	12	/	/	/	/	0	0	/	/	0	0	0
	BSLCeiled	134	55	/	12	/	/	/	/	/	0	/	/	0	0	0
	Cap	134	55	/	12	/	/	/	/	/	0	/	/	0	0	0
LW	Bin	156	70	/	/	/	/	/	/	8	9	/	/	8	9	8
	SS	155	73	/	/	/	/	/	/	20	19	/	/	18	21	18
	SSCeiled	156	70	/	/	/	/	/	/	8	8	/	/	8	8	8
	BSL	154	75	/	17	/	/	/	/	8	8	/	/	8	8	8
	BSLCeiled	156	70	/	/	/	/	/	/	8	8	/	/	8	8	8
	Cap	156	70	/	/	/	/	/	/	8	8	/	/	8	8	8
MW	Bin	155	62	/	24	18	/	18	19	18	20	/	18	21	18	19
	SS	151	71	42	32	27	/	/	26	27	27	/	28	26	25	26
	SSCeiled	155	62	/	24	18	/	18	18	18	19	/	18	21	18	19
	BSL	148	64	/	23	18	/	18	18	18	18	/	18	18	18	18
	BSLCeiled	151	64	/	23	18	/	18	18	18	18	/	18	18	18	18
	Cap	151	64	/	23	18	/	18	18	18	18	/	18	18	18	18
SW	Bin	161	67	38	31	31	/	31	31	32	31	/	31	31	32	31
	SS	159	67	45	38	40	/	38	37	39	38	/	39	38	37	38
	SSCeiled	161	67	38	31	31	/	31	31	32	31	/	31	31	32	31
	BSL	153	67	38	31	31	/	31	31	31	31	/	31	31	31	31
	BSLCeiled	158	64	38	31	31	/	31	31	31	31	/	31	31	31	31
	Cap	158	64	38	31	31	/	31	31	31	31	/	31	31	31	31
LE	Bin	111	/	23	19	19	/	19	19	19	19	/	19	19	19	19
	SS	108	59	/	25	23	/	25	24	25	23	/	25	25	24	25
	SSCeiled	111	/	24	19	19	/	19	19	19	19	/	19	19	19	19
	BSL	111	65	23	19	19	/	19	19	19	19	/	19	19	19	19
	BSLCeiled	111	/	24	19	19	/	19	19	19	19	/	19	19	19	19
	Cap	112	/	21	19	19	/	19	19	19	19	/	19	19	19	19
ME	Bin	123	51	/	20	20	/	20	20	20	20	/	20	20	20	20
	SS	122	59	38	31	31	/	31	31	31	30	/	30	29	31	31
	SSCeiled	123	51	/	20	20	/	20	20	20	20	/	20	20	20	20
	BSL	116	52	23	20	20	/	20	20	20	20	/	20	20	20	20
	BSLCeiled	123	53	/	20	20	/	20	20	20	20	/	20	20	20	20
	Cap	114	48	23	20	20	/	20	20	20	20	20	20	20	20	20
SE	Bin	142	63	/	35	35	/	35	35	35	35	35	35	35	35	35
	SS	144	68	43	39	39	/	39	39	39	38	39	39	39	39	39
	SSCeiled	142	63	/	35	35	/	35	35	35	35	35	35	35	35	35
	BSL	144	68	/	35	35	/	35	35	35	35	35	35	35	35	35
	BSLCeiled	142	62	/	35	35	/	35	35	35	35	35	35	35	35	35
	Cap	142	63	/	35	35	/	35	35	35	35	35	35	35	35	35

Table 16: Total number of stockouts over 6 pick waves, listed per parameter combination. Results are shown for 50 SKUs, $T = 1$, Demand set = D''.

