Evaluation of the end-to-end response times in an energy harvesting wireless sensor network using a receiver-initiated MAC protocol

Reference:
Blondia Christian.- Evaluation of the end-to-end response times in an energy harvesting wireless sensor network using a receiver-initiated MAC protocol
Ad hoc networks - ISSN 1570-8713 - 136(2022), 102971
Full text (Publisher's DOI): https://doi.org/10.1016/J.ADHOC.2022.102971
To cite this reference: https://hdl.handle.net/10067/1905570151162165141

Evaluation of the End-to-End Response Times in an Energy Harvesting Wireless Sensor Network using a Receiver-initiated MAC protocol

Chris Blondia
IDLab, University of Antwerp – imec, Antwerp, Belgium
chris.blondia@uantwerpen.be

Abstract

In this paper we propose an analytical model for the evaluation of the end-to-end response time in an energy-harvesting wireless sensor network where the nodes use a receiver-initiated MAC protocol. Each individual node is modeled by means of a finite capacity queue with repeated server vacations of different types. The available energy is modeled by means of an extra variable. The system occupancy in a tagged node together with the available energy are observed at inspection instants, leading to a discrete-time Markov Chain. We derive closed form formulas for the system occupancy distribution at inspection instants and at arbitrary time instants together with the mean response time in this tagged node. The model is applied for the evaluation of the end-to-end response time a packet experiences when crossing different nodes in an energy-harvesting wireless sensor network. The model allows to determine important system parameters, such as the time a node listens to the medium to receive a beacon, the frequency of the transmission of beacons.

Keywords: Energy harvesting, WSN, Receiver-initiated MAC protocol, End-to-end Delay, finite capacity queue, server vacations, Markov Chain

1. Introduction

Wireless Sensor Networks (WSN) are broadly used in application areas, such as home automation, e-health, logistics, mobility, etc. They are composed of devices provided with sensing capabilities, computing power and communication means. The networking capabilities are used to receiving data from other devices and sending this data and its own sensed data to one or more sinks. These devices have important size constraints and are often rolled out in hard-to-reach areas, leading to major energy constraints. Since WSNs are typically deployed in areas where electrical energy is not available, they are usually powered by means of batteries. As WSNs are supposed to have a long operational lifetime, a lot of research has been made to propose energy efficient communication protocols that postpone the energy exhaustion of these batteries (see e.g., [1]). The advent of energy harvesting (EH) makes it possible to avoid the above battery related problems by replenishing the energy by means of external sources (e.g.,

solar power, wind power, water turbulences, etc.). For an overview of energy harvesting in WSNs, we refer to e.g. [2], [3], [4]. Contrary to battery-powered devices, the energy of EH devices does not decrease up to an empty battery, but instead will fluctuate between a minimum and maximum value, depending on the harvesting and consumption rate. In such a device, the energy is harvested from the environment (solar, thermal, wind, hydro, etc.) and stored in a capacitor. During the transmission and the reception of packets or other communication related functions, the radio uses the stored energy. Packets can only be transmitted or received if the device has harvested enough energy and reaches a certain threshold. Below this threshold, the radio is put asleep and energy is harvested. Therefore, in between transmissions or receptions of packets, the radio may switch from an active to a sleep mode, during which energy is harvested, in order to reach the required level to transmit or to receive the next packet. Hence the need for energy-efficient wake-up protocols.
In the energy consumption process of the communication tasks of the devices, the MAC protocol is crucial. The harvested energy should be utilized efficiently, while maximizing the system performance in terms of delay and throughput. Therefore, the MAC protocol should be

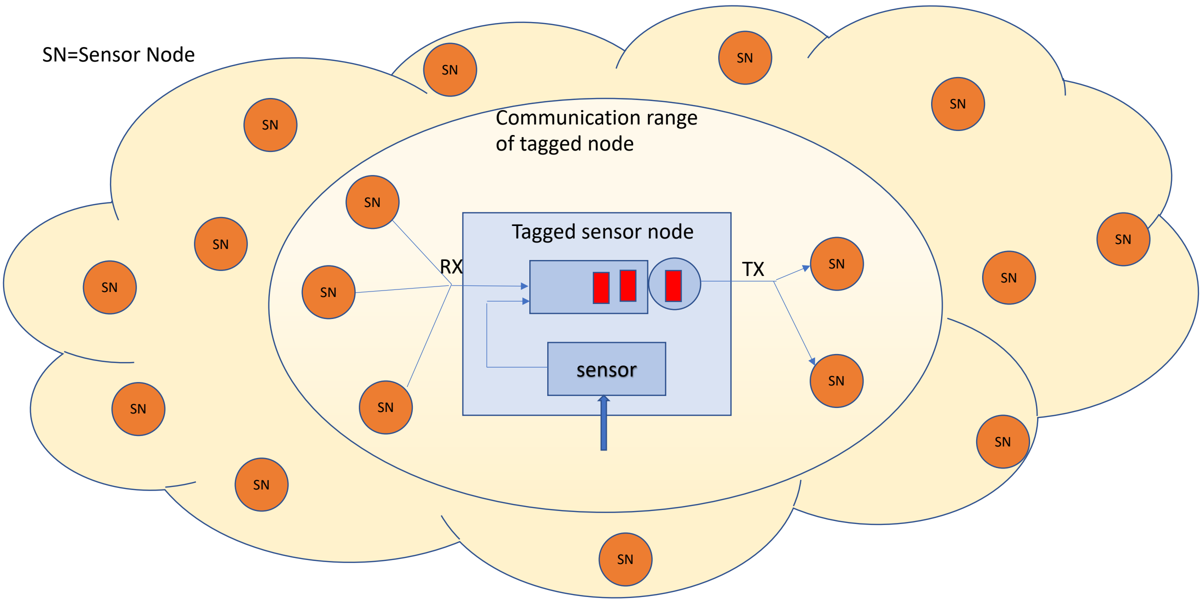

Figure 1: The Wireless Sensor Network

designed taking into account the energy consumption process. Many MAC protocols for energy harvesting WSN have been proposed. A number of papers, such as [5], [6], [7], [8] present an overview of MAC protocols for energy harvesting WSN (EH-WSN). In order to minimize the energy wastage due to idle listening, receiver-initiated MAC protocols have been proposed. Instead of letting a receiver scan the medium to detect the transmission of a packet by a neighbor sender, the receiver will broadcast a beacon, indicating to its neighbors that it is ready to accept an incoming packet. Only neighbors that have queued packets ready for transmission listen to the medium for a beacon.
The aim of this paper is to present an analytical model for an EH-WSN that uses a receiver-initiated MAC protocol. The model allows not only to compute the system occupancy and packet delay in individual nodes, but also determines the e2e delay of a packet to reach the sink of a WSN.

To study the performance of a single node, we tag a sensor node as shown in Figure 1. Packets are received from neighbor nodes (on the left of the tagged node) and stored in a queue. Moreover, the sensor of the tagged node generates packets that are also stored in the queue. Both types of packets are transmitted to neighbor nodes (on the right of the tagged node). The whole transmission process is governed by a receiver-initiated MAC protocol. This means that a packet is transmitted only if one of the candidate destination nodes broadcasts a beacon message, indicating that it is ready to receive a packet. The model of a single node is then used to determine the end-to-end response-time in an EH-WSN.
The main objectives of this paper are:

(i) To propose a model for a node in an EH-WSN that uses a receiver-initiated MAC protocol, taking into account the packets originating from neighboring nodes and packets generated by the sensor of the node itself and transmission opportunities for packets stored in the node's queue
(ii) To apply this model in an EH-WSN network and determine the end-to-end delay to reach the sink
(iii) To investigate the ratio between the energy spent to receive packets and the energy spent to transmit packets. This objective is achieved by determining a strategy for the beacon generation frequency.
(iv) To check if the results depend on used the energy harvesting function.

In Section 3, a more detailed description of the system is given, including the energy harvesting process and the different states the radio can take. Section 4 presents the analytical model for a single node and uses the model to evaluate the average end-to-end response time in a WSN. Numerical results illustrating the possible application of the model and related discussions are given in Section 5. Finally, conclusions are drawn in Section 6.
The contributions of this paper are twofold: the development of a new model to compute the end-to-end response-time in an EH-WSN and the use of the model to investigate the impact of certain system parameters on the end-to-end response-time.

(i) An analytical mode is developed for a node in an EH-WSN that uses a receiver-initiated MAC protocol. The energy consumption and harvesting process is modeled explicitly and is kept track of by means of an extra variable. The interaction with neighbor nodes of the EH-WSN is taken into account using two parameters. These parameters can be computed and allow to determine the end-to-end response-time a packet experiences to reach the sink of the network.
(ii) The model is used to investigate the relationship between the external arrival rate, the beacon frequency, and the end-to-end response-time: when the arrival rate of external packets is higher than a threshold λ_{th}, the beacon generation frequency drastically increases, leading to high delays. This threshold is computable and equal to the external arrival rate such that the system capacity is used to receive these external packets whenever no packets are ready for transmission. It is shown that this property is not due to the energy harvesting function but is related to the MAC protocol and the behavior of the radio. Furthermore, using the model, we show that in order to minimize the end-to-end response-time, the parameters of a node should be determined based on their location with respect to the sink of the network.

2. Related Work

The area of EH-WSNs has received a lot of attention in the literature. From these papers, it is clear that MAC protocols are crucial in finding the means of efficiently use the harvested energy to maximize throughput and minimize delays. Based on the node that takes the initiative to transmit packets, asynchronous MAC protocols can be grouped into two classes: sender-initiated protocols and receiver-initiated protocols. In this paper we focus on receiver-initiated protocols. Examples of other protocols belonging to this class are OD-MAC [9], EH-MAC [10], LEB-MAC [11], ERI-MAC [12], QAEE-MAC [13], ED-MAC [14], PP-MAC [15], a MAC for cloud-based applications [16].