FA Config \ $R=$		1	1	1	1	1	3	3	3	3	3	5	5	5	5	5
	Inv policy \ T_{max}	30	60	90	120	240	30	60	90	120	240	30	60	90	120	240
LI	Bin	/	/	/	/	/	/	/	/	/	1	/	/	4	/	/
	SS	/	/	/	/	/	/	/	/	/	7	/	/	7	7	/
	SSCeiled	/	/	/	/	/	/	/	/	/	/	/	/	1	/	/
	BSL	/	/	/	/	/	/	/	/	/	1	/	/	1	1	1
	BSLCeiled	/	/	/	/	/	/	/	/	/	1	/	/	/	1	/
	Cap	/	/	/	/	/	/	/	/	/	1	/	/	/	1	1
LW	Bin	/	/	/	/	/	/	/	/	12	11	/	/	11	12	11
	SS	/	/	/	/	/	/	/	/	17	15	/	/	17	18	16
	SSCeiled	/	/	/	/	/	/	/	/	12	11	/	/	11	12	11
	BSL	/	/	/	/	/	/	/	/	11	/	/	/	11	11	11
	BSLCeiled	/	/	/	/	/	/	/	/	11	11	/	/	11	11	11
	Cap	/	/	/	/	/	/	/	/	11	11	/	/	11	11	11
MW	Bin	/	/	/	/	22	/	/	22	22	22	/	22	24	22	23
	SS	/	/	/	/	26	/	/	25	26	25	/	24	25	25	25
	SSCeiled	/	/	/	/	22	/	/	22	22	22	/	22	24	22	23
	BSL	/	/	/	/	22	/	/	22	22	22	/	22	22	22	22
	BSLCeiled	/	/	/	/	22	/	/	22	22	22	/	22	22	22	22
	Cap	/	/	/	/	22	/	/	22	22	22	/	22	22	22	/
SW	Bin	/	/	/	39	39	/	39	39	40	40	/	39	39	40	39
	SS	/	/	/	45	45	/	46	45	45	45	/	44	46	44	44
	SSCeiled	/	/	/	39	39	/	39	39	40	40	/	39	39	40	39
	BSL	/	/	/	39	39	/	39	39	39	39	/	39	39	39	39
	BSLCeiled	/	/	/	39	39	/	39	39	39	39	/	39	39	39	39
	Cap	/	/	/	39	39	/	39	39	39	39	/	39	39	39	39
LE	Bin	/	/	/	/	14	/	14	14	14	14	/	14	14	14	/
	SS	/	/	/	17	18	/	18	19	18	16	/	18	19	18	19
	SSCeiled	/	/	/	14	14	/	14	14	14	14	/	14	14	14	14
	BSL	/	/	/	14	14	/	14	14	14	14	/	14	14	14	14
	BSLCeiled	/	/	/	14	14	/	/	14	14	14	/	14	14	14	14
	Cap	/	/	/	/	14	/	14	14	14	14	/	/	14	/	/
ME	Bin	/	/	/	/	16	/	16	16	16	16	/	16	16	/	/
	SS	/	/	/	20	20	/	20	19	20	20	/	20	20	20	20
	SSCeiled	/	/	/	/	16	/	16	16	16	/	/	16	16	16	16
	BSL	/	/	/	16	16	/	16	16	16	16	/	16	16	16	16
	BSLCeiled	/	/	/	/	16	/	16	16	16	16	/	16	16	16	16
	Cap	/	/	/	16	16	/	16	16	16	/	/	16	16	16	/
SE	Bin	/	/	/	36	36	/	36	36	36	36	36	36	36	36	36
	SS	/	/	37	37	37	/	37	37	37	37	37	37	37	37	37
	SSCeiled	/	/	/	36	36	/	36	36	36	36	36	36	36	36	36
	BSL	/	/	36	36	36	/	36	36	36	36	36	36	36	36	/
	BSLCeiled	/	/	/	36	36	/	36	36	36	36	/	36	36	36	/
	Cap	/	/	/	36	36	/	36	36	36	36	36	36	36	36	/

Table 17: Total number of stockouts over 6 pick waves, listed per parameter combination. Results are shown for 50 SKUs, $T = 3$, Demand set = D.

FA Config \ $R=$		1	1	1	1	1	3	3	3	3	3	5	5	5	5	5
	Inv policy \ T_{max}	**30**	**60**	**90**	**120**	**240**	**30**	**60**	**90**	**120**	**240**	**30**	**60**	**90**	**120**	**240**
LI	Bin	/	/	/	/	/	/	/	/	/	0	/	/	2	1	0
	SS	/	/	/	/	/	/	/	/	/	5	/	/	6	7	6
	SSCeiled	/	/	/	/	/	/	/	/	/	0	/	/	0	0	0
	BSL	/	/	/	/	/	/	/	/	/	0	/	/	0	0	0
	BSLCeiled	/	/	/	/	/	/	/	/	/	0	/	/	/	0	0
	Cap	/	/	/	/	/	/	/	/	/	0	/	/	0	0	/
LW	Bin	/	/	/	/	/	/	/	/	/	/	/	/	8	/	/
	SS	/	/	/	/	/	/	/	/	15	15	/	/	/	15	14
	SSCeiled	/	/	/	/	/	/	/	/	/	8	/	/	/	9	9
	BSL	/	/	/	/	/	/	/	/	/	8	/	/	/	8	8
	BSLCeiled	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
	Cap	/	/	/	/	/	/	/	/	/	/	/	/	8	/	8
MW	Bin	/	/	/	/	16	/	/	16	16	16	/	16	16	16	16
	SS	/	/	/	/	20	/	/	20	20	21	/	20	21	20	20
	SSCeiled	/	/	/	/	16	/	/	16	16	16	/	16	16	16	16
	BSL	/	/	/	/	16	/	/	16	16	16	/	16	16	16	16
	BSLCeiled	/	/	/	/	16	/	/	16	16	16	/	16	16	16	16
	Cap	/	/	/	/	16	/	/	16	16	16	/	16	16	16	16
SW	Bin	/	/	/	31	31	/	31	31	31	31	/	32	32	31	31
	SS	/	/	/	33	33	/	33	33	33	33	/	32	33	32	33
	SSCeiled	/	/	/	31	31	/	31	31	31	31	/	32	32	31	31
	BSL	/	/	/	31	31	/	31	31	31	31	/	31	31	31	31
	BSLCeiled	/	/	/	31	31	/	31	31	31	31	/	31	31	31	31
	Cap	/	/	/	/	31	/	31	31	31	31	/	31	31	31	31
LE	Bin	/	/	/	/	15	/	15	15	15	15	/	15	15	15	15
	SS	/	/	/	/	22	/	23	23	22	22	/	24	25	24	23
	SSCeiled	/	/	/	/	15	/	15	15	15	/	/	15	15	/	/
	BSL	/	/	/	15	15	/	15	15	15	15	/	15	15	15	15
	BSLCeiled	/	/	/	/	15	/	15	15	15	15	/	15	15	15	/
	Cap	/	/	/	/	15	/	15	/	15	/	/	15	15	/	/
ME	Bin	/	/	/	/	15	/	15	15	15	15	/	15	15	15	15
	SS	/	/	/	22	21	/	21	21	21	21	/	22	22	22	21
	SSCeiled	/	/	/	/	15	/	15	15	15	15	/	15	15	/	/
	BSL	/	/	/	15	15	/	15	15	15	15	/	15	15	15	15
	BSLCeiled	/	/	/	/	15	/	15	15	15	/	/	15	15	/	15
	Cap	/	/	/	/	15	/	15	15	15	/	/	15	15	15	15
SE	Bin	/	/	32	32	32	/	32	32	32	32	32	32	32	32	32
	SS	/	/	36	36	36	/	36	36	36	36	36	36	36	36	36
	SSCeiled	/	/	32	32	32	/	32	32	32	32	32	32	32	32	32
	BSL	/	/	32	32	32	/	32	32	32	32	32	32	32	32	32
	BSLCeiled	/	/	32	32	32	/	32	32	32	32	/	32	32	32	32
	Cap	/	/	32	32	32	/	32	32	32	32	32	32	32	32	32

Table 18: Total number of stockouts over 6 pick waves, listed per parameter combination. Results are shown for 50 SKUs, $T = 3$, Demand set = D'.

FA Config \ $R=$		1	1	1	1	1	3	3	3	3	3	5	5	5	5	5
	Inv policy \ T_{max}	**30**	**60**	**90**	**120**	**240**	**30**	**60**	**90**	**120**	**240**	**30**	**60**	**90**	**120**	**240**
LI	Bin	/	/	/	/	/	/	/	/	/	0	/	/	2	2	/
	SS	/	/	/	/	/	/	/	/	/	9	/	/	10	10	10
	SSCeiled	/	/	/	/	/	/	/	/	/	0	/	/	0	1	0
	BSL	/	/	/	/	/	/	/	/	/	0	/	/	0	0	0
	BSLCeiled	/	/	/	/	/	/	/	/	/	0	/	/	0	0	0
	Cap	/	/	/	/	/	/	/	/	/	0	/	/	/	0	0
LW	Bin	/	/	/	/	/	/	/	/	8	8	/	/	8	8	8
	SS	/	/	/	/	/	/	/	/	17	14	/	/	15	17	18
	SSCeiled	/	/	/	/	/	/	/	/	8	8	/	/	8	9	9
	BSL	/	/	/	/	/	/	/	/	/	8	/	/	8	/	8
	BSLCeiled	/	/	/	/	/	/	/	/	8	8	/	/	8	/	8
	Cap	/	/	/	/	/	/	/	/	8	8	/	/	8	8	8
MW	Bin	/	/	/	/	18	/	/	18	18	18	/	18	20	18	19
	SS	/	/	/	/	24	/	/	23	23	24	/	23	22	22	23
	SSCeiled	/	/	/	/	18	/	/	18	18	18	/	18	20	18	19
	BSL	/	/	/	/	18	/	/	18	18	18	/	18	18	18	18
	BSLCeiled	/	/	/	/	18	/	/	18	18	18	/	18	18	18	18
	Cap	/	/	/	/	18	/	/	18	18	18	/	18	18	18	18
SW	Bin	/	/	/	31	31	/	31	31	32	31	/	31	31	32	31
	SS	/	/	/	/	36	/	38	37	37	38	/	38	37	37	36
	SSCeiled	/	/	/	31	31	/	31	31	32	31	/	31	31	32	31
	BSL	/	/	/	31	31	/	31	31	31	31	/	31	31	31	31
	BSLCeiled	/	/	/	31	31	/	31	31	31	31	/	31	31	31	31
	Cap	/	/	/	31	31	/	31	31	31	31	/	31	31	31	31
LE	Bin	/	/	/	/	/	/	19	19	19	19	/	19	19	19	19
	SS	/	/	/	22	22	/	22	22	22	21	/	22	22	22	22
	SSCeiled	/	/	/	/	19	/	19	19	19	19	/	19	/	19	19
	BSL	/	/	/	19	19	/	19	19	19	19	/	19	19	19	19
	BSLCeiled	/	/	/	/	19	/	19	19	19	19	/	19	19	19	19
	Cap	/	/	/	/	19	/	/	19	19	/	/	19	/	19	/
ME	Bin	/	/	/	/	20	/	20	20	20	/	/	20	20	20	/
	SS	/	/	/	28	29	/	29	30	28	28	28	28	28	29	29
	SSCeiled	/	/	/	/	20	/	20	20	20	20	/	20	20	20	/
	BSL	/	/	/	20	20	/	20	20	20	20	20	20	20	20	/
	BSLCeiled	/	/	/	/	20	/	20	20	20	20	/	20	20	/	/
	Cap	/	/	/	/	20	/	/	20	20	/	/	20	20	20	/
SE	Bin	/	/	/	35	35	/	35	35	35	35	35	35	35	35	/
	SS	/	/	38	38	38	/	38	38	39	38	38	39	39	39	38
	SSCeiled	/	/	/	35	35	/	35	35	35	35	35	35	35	35	/
	BSL	/	/	/	35	35	/	35	35	35	35	35	35	35	35	35
	BSLCeiled	/	/	/	35	35	/	35	35	35	35	/	35	35	35	35
	Cap	/	/	/	35	35	/	35	35	35	35	/	35	35	35	35