Analytical models for EH-WSN have been proposed in various papers. In [17], sender-initiated and receiver-initiated are compared analytically for EH-WSNs, with respect to energy consumption. Their results suggest that receiver-initiated protocols can be tuned to consume less energy. For delay-sensitive applications in environments where the energy input is sufficient, sender-initiated protocols may provide better performance. [18] integrates energy harvesting in the network calculus framework to stochastically bound the worst-case performance of a single node in a tree-based WSN. [19] uses stochastic network calculus to evaluate systems with a stochastic energy supply, by capturing the dynamics in the energy charging and discharging processes. The queue performance for energy-harvesting cognitive radio sensor networks where the cooperation between the primary user and the sensor is applied, is studied in [20] using a two-dimensional Markov chain. In [21], an analytical model for an EH-WSN node is presented based on a threshold-controlled vacation policy. Vacations are repeated until N packets are accumulated in the queue. [22] presents a Markov model that integrates energy harvesting with the slotted CSMA/CA mechanism of IEEE802.15.4. In [23] a Markovian queueing model is proposed to investigate the impact of uncertainty in the energy harvesting process, the energy expenditure, the data acquisition and the data transmission. A delay analysis for a node in an EH-WSN considering energy costs of sensing and transmission is presented in [24]. A Markov model for the joint energy harvesting and communication analysis is presented in [25]. In this paper the evolution of the available energy is described by means of a continuous-time Markov process. The network traffic is characterized by means of a time-varying Poisson distribution. In [26] an extended Markov fluid model is used to evaluate the energy consumption, the queue length and the packet delay in an EH-WSN with reliable energy backup.

In what follows we will use some of the results obtained in [27] and [28]. In [27] a queueing system is proposed with repeated server vacations and an additional variable that keeps track of the available energy. However, as no input from other nodes in the network is taken into account and no MAC protocol is specified, the model applies for EH-WSN where the nodes are directly connected to the sink. In [28], a queueing model for a class of receiver-initiated MAC protocols for EH-WSN is proposed. The behavior of a node in [28] differs from the one described in the next section, in both the transmission and the reception of packets. In [28] it is assumed that when the node has a packet ready for transmission, it always receives an invitation from a neighbor that is ready to receive a packet (i.e., it always receives a beacon). In the next section we assume that the reception of such a beacon happens with a certain probability. When evaluating the end-to-end response-time, this probability will be determined based on the state of the neighbors. Moreover, in [28] it is assumed that the node issues a beacon only when the queue in the node is empty (i.e., if no packet is ready for transmission). In the next section we assume that a beacon is always generated when the queue is empty but is also generated when the queue is not empty, with a certain probability. This allows an increase of the rate at which beacons are generated. Since the structure of the Markov chain and the way it is solved is similar to [27] and [28], the mathematical derivation of the steady state will therefore be skipped.

Many papers focus only on one or a few of the characteristics of an EH-WSN, namely the energy harvesting and consumption process, the MAC protocol, transmission of packets, reception of packets, but no papers consider an analytical model that deal with all these characteristics simultaneously. Moreover, most papers only consider a single EH-WN node and do not analyse the network performance. Furthermore, papers addressing receiver-initiated MAC protocols mention a periodic generation of beacons, but do not give an analysis of the impact of this periodicity on the system performance. These are exactly the objectives of the research reported in this paper.

3. System Description

Tag a sensor node in a WSN consisting of the following components:

- An *energy harvesting system* and *capacitor* that provides the required energy to transmit and receive packets. We do not describe the realization of the energy harvesting process in detail, but rather characterize its operation by a number of functions that model the energy consumption and harvesting while the node is in operation. For a more detailed description of this harvesting model, we refer to [29]. Moreover, we do not take into account the energy consumption due to other tasks of the sensor node (e.g., measurements, processing of data, etc.).
- A *packet queue* that stores packets that contain data obtained from measurements made by the sensor of the node and packets that are received from other nodes of the WSN; both types of packets need to be transmitted to a neighbor node of the WSN further down to the sink.
- A *radio* that transmits and receives packets using the energy available in the capacitor and according to a receiver-initiated MAC protocol. A detailed description of the different states the radio can take is described in what follows.

Data packets may arrive at the node from two different origins. First the sensor gathers data itself and the resulting packets need to be transmitted by the radio to a neighbor node further down to the sink. We refer to these packets as *internal packets*. Since the node belongs to a WSN, it will also receive packets from other neighbor nodes as well, that also need to be transmitted by the tagged node to a neighbor further down to the sink. We refer to these packets as *external packets*. As will be described in detail later, the arrival of external packets is under control of the tagged node, since we focus on receiver-initiated MAC protocols.

3.1. The Energy Harvesting Function

In order to model the energy harvesting and consumption, we use a function that has been proposed in [29] and used in [27] and [28]. Remark however that this is to be considered as an example and that other functions will be considered in Section 5.2.

Assume that the device is in mode X, (where X may by SLEEP, TRANSMIT, RECEIVE, LISTEN or MAC) during an interval T. The evolution in time of the available energy during the interval T depends on this mode X and the available energy at time t=0. More formally, it is described by the following function:

$$F_X(i_0,t) = c(X) \cdot \left(1 - e^{-\frac{t}{a(X)}}\right) + i_0 \cdot e^{-\frac{t}{a(X)}}, \qquad t \in T \tag{1}$$

where $c(X)$ and $a(X)$ are parameters depending on the state X, and i_0 is the energy available at time $t = 0$ and t is a time instant of the interval T.

Depending on the device (capacitor, harvesting function) and the mode X, the slope of F_X at time t may be positive, zero or negative, depending on the values of the parameters $c(X)$ and $a(X)$. Clearly the function F_{SLEEP} is increasing while in the other modes the function F_X is decreasing. Remark that if the device remains in mode X, then the energy level tends to $c(X)$ for increasing t.
A more detailed description of this harvesting system and the corresponding circuitry to realize it can be found in [29].

3.2. States of the Radio

In what follows we describe the operation of the radio of the tagged node and the associated energy consumption. Assume that the energy in the capacitor takes discrete integer values $1, \cdots, i_{max}$. This assumption is made for modeling purposes in Section 4. The radio will inspect the content of the queue and the available energy at well-defined instants, called *inspection instants*. The radio can be in different modes: SLEEP, during which energy is harvested and the following states, during which energy is consumed: LISTEN (monitor the medium), MAC (time needed to access the medium), RX (receive a data packet or beacon) and TX (transmit a data packet or beacon). The energy evolution during each of these modes is governed by a different function. Let $F_X(i, T)$, where X takes values SLEEP, LISTEN, MAC, RX and TX, be the available energy after T time units if at time instant 0 the energy level was i, $1 \leq i \leq i_{max}$, and the radio was in mode X and remained in mode X during the time interval $[0\ T]$.
The radio can be in four different states (see Figure 2), each state defined by means of a number of modes (SLEEP, LISTEN, MAC, RX, TX).

1. Transmit state
The radio switches to this state with a probability p_t, if at an inspection instant the queue in the tagged node is not empty. The transmit state consists of different activities, referred to as modes above. In order to execute these functions without depletion of the capacitor, the available energy at the start of the Transmit state should be at least i_{thtx}. This energy threshold will be computed later.
Assume that at the start of the Transmit state, the available energy equals i_0. If $i_0 < i_{thtx}$, then the node will first switch to the SLEEP mode (during which energy is harvested) until i_{thtx} is reached. This time can be computed by applying the function I_{SLEEP} defined using the function $F_{SLEEP}(i, T)$:

$$I_{SLEEP}(i_0, i_{thtx}) = T \text{ iff } F_{SLEEP}(i_0, T) = i_{thtx} \quad (2)$$

At the moment the threshold is reached the tagged node switches to the LISTEN mode for a certain time $T_{LISTENTX}$.

1.a Transmit state with packet transmission
We assume that during this LISTEN mode a beacon is received with probability β. Remark that β will be used to model the interaction with neighbor nodes towards the sink. The case where no beacon is received will be treated 1.b Transmit state without packet reception. After the reception of the beacon, the radio switches to the MAC mode, during which the access to the medium is obtained. We do not detail this MAC mode operation. After this mode the data packet is transmitted, referred to as TX mode. If the duration of the different modes is known, applying the functions $F_X(i, T)$ for the various values of X, leads to the total duration of the transmit state and also the available energy at the end of this state (i.e. at the next inspection instant). Let $T_{LISTENTX}, T_B, T_{MAC}, T_P$ be the time the radio remains in LISTEN mode,

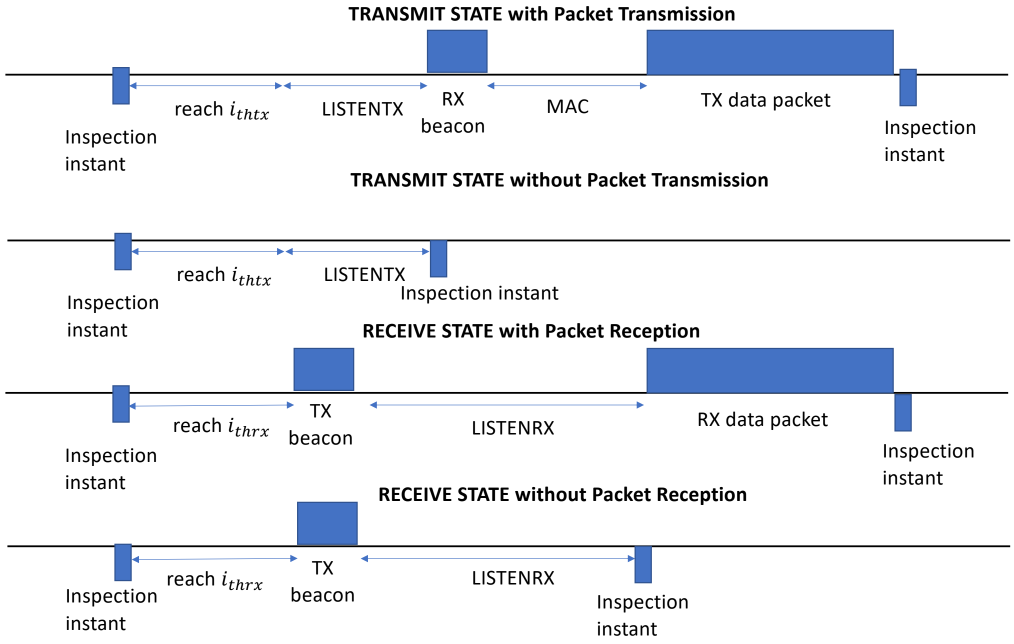

Figure 2: States of the radio of the tagged sensor node

RX beacon mode, MAC mode and TX or RX of a packet mode respectively. Then the energy left after the transmission of the packet in case $i_0 < i_{th}$, is given by

$$F_{TX}(F_{MAC}(F_{RX}(F_{LISTEN}(i_{thtx}, T_{LISTENTX}), T_B), T_{MAC}), T_P) \quad (3)$$

and the total duration of the transmit state equals

$$I_{SLEEP}(i_0, i_{thtx}) + T_{LISTENTX} + T_B + T_{MAC} + T_P \quad (4)$$

If $i_0 \geq i_{thtx}$, the SLEEP mode can be skipped since there is enough energy available to successfully transmit a packet. The energy left after the transmission of the packet is given by

$$F_{TX}(F_{MAC}(F_{RX}(F_{LISTEN}(i_0, T_{LISTENTX}), T_B), T_{MAC}), T_P) \quad (5)$$

and the total duration of the transmit state equals

$$T_{LISTENTX} + T_B + T_{MAC} + T_P \quad (6)$$

1.b Transmit state without packet transmission

Assume that at the end of the LISTEN mode no beacon is received. This happens with probability $1 - \beta$. The energy left at the end of this state if $i_0 < i_{thtx}$, is given by

$$F_{LISTEN}(i_{thtx}, T_{LISTEN}) \quad (7)$$

and the total duration of the transmit state equals

$$I_{SLEEP}(i_0, i_{thtx}) + T_{LISTENTX} \tag{8}$$

If $i_0 \geq i_{th}$, the SLEEP mode can be skipped since there is enough energy available to successfully transmit a packet. The energy left after the transmission of the packet is given by

$$F_{LISTEN}(i_0, T_{LISTENTX}) \tag{9}$$

and the total duration of the transmit state equals $T_{LISTENTX}$.

2. Receive state

The radio switches to the Receive state in two cases: with probability 1 if the queue is empty and with probability $p_r = 1 - p_t$ when the queue is not empty. The receive state consists of different functions. In order to execute these functions, the available energy should be at least a threshold i_{thrx}, computed in a similar way as i_{thtx}.

Let at an inspection instant the available energy be i_0. First assume that $i_0 < i_{thrx}$. Similar to what happened in the Transmit state, the node will first switch to the SLEEP mode (during which energy is harvested) until i_{thrx} is reached. This time is again computed using the function I_{SLEEP}. Then a TX beacon is transmitted, informing its neighbors (potential sensor nodes that have packets ready for transmission to our tagged node) that the node is ready to receive a packet. Then the node goes into LISTEN mode.

2.a Receive state with packet reception

Upon reception of the TX beacon, a neighbor node has a packet ready for transmission to the tagged node with probability α. Remark that α will be used to model the interaction with neighbor nodes that reach the sink via the tagged node. Then at the end of the LISTEN mode, the tagged node switches to the RX mode and receives a packet from that neighbor node. If the queue is not full, this packet is stored in the queue and will be scheduled for transmission later on.

If the duration of the different modes is known, applying the functions $F_X(i, T)$ for the various values of *X*, leads to the total duration of the Receive state with packet reception and also the available energy at the end of this state (i.e., at the next inspection instant). If at the start of this state $i_0 < i_{thrx}$, the energy left after the reception of the packet is given by

$$F_{RX}(F_{LISTEN}(F_{TX}(i_{thrx}, T_B), T_{LISTENRX}), T_P) \tag{10}$$

and the total duration of the receive state equals

$$I_{SLEEP}(i_0, i_{thrx}) + T_B + T_{LISTENRX} + T_P \tag{11}$$

If $i_0 \geq i_{thrx}$, the SLEEP mode is skipped and the energy left after the reception of the packet is given by

$$F_{RX}(F_{LISTEN}(F_{TX}(i_0, T_B), T_{LISTENRX}), T_P) \tag{12}$$

and the total duration of the receive state equals

$$T_B + T_{LISTENRX} + T_P \tag{13}$$

2.b Receive state without packet reception
Assume that with probability $1-\alpha$ no neighbor node has a packet ready for transmission, the tagged node does not receive a packet at the end of the LISTEN mode.
If $i_0 < i_{thrx}$ the energy left at the end of this state is given by

$$F_{LISTEN}(F_{TX}(i_{thrx}, T_B), T_{LISTENRX}) \quad (14)$$

and the total duration of the receive state equals

$$I_{SLEEP}(i_0, i_{thrx}) + T_B + T_{LISTENRX} \quad (15)$$

If $i_0 \geq i_{th}$, then the energy left is given by

$$F_{LISTEN}(F_{TX}(i_0, T_B), T_{LISTENRX}) \quad (16)$$

and the total duration of the receive state equals

$$T_B + T_{LISTENRX} \quad (17)$$

In what follows we compute the threshold value i_{thtx}. First, let i_{min} be a value of the energy level, $1 \leq i_{min} < i_{max}$, a node should never drop below. Then i_{thtx} should be such that when executing a complete Transmit state, at the end, the energy level should at least be i_{min}. To obtain the value of i_{thtx}, we use the inverse function of $F_X(i,T)$, defined as

$$F_X^{-1}(i,T) = i_0 \text{ iff } F_X(i_0,T) = i \quad (18)$$

Then

$$i_{thtx} = F_{LISTEN}^{-1}(F_{RX}^{-1}(F_{MAC}^{-1}(F_{TX}^{-1}(i_{min}, T_P), T_{MAC}), T_B), T_{LISTENTX}) \quad (19)$$

The threshold value i_{thrx} can be computed in the same way.

Remarks

(i) From the above description of the states of the radio, it is clear that the available energy is fully used as no unnecessary SLEEP periods are introduced.

(ii) The description of the system uses three parameters: α, β, p_t (or $p_r = 1 - p_t$). The parameter α is introduced to model the interaction with neighbors that are transmitting packets to the tagged node (i.e., nodes that are located left from the tagged node in Figure 1). The parameter β is introduced to model the interaction with neighbors that are potential receivers of packets transmitted by the tagged node (i.e., nodes that are located right from the tagged node in Figure 1). The parameters α and β will be used in Section 4.4. when the end-to-end response-time in a WSN is determined. Finally, the parameter p_t (or p_r) is introduced to control the frequency by which beacons are generated by the tagged node.

4. System Model

4.1. The Queueing Model for a Single Node

We model the tagged node described in the previous section by means of a finite capacity queueing system with three types of server vacations.
Internal packets arrive according to a Poisson process with rate λ. The tagged node may hold at most N packets (including the packet being transmitted). The node applies a drop-tail admission strategy, i.e., packets arriving at a full queue are dropped. The energy level is modeled by means of a variable I taking values $1, \cdots, i_{max}$. The way I evolves in time will be defined later on by a number of functions based on the functions $F_X(i,T)$.
Packets are transmitted in a FIFO order. If at an inspection instant the queue in the tagged node is not empty, then with a probability p_t, the first packet in the queue will be selected for transmission. The actual transmission of the packet takes place if a beacon is received, and this happens with probability β. The total duration of the transmission of a packet is a random variable that depends on the value i of I at its start and is denoted by S_i, with distribution $S_i(t)$ and Laplace Transform (LST) $S_i^*(\vartheta)$. If a service time starts with $I = i$ and has length S_i, then the value of I at the end of the service is given by $F(i, S_i)$. Remark that S_i models the length of the complete Transmit state with packet transmission and not only the transmission of the packet itself. $F(i, S_i)$ can be computed using the functions $F_X(i,T)$, as shown in the previous Section. With probability $1 - \beta$ no beacon is received, and the state ends after the LISTEN mode. The total duration of the transmission without transmission of a packet is a random variable that depends on the value i of I at its start and is denoted by W_i, with distribution $W_i(t)$ and Laplace Transform $W_i^*(\vartheta)$. If a Transmit state starts with $I = i$ and has length W_i, then the value of I at the end of the Transmit state is given by $G(i, W_i)$. Again $G(i, W_i)$ can be computed using the functions $F_X(i,T)$, as shown in the previous Section.
If at an inspection instant the queue in the tagged node is not empty, then with a probability $p_r = 1 - p_t$, a switch to the Receive state is made. With a probability α a neighbor node will transmit a packet that is received by the tagged node and with a probability 1-α no neighbor node has a packet ready for transmission. If a Receive state with packet reception starts with $I = i$, then its length is a random variable denoted by R_i, with distribution $R_i(t)$ and LST $R_i^*(\vartheta)$. At the end of this Receive State the value of I is given by $H(i, R_i)$. For a Receive State without packet reception its length is denoted by V_i, with distribution $V_i(t)$ and LST $V_i^*(\vartheta)$. At the end of this Receive State the value of I is given by $K(i, V_i)$. Both $H(i, R_i)$ and $K(i, V_i)$ can be computed using the functions $F_X(i,T)$. If at an inspection instant the queue in the tagged node is empty, then a switch to the Receive state is always made. Again, with a probability α a neighbor node will transmit a packet that is received by the tagged node and with a probability 1-α no neighbor node has a packet ready for transmission. The lengths of the Receive states and the energy at the end of this state are defined as in case the queue is not empty.
Remark that from a queuing theoretical point of view, the Transmit state without packet transmission and both Receive states can be seen as a server vacation while the Transmit state with packet transmission can be seen as a customer being served. The Transmit state without packet reception is referred to as a vacation of type 1, while the Receive state with packet reception is referred to as a vacation of type 2 and the Receive state without packet reception is called a vacation of type 3.

4.2. The Embedded Markov Chain

It is clear that given the system occupancy in the tagged node and the energy level I at an inspection instant, it is possible to decide whether the time interval till the next inspection instant will be a service, a vacation of type 1, a vacation of type 2 or a vacation of type 3.