Table 19: Total number of stockouts over 6 pick waves, listed per parameter combination. Results are shown for 50 SKUs, $T = 3$, Demand set = D''.

FA Config \ $R=$		1	1	1	1	1	3	3	3	3	3	5	5	5	5	5
	Inv policy \ T_{max}	**30**	**60**	**90**	**120**	**240**	**30**	**60**	**90**	**120**	**240**	**30**	**60**	**90**	**120**	**240**
LI	Bin	152	85	/	/	/	/	/	/	2	4	/	/	2	2	3
	SS	153	88	/	30	/	/	/	/	9	8	/	/	11	9	8
	SSCeiled	152	85	/	/	/	/	/	/	2	4	/	/	2	2	3
	BSL	155	84	/	22	/	/	/	/	2	2	/	/	2	2	2
	BSLCeiled	152	/	/	/	/	/	/	/	2	2	/	/	2	2	2
	Cap	152	/	/	/	/	/	/	/	/	2	/	/	/	2	2
LW	Bin	177	108	/	/	/	/	/	/	/	7	/	/	5	8	7
	SS	178	116	/	/	/	/	/	/	/	14	/	/	11	11	15
	SSCeiled	182	108	/	/	/	/	/	/	/	7	/	/	5	8	7
	BSL	183	99	/	/	/	/	/	/	/	5	/	/	5	5	5
	BSLCeiled	185	113	/	/	/	/	/	/	/	5	/	/	5	5	5
	Cap	185	/	/	/	/	/	/	/	/	5	/	/	5	5	5
MW	Bin	218	/	/	/	/	/	/	/	/	14	/	/	13	16	17
	SS	210	129	/	43	/	/	/	/	/	21	/	/	20	18	21
	SSCeiled	218	/	/	/	/	/	/	/	/	14	/	/	13	12	15
	BSL	212	130	/	/	/	/	/	/	/	12	/	/	12	12	12
	BSLCeiled	213	133	/	/	/	/	/	/	/	12	/	/	12	12	12
	Cap	213	133	/	/	/	/	/	/	/	12	/	/	12	12	12
SW	Bin	215	/	/	/	28	/	/	29	28	29	/	29	30	29	29
	SS	218	138	/	/	35	/	/	35	34	36	/	35	35	34	35
	SSCeiled	215	140	/	/	28	/	/	29	28	29	/	29	30	29	29
	BSL	220	/	/	/	28	/	/	28	28	28	/	28	28	28	28
	BSLCeiled	220	/	/	/	28	/	/	28	28	28	/	28	28	28	28
	Cap	220	/	/	/	28	/	/	28	28	28	/	28	28	28	28
LE	Bin	149	/	/	/	26	/	25	25	26	25	/	25	25	25	26
	SS	152	/	/	32	31	/	31	31	31	31	/	31	30	31	31
	SSCeiled	149	/	/	/	26	/	25	25	26	25	/	25	25	25	26
	BSL	149	/	/	/	25	/	25	25	25	25	/	25	25	25	25
	BSLCeiled	149	/	/	/	25	/	25	25	25	25	/	25	25	25	25
	Cap	146	/	/	/	25	/	25	25	25	25	/	25	25	25	25
ME	Bin	161	/	/	36	36	/	36	36	36	36	/	36	36	36	36
	SS	163	/	/	41	41	/	41	41	41	41	/	41	41	41	41
	SSCeiled	161	/	/	36	36	/	36	36	36	36	/	36	36	36	36
	BSL	163	/	/	36	36	/	36	36	36	36	/	36	36	36	36
	BSLCeiled	161	/	/	36	36	/	36	36	36	36	/	36	36	36	36
	Cap	161	/	/	/	36	/	36	36	36	36	/	36	36	36	36
SE	Bin	205	/	/	50	50	/	50	50	50	50	/	50	50	50	50
	SS	201	/	/	52	52	/	52	52	52	52	/	52	52	52	52
	SSCeiled	205	/	/	50	50	/	50	50	50	50	/	50	50	50	50
	BSL	195	/	/	50	50	/	50	50	50	50	/	50	50	50	50
	BSLCeiled	205	/	/	50	50	/	50	50	50	50	/	50	50	50	50
	Cap	210	/	/	50	50	/	50	50	50	50	/	50	50	50	50

Table 20: Total number of stockouts over 6 pick waves, listed per parameter combination. Results are shown for 100 SKUs, $T = 1$, Demand set = D.