Moreover, the system occupancy in the tagged node and the energy level I at the next inspection instant may be computed using the functions F, G, H and K. This leads to the definition of the following discrete-time Markov chain embedded at inspection instants.
We consider the stochastic process (Q_n, I_n) at those inspection instants t_n, where Q_n is the number of packets in the tagged node at time instant t_n and I_n is the energy capacity available in the tagged node at time instant t_n. It is easy to check that this stochastic process is a discrete-time finite Markov Chain with state space size equal to $(N+1)\cdot i_{max}$. We denote the limiting probability distribution vector by

$$\overline{\boldsymbol{p}} = (\overline{\boldsymbol{p}}_{\boldsymbol{0}}, \overline{\boldsymbol{p}}_{\boldsymbol{1}}, \dots, \overline{\boldsymbol{p}}_{\boldsymbol{N-1}}, \overline{\boldsymbol{p}}_{\boldsymbol{N}}) \tag{20}$$

where the vectors $\overline{\boldsymbol{p}}_{\boldsymbol{k}}$, $k = 0,1,\cdots,N$, of size i_{max}, are given by

$$\left(\overline{\boldsymbol{p}}_{\boldsymbol{k}}\right)_i = \lim_{n\to\infty} Prob\{Q_n = k, I_n = i\}, 0 \le k \le N,\ i = 1, \dots, i_{max} \tag{21}$$

Let, $f_{n,i}$, $g_{n,i}$, $h_{n,i}$ and $k_{n,i}$ respectively, be the probability that n packets arrive during a service with length S_i, a vacation of type 1 with length W_i, a vacation of type 2 with length R_i or a vacation of type 3 with length V_i that starts with $I = i$. Then clearly

$$f_{n,i} = \int_0^\infty \frac{(\lambda t)^n}{n!} e^{-\lambda t} dS_i(t) \tag{22}$$

$$g_{n,i} = \int_0^\infty \frac{(\lambda t)^n}{n!} e^{-\lambda t} dW_i(t) \tag{23}$$

$$h_{n,i} = \int_0^\infty \frac{(\lambda t)^n}{n!} e^{-\lambda t} dR_i(t) \tag{24}$$

$$k_{n,i} = \int_0^\infty \frac{(\lambda t)^n}{n!} e^{-\lambda t} dV_i(t) \tag{25}$$

In what follows we denote the column vectors of the LSTs of the length of the different intervals by $\overline{\boldsymbol{S}}^*(\theta)$, $\overline{\boldsymbol{W}}^*(\theta)$, $\overline{\boldsymbol{R}}^*(\theta)$ and $\overline{\boldsymbol{V}}^*(\theta)$.

$$\overline{\boldsymbol{S}}^*(\theta) = \left(S_1^*(\vartheta), \cdots, S_{i_{max}}^*(\vartheta)\right)' \tag{26}$$
$$\overline{\boldsymbol{W}}^*(\theta) = (W_1^*(\vartheta), \cdots, W_{i_{max}}^*(\vartheta)) \tag{27}$$
$$\overline{\boldsymbol{R}}^*(\theta) = (R_1^*(\vartheta), \cdots, R_{i_{max}}^*(\vartheta))' \tag{28}$$
$$\overline{\boldsymbol{V}}^*(\theta) = (V_1^*(\vartheta), \cdots, V_{i_{max}}^*(\vartheta))' \tag{29}$$

and the column vectors of the averages of the intervals by

$$\overline{\boldsymbol{E}[\boldsymbol{S}]} = (E[S_1], \cdots, E[S_{i_{max}}])' \tag{30}$$
$$\overline{\boldsymbol{E}[\boldsymbol{W}]} = (E[W_1], \cdots, E[W_{i_{max}}])' \tag{31}$$
$$\overline{\boldsymbol{E}[\boldsymbol{R}]} = (E[R_1], \cdots, E[R_{i_{max}}])' \tag{32}$$
$$\overline{\boldsymbol{E}[\boldsymbol{V}]} = (E[V_1], \cdots, E[V_{i_{max}}])' \tag{33}$$

We introduce the following $i_{max} \cdot i_{max}$ matrices

$$(\boldsymbol{F_n})_{i,j} = \begin{cases} f_{n,i}, & j = F(i, S_i), i = 1, \cdots, i_{max} \\ 0, & \text{elsewhere} \end{cases} \tag{34}$$

$$(\boldsymbol{G_n})_{i,j} = \begin{cases} g_{n,i}, & j = G(i, W_i), i = 1, \cdots, i_{max} \\ 0, & \text{elsewhere} \end{cases} \tag{35}$$

$$(\boldsymbol{H_n})_{i,j} = \begin{cases} h_{n,i}, & j = H(i, R_i), i = 1, \cdots, i_{max} \\ 0, & \text{elsewhere} \end{cases} \tag{36}$$

$$(\boldsymbol{K_n})_{i,j} = \begin{cases} k_{n,i}, & j = K(i, V_i), i = 1, \cdots, i_{max} \\ 0, & \text{elsewhere} \end{cases} \tag{37}$$

$$\boldsymbol{A_0} = p_t \cdot \beta \cdot \boldsymbol{F_0} \tag{38}$$

$$\boldsymbol{A_1} = p_t \cdot \beta \cdot \boldsymbol{F_1} + p_t \cdot (1-\beta) \cdot \boldsymbol{G_0} + p_r \cdot (1-\alpha) \cdot \boldsymbol{K_0} \tag{39}$$

$$\boldsymbol{A_n} = p_t \cdot \beta \cdot \boldsymbol{F_n} + p_t \cdot (1-\beta) \cdot \boldsymbol{G_{n-1}} + p_r \cdot [\alpha \cdot \boldsymbol{H_{n-2}} + (1-\alpha) \cdot \boldsymbol{K_{n-1}}], \quad 2 \le n \le N-2 \tag{40}$$

$$\boldsymbol{B_0} = (1-\alpha) \cdot \boldsymbol{K_0} \tag{41}$$

$$\boldsymbol{B_n} = \alpha \cdot \boldsymbol{H_{n-1}} + (1-\alpha) \cdot \boldsymbol{K_n}, 1 \le n \le N-1 \tag{42}$$

$$\boldsymbol{A'_0} = p_t \cdot \beta \cdot \sum_{k=0}^{\infty} \boldsymbol{F_k} \tag{43}$$

$$\boldsymbol{A'_1} = p_t \cdot \beta \cdot \sum_{k=1}^{\infty} \boldsymbol{F_k} + p_t \cdot (1-\beta) \cdot \boldsymbol{G_0} + p_r \cdot (1-\alpha) \cdot \boldsymbol{K_0} \tag{45}$$

$$\boldsymbol{A'_n} = p_t \cdot \beta \cdot \sum_{k=n}^{\infty} \boldsymbol{F_k} + p_t \cdot (1-\beta) \cdot \boldsymbol{G_{n-1}} + p_r \cdot [\alpha \cdot \boldsymbol{H_{n-2}} + (1-\alpha) \cdot \boldsymbol{K_{n-1}}], \quad 2 \le n \le N-1 \tag{46}$$

$$\boldsymbol{A''_1} = p_t \cdot (1-\beta) \cdot \sum_{k=0}^{\infty} \boldsymbol{G_k} + p_r \cdot \left[\alpha \cdot \sum_{k=0}^{\infty} \boldsymbol{H_k} + (1-\alpha) \cdot \sum_{k=0}^{\infty} \boldsymbol{K_k}\right] \tag{47}$$

$$\boldsymbol{A''_n} = p_t \cdot (1-\beta) \cdot \sum_{k=n-1}^{\infty} \boldsymbol{G_k} + p_r \cdot \left[\alpha \cdot \sum_{k=n-2}^{\infty} \boldsymbol{H_k} + (1-\alpha) \cdot \sum_{k=n-1}^{\infty} \boldsymbol{K_k}\right], \quad 2 \le n \le N \tag{48}$$

The matrices $\boldsymbol{A_n}$ define the transitions from states with a non-empty queue, while the matrices $\boldsymbol{B_n}$ define the transitions from states with an empty queue in the tagged node.
Then the transition matrix of this Markov Chain is given by

$$P = \begin{pmatrix} B_0 & B_1 & B_2 & \cdots & B_{N-2} & B_{N-1} & \sum_{n=N}^{\infty} B_n \\ A_0 & A_1 & A_2 & \cdots & A_{N-2} & A'_{N-1} & A''_N \\ 0 & A_0 & A_1 & \cdots & A_{N-3} & A'_{N-2} & A''_{N-1} \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & A_0 & A'_1 & A''_2 \\ 0 & 0 & 0 & \cdots & 0 & A'_0 & A''_1 \end{pmatrix} \quad (49)$$

The distribution vector $\overline{\boldsymbol{p}}$ satisfies the following equations

$$\overline{\boldsymbol{p}} = \overline{\boldsymbol{p}} \cdot \boldsymbol{P} \quad (50)$$
$$\overline{\boldsymbol{p}} \cdot \overline{\boldsymbol{e}} = 1 \quad (51)$$

where $\overline{\boldsymbol{e}}$ is the $(N+1) \cdot i_{max}$ unit column vector.
The system occupancy distribution of the tagged node at inspection epochs is then given by

$$\nu_n = \overline{\boldsymbol{p}}_{\boldsymbol{n}} \cdot \overline{\boldsymbol{e}}, \qquad 0 \leq n \leq N \quad (52)$$

4.3. System Occupancy Probability at Arbitrary Instants

Once the probability vector $\overline{\boldsymbol{p}}$ is determined, it is possible to compute the steady state probability of the system occupancy at arbitrary time instants.
Let

$$D = \sum_{j=1}^{i_{max}} \{ \sum_{n=1}^{N} p_t \cdot \beta \cdot (\overline{\boldsymbol{p}}_{\boldsymbol{n}})_j \cdot E[S_j] + \sum_{n=1}^{N} p_t \cdot (1-\beta) \cdot (\overline{\boldsymbol{p}}_{\boldsymbol{n}})_j \cdot E[W_j] + [\sum_{n=1}^{N} p_r \cdot \alpha \cdot (\overline{\boldsymbol{p}}_{\boldsymbol{n}})_i]$$
$$+ \alpha \cdot (\overline{\boldsymbol{p}}_{\boldsymbol{0}})_i] \cdot E[R_i] + [\sum_{n=1}^{N} p_r \cdot (1-\alpha) \cdot (\overline{\boldsymbol{p}}_{\boldsymbol{n}})_i + (1-\alpha) \cdot (\overline{\boldsymbol{p}}_{\boldsymbol{0}})_i \cdot E[V_i]\} \quad (53)$$

Remark that D is nothing else but the average time between two inspection instants.

Using similar mathematical computations as in [27] and [28], it is possible to show that the distribution of the number of packets in the tagged node at an arbitrary time instant is given by

$$\eta_0 = \frac{1}{\lambda \cdot D} \cdot [\overline{\boldsymbol{p}}_{\boldsymbol{0}} \cdot \overline{\boldsymbol{e}} - \overline{\boldsymbol{p}}_{\boldsymbol{0}} \cdot (\boldsymbol{B}_{\boldsymbol{0}} + \alpha \cdot \boldsymbol{H}_{\boldsymbol{0}}) \cdot \overline{\boldsymbol{e}}] \quad (54)$$

$$\eta_n = \frac{1}{\lambda \cdot D} \cdot \{\overline{\boldsymbol{p}}_{\boldsymbol{n}} \cdot \overline{\boldsymbol{e}} - \sum_{k=1}^{n} \overline{\boldsymbol{p}}_{\boldsymbol{k}} \cdot [p_r \cdot \boldsymbol{B}_{\boldsymbol{n-k}} + p_t \cdot (1-\beta) \cdot \boldsymbol{G}_{\boldsymbol{n-k}} + p_r \cdot \alpha \cdot \boldsymbol{H}_{\boldsymbol{n-k}}] \cdot \overline{\boldsymbol{e}} - \overline{\boldsymbol{p}}_{\boldsymbol{0}} \cdot (\boldsymbol{B}_{\boldsymbol{n}} + \alpha \cdot \boldsymbol{H}_{\boldsymbol{n}}) \cdot \overline{\boldsymbol{e}}\}, n = 1, \dots, N-1 \quad (55)$$

$$\eta_N = 1 - \sum_{n=0}^{N-1} \eta_n \quad (56)$$

Remark that if no packets are received from neighbors, i.e., when $p_r = 0$ and $\alpha = 0$, and if the node always receives a beacon when a packet is ready for transmission, i.e., $\beta = 1$, then this result reduces to Formula (29) of [27].
The average system occupancy is given by

$$E[\eta] = \sum_{n=1}^{N} n \cdot \eta_n \quad (57)$$

The probability that an internal packet finds the system full upon arrival (and hence is dropped) is given by

$$P_{b,i} = \eta_N \quad (58)$$

The probability of having an interval during which a packet will be received is given by

$$P_{rec} = \alpha \cdot (p_r \cdot \sum_{k=0}^{N} \overline{\boldsymbol{p}}_{\boldsymbol{k}} + \overline{\boldsymbol{p}}_{\boldsymbol{0}}) \cdot \overline{\boldsymbol{e}} = \alpha \cdot [p_r \cdot (1 - \overline{\boldsymbol{p}}_{\boldsymbol{0}} \cdot \overline{\boldsymbol{e}}) + \overline{\boldsymbol{p}}_{\boldsymbol{0}} \cdot \overline{\boldsymbol{e}}] \quad (59)$$

Hence, the external packet arrival rate at the tagged node is given by

$$\lambda_e = \frac{\alpha}{D} \cdot [p_r \cdot (1 - \overline{\boldsymbol{p}}_{\boldsymbol{0}} \cdot \overline{\boldsymbol{e}}) + \overline{\boldsymbol{p}}_{\boldsymbol{0}} \cdot \overline{\boldsymbol{e}}] \quad (60)$$

and the probability that an external packet finds the system full (and hence is dropped) is given by

$$P_{b,e} = \{[p_r \cdot \sum_{k=0}^{N} \overline{\boldsymbol{p}}_{\boldsymbol{k}} \cdot \sum_{l=N-k}^{\infty} \boldsymbol{K}_l + \overline{\boldsymbol{p}}_{\boldsymbol{0}} \cdot \sum_{l=N}^{\infty} \boldsymbol{K}_l] \cdot \overline{\boldsymbol{e}}\} / [p_r \cdot (1 - \overline{\boldsymbol{p}}_{\boldsymbol{0}} \cdot \overline{\boldsymbol{e}}) + \overline{\boldsymbol{p}}_{\boldsymbol{0}} \cdot \overline{\boldsymbol{e}}] \quad (61)$$

The actual packet arrival rate is given by

$$\lambda_a = (1 - P_{bi}) \cdot \lambda + (1 - P_{be}) \cdot \lambda_e \quad (62)$$

Using Little's result, the average response time is given by

$$E[R] = \frac{1}{\lambda_a} \cdot E[\eta] \quad (63)$$

4.4. The Average End-to-End Response Time in a Wireless Sensor Network

Now we evaluate the end-to-end average response time in a WSN that uses the receiver-initiated MAC protocol described and evaluated in the previous sections. Consider a network as depicted in Figure 3. Assume that the network consists of M rows. Packets are sent towards the sink in the following way. Tag a node in the network. This node receives packets from l neighbor nodes. These nodes are situated in the row above the tagged node (In Figure 4 we assume $l = 3$). The packets originating from the nodes on the row above are called external packets from the point of view of the tagged node. Apart from these l packet input streams, the node itself generates packets (referred to as internal packets in Section 3) with a rate λ. The tagged node will forward all received packets (internal as well as external) to l neighbor nodes, situated in the row below the tagged node in Figure 4 (again we assume $l = 3$). This process continues until the sink is reached (below row M).
In what follows, we use the super-index $.^{(m)}$ for parameters related to a node belonging to row m.

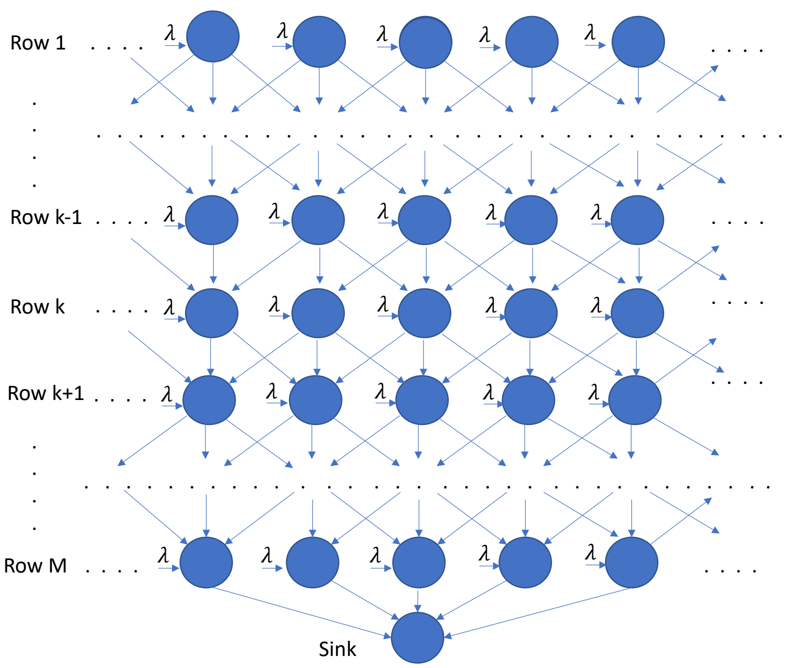

Figure 3: The Wireless Sensor Network

We assume that the traffic originating from each of the l nodes in the row above our tagged node is transmitted to l nodes in the row of our tagged node in an equal way. The same applies for the transmission of packets to the l nodes in the row below our tagged node.
Hence, from a modeling point of view, it is as if all external packets arriving in our tagged node originate from a single node of the row above, and that all packets transmitted by the tagged node have destination a single node in the row below.
In the single node model, we have assumed that the parameters α and β are known. However, since the EH-WSN uses a receiver-initiated MAC protocol, the values of these parameters depend on parameters of nodes belonging to higher rows (for β) as well as to lower rows (for α). Let us first determine these values. Tag a node in row m, $1 \leq m \leq M$.

Determination of parameter β

When at an inspection instant the queue in the tagged node is not empty, the radio switches to the Transmit state with probability p_t. This state ends with the transmission of a packet provided the tagged node has received a beacon from a node in row $(m+1)$, which happens with probability $\beta^{(m)}$. The value of the parameter $\beta^{(m)}$ will depend on the frequency of the generation of beacons in nodes of row $m+1$ and the duration of the LISTEN mode of our tagged node when in Transmit state (i.e., $LISTENTX^{(m)}$). The probability that in a node of row $m+1$ an inspection instant is the start of an interval where a beacon is generated is given by

$$P_{bc}^{(m+1)} = p_r^{(m+1)} \cdot (1 - \overline{\boldsymbol{p}}_{\boldsymbol{0}}^{(\boldsymbol{m+1})} \cdot \overline{\boldsymbol{e}}) + \overline{\boldsymbol{p}}_{\boldsymbol{0}}^{(\boldsymbol{m+1})} \cdot \overline{\boldsymbol{e}} \tag{64}$$

Assume that the time between the generation of two consecutive beacons is exponentially distributed with mean $\mu^{(m+1)} = \frac{D^{(m+1)}}{P_{bc}^{(m+1)}}$. Since the residual life-time of a variable with an exponential distribution is also exponentially distributed with the same mean, and assuming that our tagged node starts to listen at an arbitrary time instant for a duration of $LISTENTX^{(m)}$, we obtain

$$\beta^{(m)} = 1 - e^{-\frac{LISTENTX^{(m)}}{\mu^{(m+1)}}} \tag{65}$$

Remark that since the nodes of row M are directly connected to the sink, no reception of a beacon is needed for a node of row M to send a packet to the sink, and hence

$$LISTENTX^{(M)} = 0\,,\;\; \beta^{(M)} = 1 \tag{66}$$

Determination of parameter α

The parameters α and p_r are closely related since they both are concerned with the reception of packets originating from a node of row $m-1$. Suppose that there is very little or no packet loss. Then the total rate of packets originating from row $m-1$ is given by $\lambda_e^{(m)} = (m-1) \cdot \lambda$.

First, we determine $\lambda_{th}^{(m)}$, the value of $\lambda_e^{(m)}$ such that all packets originating from row $m-1$ can be received when the tagged node is in a Receive state that started with an empty queue. In other words, the frequency by which the queue is empty is high enough to let the node in row $m-1$ send all its packets.

For this value of $\lambda_e^{(m)}$ the probability p_r can be chosen 0 (or equivalently $p_t = 1$), resulting in no waste of energy when the queue is not empty, and the radio executes the Transmit state. Moreover, for $\lambda_e^{(m)} = \lambda_{th}^{(m)}$, we have that $\alpha^{(m)} = 1$, since there is in average always a packet ready for transmission in row $m-1$ to the tagged node. Let us now compute $\lambda_{th}^{(m)}$.