FA Config \ $R=$		1	1	1	1	1	3	3	3	3	3	5	5	5	5	5
	Inv policy \ T_{max}	**30**	**60**	**90**	**120**	**240**	**30**	**60**	**90**	**120**	**240**	**30**	**60**	**90**	**120**	**240**
LI	Bin	161	94	/	/	/	/	/	/	/	5	/	/	/	5	6
	SS	160	98	54	/	/	/	/	/	20	19	/	/	20	20	18
	SSCeiled	161	94	/	/	/	/	/	/	/	5	/	/	/	5	6
	BSL	158	96	/	/	/	/	/	/	/	5	/	/	5	5	5
	BSLCeiled	161	92	/	/	/	/	/	/	/	5	/	/	/	5	5
	Cap	161	92	/	/	/	/	/	/	/	5	/	/	/	5	5
LW	Bin	185	/	/	/	/	/	/	/	/	10	/	/	9	12	14
	SS	191	122	/	/	/	/	/	/	/	21	/	/	20	20	23
	SSCeiled	181	/	/	/	/	/	/	/	/	10	/	/	9	13	12
	BSL	182	121	/	/	/	/	/	/	/	9	/	/	9	9	9
	BSLCeiled	182	/	/	/	/	/	/	/	/	9	/	/	9	9	9
	Cap	189	120	/	/	/	/	/	/	/	9	/	/	9	9	9
MW	Bin	213	140	/	/	/	/	/	/	15	16	/	/	16	16	21
	SS	209	138	/	/	/	/	/	/	/	29	/	/	29	29	29
	SSCeiled	213	140	/	/	/	/	/	/	/	16	/	/	16	16	20
	BSL	210	142	/	/	/	/	/	/	/	15	/	/	15	15	15
	BSLCeiled	212	145	/	/	/	/	/	/	/	15	/	/	15	15	15
	Cap	212	145	/	/	/	/	/	/	/	15	/	/	15	15	15
SW	Bin	227	146	/	/	35	/	/	35	35	35	/	35	36	36	35
	SS	224	146	/	/	44	/	/	43	45	44	/	44	44	46	43
	SSCeiled	227	146	/	/	35	/	/	35	35	36	/	35	36	36	37
	BSL	231	145	/	/	/	/	/	35	35	35	/	/	35	35	35
	BSLCeiled	230	146	/	/	35	/	/	35	35	35	/	/	35	35	35
	Cap	230	146	/	/	35	/	/	35	35	35	/	35	35	35	35
LE	Bin	151	/	/	/	24	/	/	24	24	24	/	24	24	24	24
	SS	152	91	/	/	40	/	/	40	40	40	/	40	41	40	40
	SSCeiled	151	/	/	/	24	/	/	24	24	24	/	24	24	24	24
	BSL	154	/	/	/	25	/	/	25	25	25	/	25	25	25	25
	BSLCeiled	151	/	/	/	24	/	/	24	24	24	/	24	24	24	24
	Cap	156	/	/	/	24	/	/	24	24	24	/	24	24	24	24
ME	Bin	167	/	/	/	35	/	/	35	35	35	/	35	35	35	35
	SS	170	/	/	/	46	/	/	45	45	46	/	46	46	46	45
	SSCeiled	167	/	/	/	35	/	/	35	35	35	/	35	35	35	35
	BSL	172	/	/	38	35	/	/	35	35	35	/	35	35	35	35
	BSLCeiled	167	/	/	/	35	/	/	35	35	35	/	35	35	35	35
	Cap	167	/	/	/	35	/	/	35	35	35	/	/	35	35	35
SE	Bin	204	/	/	59	59	/	59	59	59	59	/	59	59	59	59
	SS	215	/	/	64	64	/	64	64	64	64	/	64	64	64	63
	SSCeiled	204	/	/	59	59	/	59	59	59	59	/	59	59	59	59
	BSL	201	/	/	59	59	/	59	59	59	59	/	59	59	59	59
	BSLCeiled	204	/	/	59	59	/	59	59	59	59	/	59	59	59	59
	Cap	205	/	/	59	59	/	59	59	59	59	/	59	59	59	59

Table 21: Total number of stockouts over 6 pick waves, listed per parameter combination. Results are shown for 100 SKUs, $T = 1$, Demand set = D'.

FA Config \ $R=$		1	1	1	1	1	3	3	3	3	3	5	5	5	5	5
	Inv policy \ T_{max}	30	60	90	120	240	30	60	90	120	240	30	60	90	120	240
LI	Bin	150	/	/	/	/	/	/	/	/	7	/	/	5	5	5
	SS	156	87	/	/	/	/	/	/	14	12	/	/	12	14	13
	SSCeiled	150	/	/	/	/	/	/	/	7	7	/	/	/	5	5
	BSL	155	/	/	/	/	/	/	/	6	6	/	/	6	6	6
	BSLCeiled	150	/	/	/	/	/	/	/	5	5	/	/	5	5	5
	Cap	150	/	/	/	/	/	/	/	/	5	/	/	/	5	5
LW	Bin	175	/	/	/	/	/	/	/	/	10	/	/	7	9	9
	SS	184	/	/	/	/	/	/	/	/	15	/	/	13	15	15
	SSCeiled	175	/	/	/	/	/	/	/	/	8	/	/	7	8	8
	BSL	174	109	/	/	/	/	/	/	/	7	/	/	7	7	7
	BSLCeiled	181	/	/	/	/	/	/	/	/	6	/	/	6	6	6
	Cap	181	106	/	/	/	/	/	/	/	6	/	/	6	6	6
MW	Bin	207	115	/	/	/	/	/	/	12	17	/	/	13	13	14
	SS	200	116	/	47	/	/	/	/	23	29	/	/	23	24	24
	SSCeiled	207	115	/	/	/	/	/	/	16	17	/	/	13	13	14
	BSL	202	118	/	/	/	/	/	/	13	13	/	/	13	13	13
	BSLCeiled	203	129	/	/	/	/	/	/	/	12	/	/	12	12	12
	Cap	203	129	/	/	/	/	/	/	/	12	/	/	12	12	12
SW	Bin	210	136	/	/	/	/	/	30	29	30	/	29	29	30	29
	SS	212	131	/	/	33	/	/	/	33	34	/	32	34	34	34
	SSCeiled	210	136	/	/	/	/	/	30	29	30	/	29	29	30	29
	BSL	218	136	/	/	/	/	/	/	29	29	/	29	29	29	29
	BSLCeiled	214	136	/	/	/	/	/	29	29	29	/	29	29	29	29
	Cap	214	136	/	/	29	/	/	29	29	29	/	29	29	29	29
LE	Bin	142	/	/	/	25	/	25	25	25	25	/	25	25	25	25
	SS	145	/	/	/	34	/	34	32	34	33	/	34	33	34	34
	SSCeiled	142	/	/	/	25	/	25	25	25	25	/	25	25	25	25
	BSL	147	/	/	/	26	/	26	26	26	26	/	26	26	26	26
	BSLCeiled	142	/	/	/	25	/	25	25	25	25	/	25	25	25	25
	Cap	147	/	/	/	25	/	/	25	25	25	/	25	25	25	25
ME	Bin	163	/	/	/	37	/	37	37	37	37	/	37	37	37	37
	SS	165	100	/	/	43	/	42	42	43	42	/	42	43	41	43
	SSCeiled	163	/	/	37	37	/	37	37	37	37	/	37	37	37	37
	BSL	166	96	/	38	38	/	38	38	38	38	/	38	38	38	38
	BSLCeiled	163	/	/	/	37	/	37	37	37	37	/	37	37	37	37
	Cap	163	/	/	37	37	/	37	37	37	37	/	37	37	37	37
SE	Bin	195	112	/	52	52	/	52	52	52	52	/	52	52	52	52
	SS	195	114	/	53	53	/	53	53	53	53	/	53	53	53	53
	SSCeiled	195	112	/	52	52	/	52	52	52	52	/	52	52	52	52
	BSL	190	/	/	52	52	/	52	52	52	52	/	52	52	52	52
	BSLCeiled	195	112	/	52	52	/	52	52	52	52	/	52	52	52	52
	Cap	192	112	/	52	52	/	52	52	52	52	/	52	52	52	52

Table 22: Total number of stockouts over 6 pick waves, listed per parameter combination. Results are shown for 100 SKUs, $T = 1$, Demand set = D''.