Since the probability that an inspection instant is the start of an interval where a packet is received is given by

$$P_{rec}^{(m)} = \alpha^{(m)}[p_r \cdot (1 - \overline{\boldsymbol{p}}_{\boldsymbol{0}}^{(\boldsymbol{m})} \cdot \overline{\boldsymbol{e}}) + \overline{\boldsymbol{p}}_{\boldsymbol{0}}^{(\boldsymbol{m})} \cdot \overline{\boldsymbol{e}}] \tag{67}$$

the condition that the rate of received packets in the tagged node of row m equals the total rate of packets originating from row $m-1$ is

$$\lambda_e^{(m)} = \frac{\alpha^{(m)}[p_r \cdot (1 - \overline{\boldsymbol{p}}_{\boldsymbol{0}}^{(\boldsymbol{m})} \cdot \overline{\boldsymbol{e}}) + \overline{\boldsymbol{p}}_{\boldsymbol{0}}^{(\boldsymbol{m})} \cdot \overline{\boldsymbol{e}}]}{D^{(m)}} \tag{68}$$

Since for the threshold $\lambda_{th}^{(m)}$, $p_r = 0$ and $\alpha = 1$, we obtain

$$\lambda_{th}^{(m)} = \frac{\overline{\boldsymbol{p}}_{\boldsymbol{0}}^{(\boldsymbol{m})} \cdot \overline{\boldsymbol{e}}}{D^{(m)}} \tag{69}$$

Remark that the value of $\lambda_{th}^{(m)}$is independent from $\lambda_e^{(m)}$.
Now assume that $\lambda_e^{(m)} \leq \lambda_{th}^{(m)}$, then $p_r = 0$, since the packets from row $m-1$ can be sent when the queue in our tagged node is empty. Then according to (68), we have that

$$\alpha^{(m)} = \frac{\lambda_e^{(m)} \cdot D^{(m)}}{\overline{\boldsymbol{p}}_{\boldsymbol{0}}^{(\boldsymbol{m})} \cdot \overline{\boldsymbol{e}}} \tag{70}$$

Since the values of $D^{(m)}$ and $\overline{\boldsymbol{p}}_{\boldsymbol{0}}^{(\boldsymbol{m})}$ depend on $\alpha^{(m)}$ itself, the value of $\alpha^{(m)}$ is to be obtained iteratively.
If on the other hand $\lambda_e^{(m)} > \lambda_{th}^{(m)}$, then we can determine p_r, such that $\alpha^{(m)} = 1$, i.e. determine the probability that the system switches to the Receive state when the queue is not empty, in order that the tagged node always receives a packet when it is in the Receive state. Then the value of p_r follows from (69)

$$p_r = \frac{\lambda_e^{(m)} \cdot D^{(m)} - \overline{\boldsymbol{p}}_{\boldsymbol{0}}^{(\boldsymbol{m})} \cdot \overline{\boldsymbol{e}}}{1 - \overline{\boldsymbol{p}}_{\boldsymbol{0}}^{(\boldsymbol{m})} \cdot \overline{\boldsymbol{e}}} \tag{71}$$

Again p_r needs to be computed iteratively.
Remark that since row $m = 1$ has no neighbors that generate packets we have that

$$\alpha^{(1)} = 0. \tag{72}$$

Computation of the average end-to-end response time in a WSN
Consider the network depicted in Figure 4. For the computation of the average end-to-end response time in the network, we assume that the packet loss is small and hence that the external packet arrival rate in a node of row m can be approximated by $\lambda_e^{(m)} = (m-1) \cdot \lambda$. We determine the response time in a node of each row, starting with row M.
Tag a node in row M and let $LISTENTX^{(M)} = 0$. Let $\beta = 1$, $\alpha = 1$ and $p_r = 0$. Determine the steady state distribution of this tagged node in row M , and compute the threshold $\lambda_{th}^{(M)} = \frac{\overline{\boldsymbol{p}}_{\boldsymbol{0}}^{(\boldsymbol{M})} \cdot \overline{\boldsymbol{e}}}{D^{(M)}}$.
If $\lambda_e^{(M)} \leq \lambda_{th}^{(M)}$, then $p_r = 0$ and $\alpha^{(M)}$is obtained using (70) iteratively. If on the other hand $\lambda_e^{(M)} > \lambda_{th}^{(M)}$, then $\alpha^{(M)} = 1$ and p_r can be computed using (71) iteratively. Once these parameters are known, it is possible to determine the average delay $E[R_M]$ in a node of row M using Formula (63).
Consider a node belonging to row m, $2 \leq m \leq M-1$. First, compute $\beta^{(m)}$ using Formula (65). Let $\alpha = 1$ and $p_r = 0$. Determine the steady state distribution of this tagged node in row

m , and compute the threshold $\lambda_{th}^{(m)} = \frac{\overline{\boldsymbol{p}}_{\mathbf{0}}^{(m)} \cdot \bar{e}}{D^{(m)}}$. If $\lambda_e^{(m)} \leq \lambda_{th}^{(m)}$, then $p_r = 0$ and $\alpha^{(m)}$is obtained using (70) iteratively. If on the other hand $\lambda_e^{(m)} > \lambda_{th}^{(m)}$, then $\alpha^{(m)} = 1$ and p_r can be computed using (71) iteratively. Once these parameters are known, it is possible to determine the average delay $E[R_m]$ in a node of row m using Formula (63).
For row 1, we follow a similar procedure to determine $E[R_1]$, where $p_r = 0$ and $\alpha^{(1)} = 0$. The total end-to-end delay is then given by

$$E[R] = \sum_{m=1}^{M} [R_m] \tag{73}$$

This results in the following algorithm to compute the average end-to-end delay.

```
LISTENTX^(M) = 0;
α^(1) = 0;
For m=M:1
     Compute β^(m) (for m=M, β^(M) = 1)
     Set α = 1 and p_r = 0
     Compute λ_th^(m) = p̄_0^(m)·ē / D^(m).
     If λ_e^(m) ≤ λ_th^(m) then p_r = 0 and compute α^(m) using (70)
     If λ_e^(m) > λ_th^(m) then α^(m) = 1 and compute p_r using (71)

     Compute average delay E[R_m] for node of row m using (63)
end
E2EResponsetime = Σ_{m=1}^{M} E[R_m];
```

5. Numerical Examples and Discussion

In this section we apply the model described in Section 4. The purpose is not to investigate in detail the performance of a communication node using energy harvesting, but rather show the impact of the different system parameters on the response time using the model presented in the previous section.

In the next examples, the maximum energy level is equal to $E_{max} = \max_X c(X)$.
We use the values for $c(X)$ and $a(X)$ as mentioned in Table 1.

	SLEEP	TX	RX	LISTEN	MAC
c	3.2828	0.6649	0.4943	0.5764	0.5764
a	108.3316	21.9410	16.3122	19.0220	19.0220

Table 1: parameters of the energy function *F*

For simplicity reasons, we let the duration of the MAC operation, the transmission and the reception of the beacon message, the transmission and the reception of a data packet and the LISTEN intervals all having a deterministic distribution (i.e., a constant value).
Remark also that in the model the variable I (the available energy) takes integer values. Application of formula (1) leads to positive real numbers of $F_X(i_0, T)$. Multiplication with an appropriate factor (e.g., 100) results into integer values for I as required in the model.
In the following examples we use the parameters listed in Table 2, unless otherwise mentioned.

Parameter	***Value***
i_{max}	330
i_{min}	8
LISTENRX	1.0 sec
LISTENTX	1.0 sec
Beacon message RX	0.002 sec
Beacon message TX	0.002 sec
MAC	0.05 sec
Data packet TX	0.0182 sec
Data packet RX	0.0275 sec

Table 2: System parameters used in the examples

5.1. Single node performance: Impact of the rate of external packets on the parameters α and p_r

In a first example we consider a single node and study the impact of the rate of external packets λ_e on the parameters α and p_r and on the average response-time. Consider a node with parameters shown in Table 2, except for LISTENTX, and let the internal packet arrival rate $\lambda = 0.05$, LISTENTX=1.5 sec. We fix the parameter $\beta = 0.75$. The external packet arrival rate varies between 0.1 and 0.9.

For these parameters, the threshold value $\lambda_{th} = \frac{\overline{p}_0 \cdot \overline{e}}{D}$ is 0.2892. In Figure 4 both the probabilities α and p_r are shown as a function of λ_e. Clearly, if $\lambda_e \leq \lambda_{th}$, then $p_r = 0$ and α increases from 0 to 1 for increasing λ_e. $\alpha = 1$ means that all Receive states of the radio result in the arrival of