FA Config \ $R=$		1	1	1	1	1	3	3	3	3	3	5	5	5	5	5
	Inv policy \ T_{max}	**30**	**60**	**90**	**120**	**240**	**30**	**60**	**90**	**120**	**240**	**30**	**60**	**90**	**120**	**240**
LI	Bin	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
	SS	/	/	/	/	/	/	/	/	9	8	/	/	9	9	8
	SSCeiled	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
	BSL	/	/	/	/	/	/	/	/	/	2	/	/	2	2	/
	BSLCeiled	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
	Cap	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
LW	Bin	/	/	/	/	/	/	/	/	/	6	/	/	/	/	6
	SS	/	/	/	/	/	/	/	/	/	10	/	/	10	10	12
	SSCeiled	/	/	/	/	/	/	/	/	/	6	/	/	/	5	6
	BSL	/	/	/	/	/	/	/	/	/	5	/	/	/	5	/
	BSLCeiled	/	/	/	/	/	/	/	/	/	5	/	/	/	5	5
	Cap	/	/	/	/	/	/	/	/	/	/	/	/	/	5	5
MW	Bin	/	/	/	/	/	/	/	/	/	12	/	/	/	12	/
	SS	/	/	/	/	/	/	/	/	/	17	/	/	18	15	18
	SSCeiled	/	/	/	/	/	/	/	/	/	12	/	/	/	12	/
	BSL	/	/	/	/	/	/	/	/	/	12	/	/	/	12	12
	BSLCeiled	/	/	/	/	/	/	/	/	/	12	/	/	/	12	12
	Cap	/	/	/	/	/	/	/	/	/	12	/	/	12	12	12
SW	Bin	/	/	/	/	/	/	/	/	28	/	/	/	28	28	/
	SS	/	/	/	/	/	/	/	/	/	35	/	/	35	/	/
	SSCeiled	/	/	/	/	/	/	/	/	/	28	/	/	28	28	/
	BSL	/	/	/	/	/	/	/	/	28	28	/	28	28	/	/
	BSLCeiled	/	/	/	/	/	/	/	/	/	28	/	/	28	28	/
	Cap	/	/	/	/	/	/	/	/	28	28	/	/	28	28	/
LE	Bin	/	/	/	/	/	/	/	/	25	/	/	/	/	/	/
	SS	/	/	/	/	31	/	32	31	31	31	/	32	30	32	32
	SSCeiled	/	/	/	/	/	/	/	25	/	/	/	/	/	25	/
	BSL	/	/	/	/	25	/	/	25	25	25	/	25	25	/	25
	BSLCeiled	/	/	/	/	/	/	/	/	/	25	/	/	25	/	/
	Cap	/	/	/	/	/	/	/	/	/	25	/	/	/	/	/
ME	Bin	/	/	/	/	/	/	/	36	36	/	/	/	36	/	/
	SS	/	/	/	41	41	/	41	41	41	41	/	41	41	41	41
	SSCeiled	/	/	/	/	/	/	/	36	36	/	/	/	36	/	/
	BSL	/	/	/	/	36	/	/	36	36	/	/	/	36	36	/
	BSLCeiled	/	/	/	/	/	/	/	/	/	/	/	/	36	/	/
	Cap	/	/	/	/	36	/	/	/	/	/	/	/	/	36	/
SE	Bin	/	/	/	/	50	/	50	50	50	50	/	50	50	50	/
	SS	/	/	/	/	52	/	52	52	52	52	/	52	52	52	52
	SSCeiled	/	/	/	/	50	/	50	50	50	50	/	50	50	50	50
	BSL	/	/	/	/	50	/	50	50	50	50	/	50	50	50	/
	BSLCeiled	/	/	/	/	50	/	50	50	50	50	/	50	50	50	50
	Cap	/	/	/	/	50	/	50	50	50	50	/	50	50	50	/

Table 23: Total number of stockouts over 6 pick waves, listed per parameter combination. Results are shown for 100 SKUs, $T = 3$, Demand set = D.

FA Config \ $R=$		1	1	1	1	1	3	3	3	3	3	5	5	5	5	5
	Inv policy \ T_{max}	**30**	**60**	**90**	**120**	**240**	**30**	**60**	**90**	**120**	**240**	**30**	**60**	**90**	**120**	**240**
LI	Bin	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
	SS	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
	SSCeiled	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
	BSL	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
	BSLCeiled	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
	Cap	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
LW	Bin	/	/	/	/	/	/	/	/	/	9	/	/	/	/	11
	SS	/	/	/	/	/	/	/	/	/	17	/	/	16	14	/
	SSCeiled	/	/	/	/	/	/	/	/	/	9	/	/	/	/	10
	BSL	/	/	/	/	/	/	/	/	/	/	/	/	/	9	9
	BSLCeiled	/	/	/	/	/	/	/	/	/	9	/	/	/	/	9
	Cap	/	/	/	/	/	/	/	/	/	9	/	/	/	9	9
MW	Bin	/	/	/	/	/	/	/	/	/	15	/	/	/	/	/
	SS	/	/	/	/	/	/	/	/	/	25	/	/	23	21	/
	SSCeiled	/	/	/	/	/	/	/	/	/	15	/	/	/	15	/
	BSL	/	/	/	/	/	/	/	/	/	15	/	/	/	15	/
	BSLCeiled	/	/	/	/	/	/	/	/	/	15	/	/	/	15	/
	Cap	/	/	/	/	/	/	/	/	/	15	/	/	/	/	15
SW	Bin	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
	SS	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
	SSCeiled	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
	BSL	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
	BSLCeiled	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
	Cap	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
LE	Bin	/	/	/	/	/	/	/	/	24	/	/	/	/	/	/
	SS	/	/	/	/	36	/	/	/	36	36	/	36	36	37	38
	SSCeiled	/	/	/	/	/	/	/	/	24	/	/	/	/	/	/
	BSL	/	/	/	/	25	/	/	/	25	/	/	25	25	/	/
	BSLCeiled	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
	Cap	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
ME	Bin	/	/	/	/	/	/	/	/	/	/	/	/	35	/	/
	SS	/	/	/	/	45	/	/	45	44	44	/	45	44	45	45
	SSCeiled	/	/	/	/	/	/	/	/	/	/	/	/	35	/	/
	BSL	/	/	/	/	35	/	/	/	35	/	/	35	35	35	/
	BSLCeiled	/	/	/	/	/	/	/	/	/	35	/	/	/	/	/
	Cap	/	/	/	/	/	/	/	/	/	35	/	/	35	35	/
SE	Bin	/	/	/	/	59	/	59	59	59	59	/	59	59	59	/
	SS	/	/	/	/	64	/	64	64	64	64	/	63	63	63	64
	SSCeiled	/	/	/	/	59	/	59	59	59	59	/	59	59	59	59
	BSL	/	/	/	/	59	/	59	59	59	59	/	59	59	59	59
	BSLCeiled	/	/	/	/	59	/	/	59	59	59	/	59	59	59	59
	Cap	/	/	/	/	59	/	59	59	59	59	/	59	59	59	59

Table 24: Total number of stockouts over 6 pick waves, listed per parameter combination. Results are shown for 100 SKUs, $T = 3$, Demand set = D'.

FA Config \ R=		1	1	1	1	1	3	3	3	3	3	5	5	5	5	5
	Inv policy \ T_{max}	30	60	90	120	240	30	60	90	120	240	30	60	90	120	240
LI	Bin	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
	SS	/	/	/	/	/	/	/	/	/	/	/	/	/	11	/
	SSCeiled	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
	BSL	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
	BSLCeiled	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
	Cap	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
LW	Bin	/	/	/	/	/	/	/	/	/	7	/	/	/	6	7
	SS	/	/	/	/	/	/	/	/	/	10	/	/	11	12	13
	SSCeiled	/	/	/	/	/	/	/	/	/	7	/	/	/	/	/
	BSL	/	/	/	/	/	/	/	/	/	7	/	/	7	7	/
	BSLCeiled	/	/	/	/	/	/	/	/	/	6	/	/	/	/	/
	Cap	/	/	/	/	/	/	/	/	/	6	/	/	/	6	6
MW	Bin	/	/	/	/	/	/	/	/	/	12	/	/	/	/	/
	SS	/	/	/	/	/	/	/	/	/	20	/	/	/	18	/
	SSCeiled	/	/	/	/	/	/	/	/	/	12	/	/	12	/	/
	BSL	/	/	/	/	/	/	/	/	/	13	/	/	/	13	/
	BSLCeiled	/	/	/	/	/	/	/	/	/	12	/	/	/	12	/
	Cap	/	/	/	/	/	/	/	/	/	12	/	/	/	12	/
SW	Bin	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
	SS	/	/	/	/	/	/	/	/	/	32	/	/	/	/	/
	SSCeiled	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
	BSL	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
	BSLCeiled	/	/	/	/	/	/	/	/	29	/	/	/	/	/	/
	Cap	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
LE	Bin	/	/	/	/	/	/	/	/	/	/	/	/	25	/	/
	SS	/	/	/	/	31	/	/	32	32	32	/	31	31	32	/
	SSCeiled	/	/	/	/	/	/	/	25	/	/	/	/	/	/	/
	BSL	/	/	/	/	26	/	/	26	26	/	/	26	26	26	/
	BSLCeiled	/	/	/	/	/	/	/	/	/	/	/	/	/	/	/
	Cap	/	/	/	/	/	/	/	/	/	/	/	/	/	25	/
ME	Bin	/	/	/	/	/	/	/	/	/	/	/	/	37	/	/
	SS	/	/	/	/	41	/	41	42	42	42	/	42	42	41	/
	SSCeiled	/	/	/	/	/	/	/	/	/	/	/	/	37	/	/
	BSL	/	/	/	/	38	/	/	38	38	38	/	38	/	38	/
	BSLCeiled	/	/	/	/	37	/	/	/	/	/	/	/	/	/	/
	Cap	/	/	/	/	/	/	/	/	/	/	/	/	37	/	/
SE	Bin	/	/	/	/	52	/	52	52	52	52	/	52	52	52	52
	SS	/	/	/	53	53	/	53	53	53	53	/	53	53	53	53
	SSCeiled	/	/	/	/	52	/	52	52	52	52	/	52	52	52	52
	BSL	/	/	/	/	52	/	52	52	52	52	/	52	52	52	52
	BSLCeiled	/	/	/	/	52	/	52	52	52	52	/	52	52	52	52
	Cap	/	/	/	/	52	/	52	52	52	52	/	52	52	52	52

Table 25: Total number of stockouts over 6 pick waves, listed per parameter combination. Results are shown for 100 SKUs, $T = 3$, Demand set = D''.

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