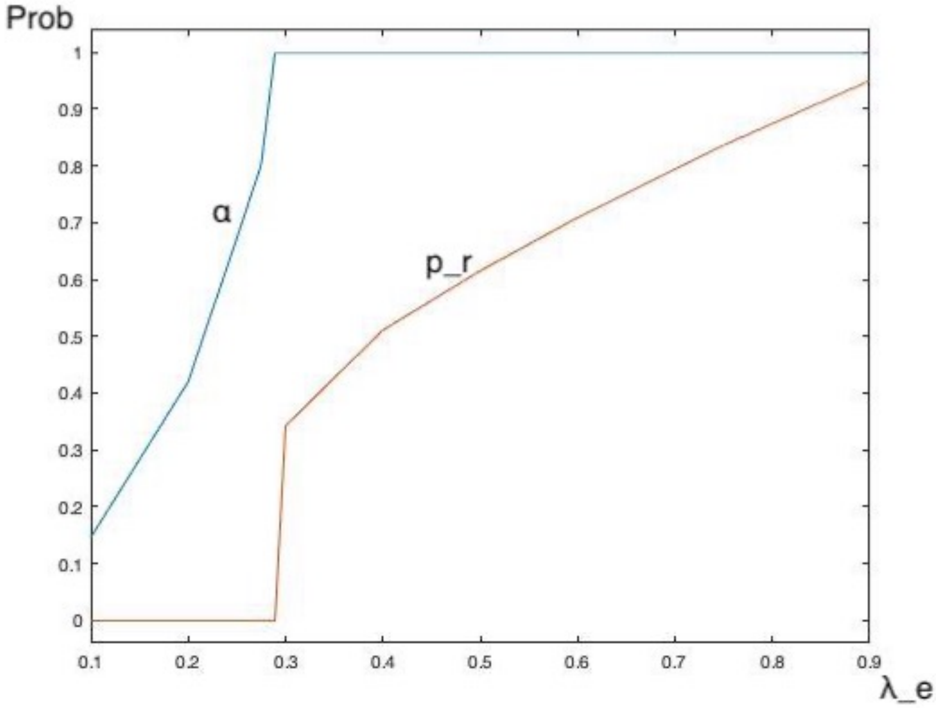

Figure 4: parameters α and p_r as a function of λ_e

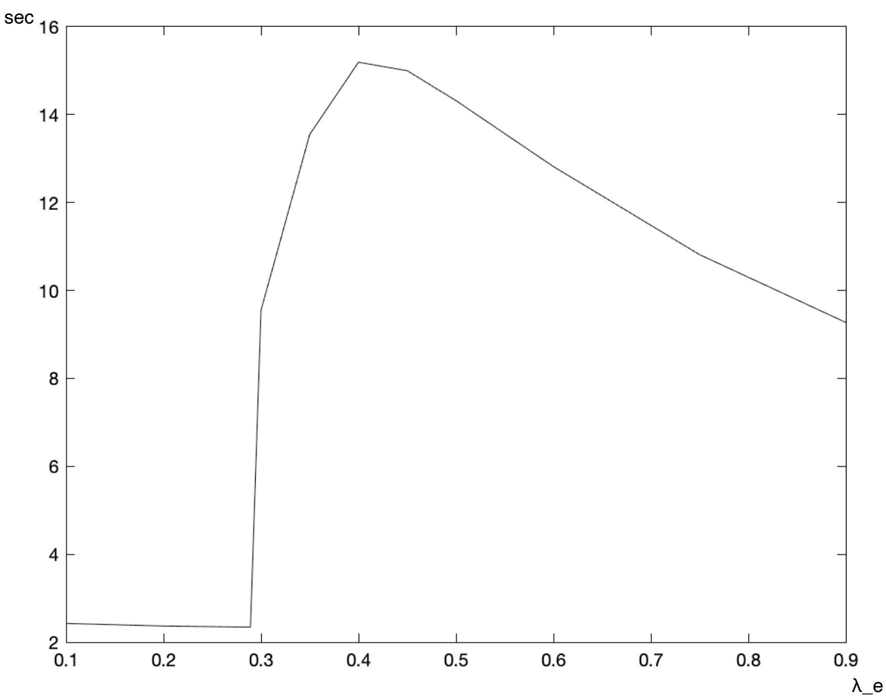

Figure 5: Average response time as a function of λ_e

an external packet. If $\lambda_e > \lambda_{th}$, then $\alpha = 1$ and p_r increases from 0 to 1, meaning that when the queue is not empty, more switches to the Receive state will be made in order to receive all

external packets. Figure 6 shows the average response time as a function of λ_e. For values of λ_e below the threshold λ_{th}, the response time is limited, while above the threshold, the response time rapidly increases. The reason for this increase is that for values of λ_e larger than the threshold λ_{th}, the value of p_r is increasing rapidly as seen in Figure 4. This leads to more frequent switches to the Receive state even if the queue is not empty. Hence, packets must wait longer in the queue before being transmitted (as p_t becomes smaller), leading to longer delays. The response time reaches a maximum for $\lambda_e = 0.4$ and then decreases. This is due to the fact that when λ_e increases, the probability that the system is empty tends faster to 0 than p_r to 1. Hence, the node gets more opportunities to transmit packets when the queue is empty, leading to shorter response times.
This result extends the conclusion of [9] that increasing the frequency of the beacon generation increases the delay, but it also shows the relationship between the external arrival rate, the beacon frequency and the delay: when the external arrival rate is higher than the threshold λ_{th}, the beacon generation frequency drastically increases, leading to high delays. Hence, in order to keep the response-time limited, the external arrival rate should be kept lower than the threshold λ_{th}, being the value of the external arrival rate such that all external packets can be received when the node is in a Receive state that started with an empty queue.

5.2. Single node performance: impact of the harvesting function on the average response-time

In this example we consider a set of linear energy harvesting functions and compare the results for the average response-time with those obtained when using the energy harvesting function defined in Section 3.1. For E=100, 125, 150, 175 and 200 define

$$Y_{\boldsymbol{E}}(t) = \begin{cases} \frac{i_{max}}{E} \cdot t, & t \leq E \\ i_{max}, & t > E \end{cases} \quad (74)$$

Let the energy harvesting while the radio is in SLEEP mode be described by $Y_{\boldsymbol{E}}(t)$. These functions are shown in Figure 6, together with the energy harvesting function defined in Section 3.1, denoted by $Y_{\boldsymbol{exp}}$. Consider a single node with the parameter values used in the previous section, with $i_{min} = 1.00$.
Figure 7 shows the average response-time for variable external input rate λ_e. The threshold value λ_{th} is different for each energy harvesting function, but the characteristic behavior is similar. Indeed, for values of λ_e below the threshold λ_{th}, the response time is limited, while above the threshold, the response time rapidly increases. Hence, this property is not due to the energy harvesting function but is related to the MAC protocol and the behavior of the radio. We see that for increasing values of E, the threshold λ_{th} decreases, leading to an earlier increase of the average response-time. The smaller the value of E, the steeper the function $Y_{\boldsymbol{E}}$, and hence the faster the radio will reach the threshold i_{thtx} or i_{thrx} while being in the SLEEP mode. We see that for energy values between 1.0 and 1.5, Y_{exp} is larger than Y_E for E=150, 175 and 200. This is exactly the range the radio is in while being in the SLEEP mode, and therefore it is not surprising that the average response-time for the energy harvesting function defined in Section 3.1 is situated between the curves corresponding to Y_{125} and Y_{150}.

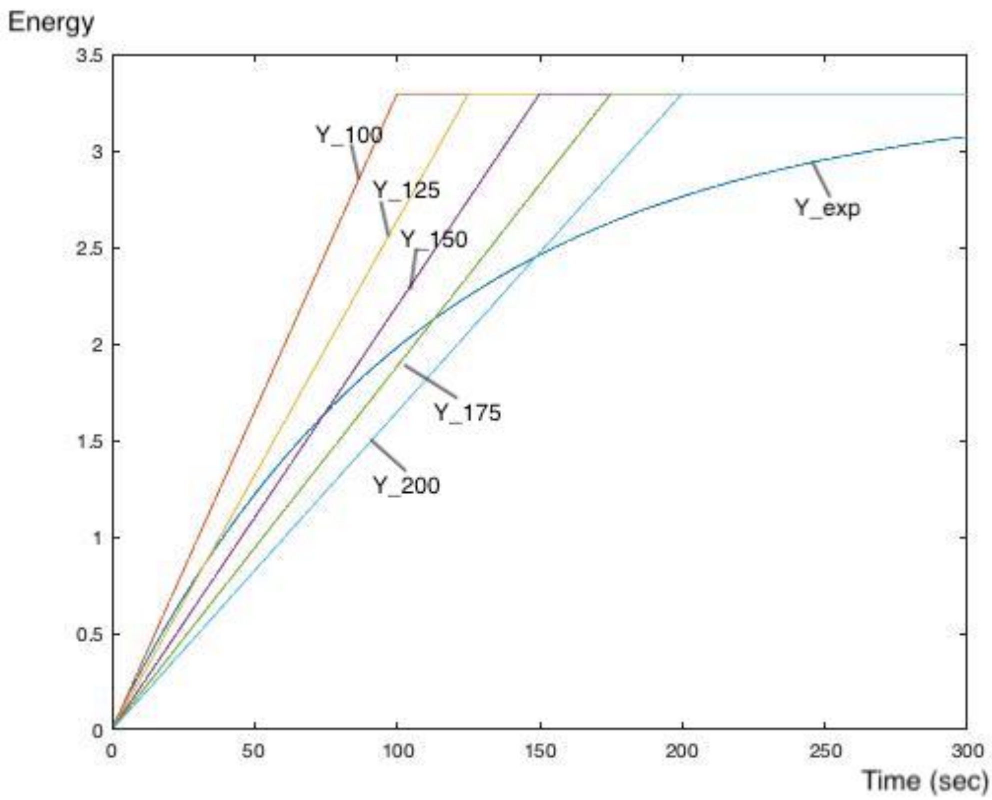

Figure 6: Energy harvesting functions

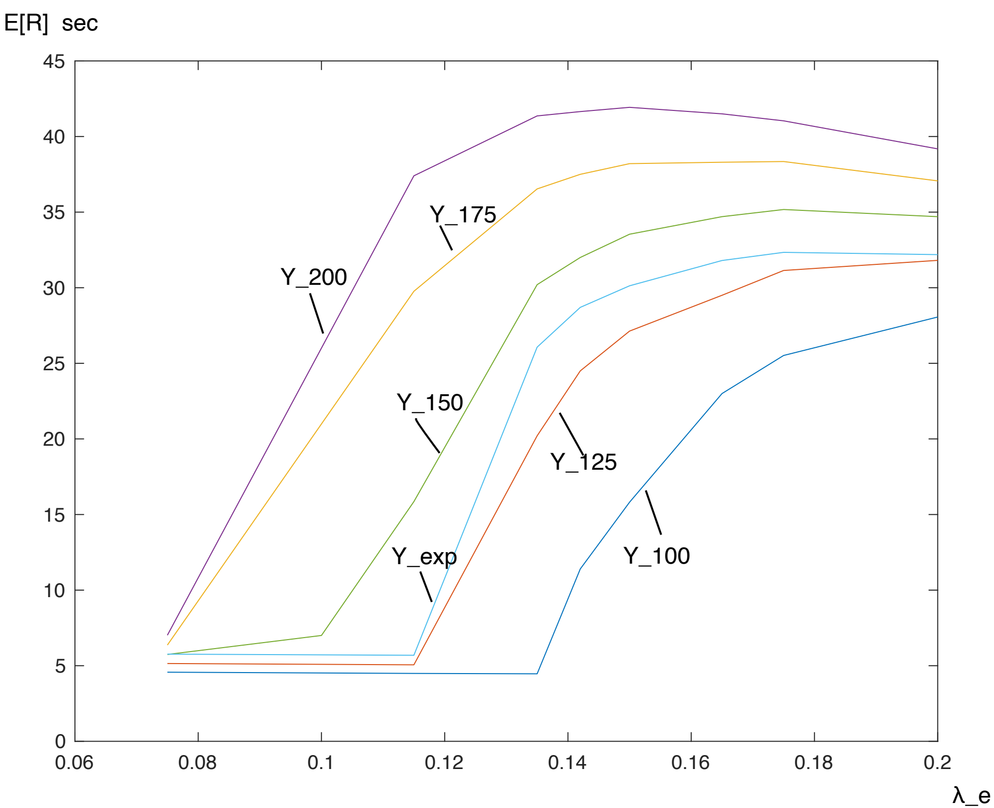

Figure 7: Average response-time for different energy harvesting functions

5.3. Single node performance: impact of the parameter LISTENTX on the average response-time

In a second example we consider a single node of row m and study the impact of the parameter LISTENTX (and hence of the parameter β) on the average response time. We assume that the parameters $D^{(m+1)}$ and $\overline{\boldsymbol{p}}_{\boldsymbol{0}}^{(\boldsymbol{m+1})} \cdot \bar{\boldsymbol{e}}$ of row (m+1) are fixed and given by 0.9385 and 0.7882 respectively. These parameters are needed to compute β (see formula (65)).

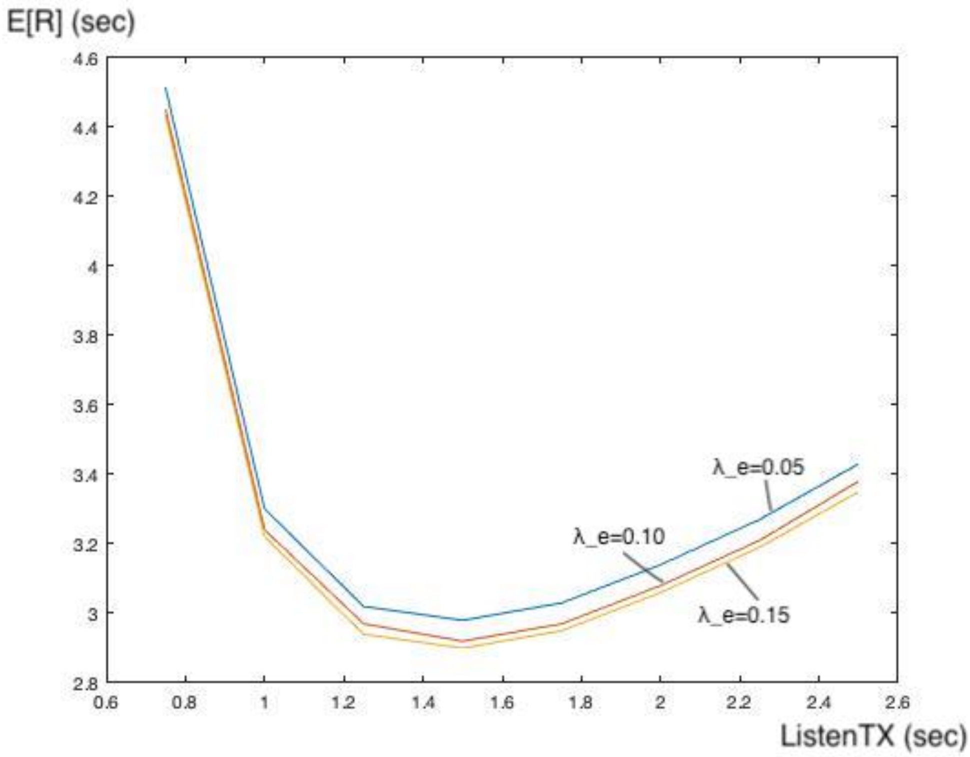

Figure 8: Average response time as a function of the LISTENTX for variable external input rate

First, we consider values of λ_e such that the threshold $\lambda_{th} = \frac{\overline{\boldsymbol{p}}_{\boldsymbol{0}}^{(\boldsymbol{m})} \cdot \bar{\boldsymbol{e}}}{D^{(m)}}$ is never surpassed. Figure 8 shows the average response time as a function of the parameter LISTENTX for different values of λ_e. It is clear that in those cases the behavior of the response time is very similar for the different values of λ_e.

Figure 6 shows that LISTENTX=1.5 sec results in a minimal value of the average response time for all values of λ_e. For small values of LISTENTX, the node often misses the beacon sent by a neighbor node 1, and hence, the packets have to wait longer. If on the other hand LISTENTX is large, then time is lost by continuing to listen after the beacon has been received, again leading to longer response times.

In Figure 9, we show similar results as in Figure 6, but now for values of λ_e such that the threshold λ_{th} is surpassed for some or for all values of the parameter LISTENTX.

For $\lambda_e = 0.275$ and for $\lambda_e = 0.3$ the threshold λ_{th} is surpassed for all values of the parameter LISTENTX, while for $\lambda_e = 0.25$ it is not the case for LISTENTX=1.25 and 1.50. For $\lambda_e = 0.2$ the threshold is only surpassed for LISTENTX=0.75. It is clear from both Figure 8 and

Figure 9, that surpassing the threshold λ_{th} has an important impact on the average response time, and therefore is a crucial system parameter when considering the average response time.

From this example, we conclude that when selecting the length of the time interval a node scans the medium for a beacon (i.e., LISTENTX), the external arrival rate needs to be taken into account. In order to keep the average response-time limited, the external arrival rate should not exceed the threshold λ_{th} that corresponds with this length LISTENTX.

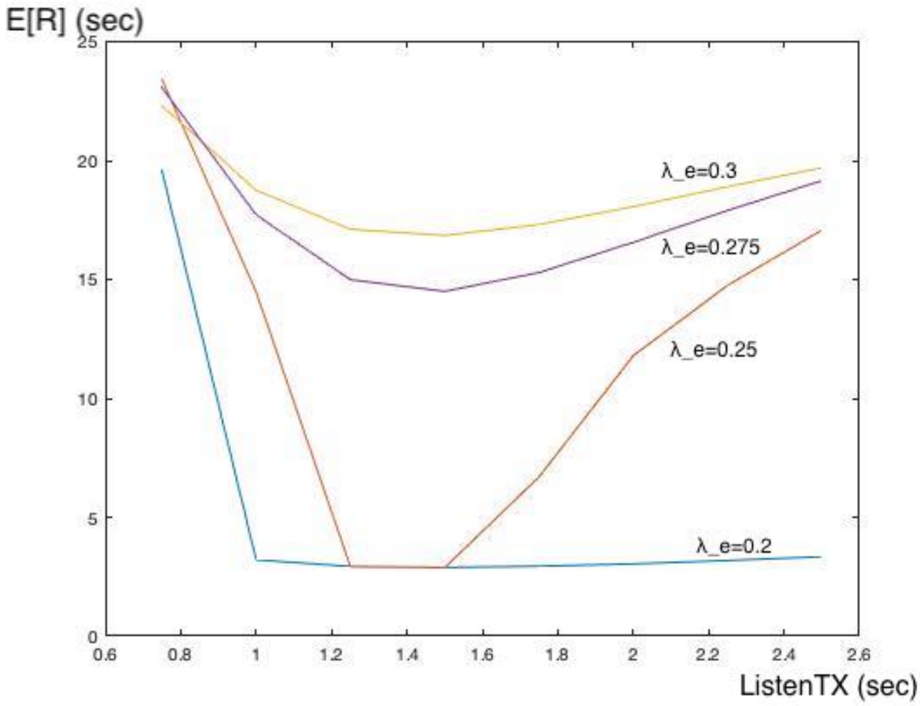

Figure 9: Average response as a function of the LISTENTX interval
for variable external input rate

5.4. WSN performance: Average Response Time in the Nodes of an EH-WSN

Consider an EH-WSN with 7 rows. We use the parameters of Table 1 and Table 2. We compute the average response time a packet experiences in each row, for variable input rate λ. In case there is no packet loss, the total rate of packets originating from row $m-1$ is given by $\lambda_e^{(m)} = (m-1) \cdot \lambda$. This condition is satisfied by selecting values of λ such that for each row m, $(m-1) \cdot \lambda < \lambda_{th}^{(m)}$. We let LISTENTX=1.5 sec. From Figure 10, it is clear that higher input rates lead to longer response-times in the various rows. While for $\lambda = 0.005$ the curve is flat (except for the node of row 7 that is directly connected to the sink and does not need a beacon to be allowed to transmit), we see that for higher values of λ the curve is no longer flat and the average response time experienced by a packet is quite different.
The average response times may be lowered by letting certain parameters depend on the row number. Let the parameter LISTENTX for row 1 to 6 take the respective values 0.7sec, 0.9sec, 1.0sec, 1.1sec, 1.2sec, 1.5 sec instead of the same value 1.5sec for each of these rows. Of course, for row 7, LISTENTX = 0 sec for both cases. Let λ =0.03. Figure 11 shows the difference in average response time for each row between the same value for the parameter

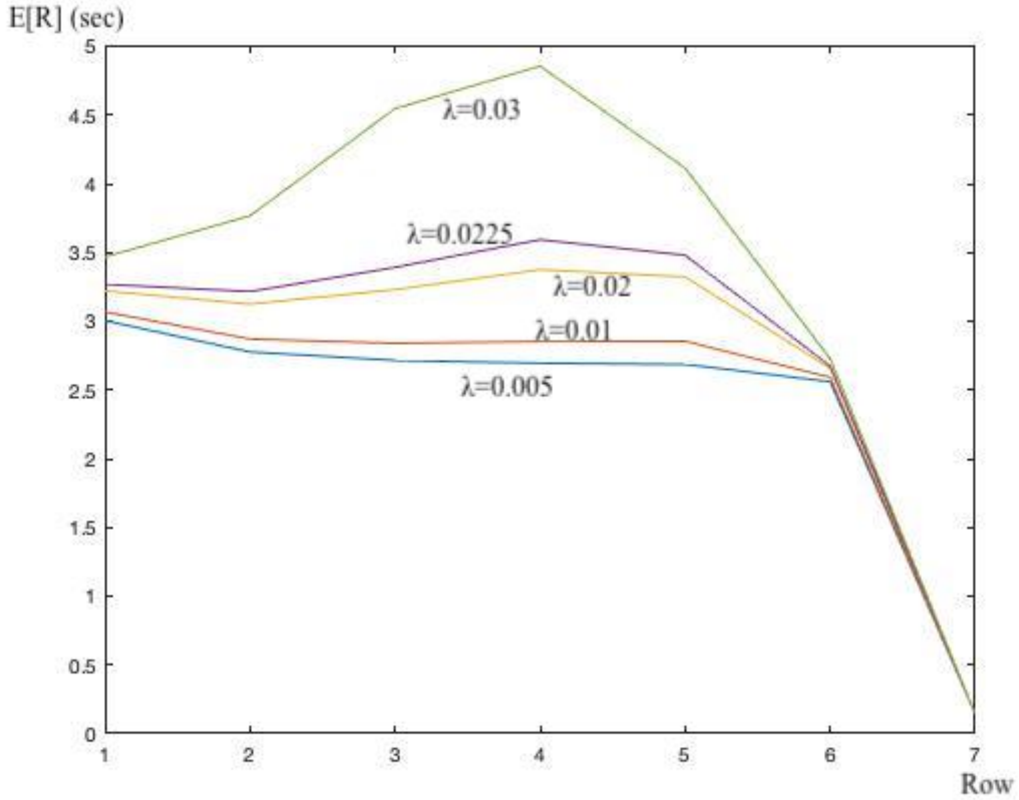

Figure 10: Average response time in each row for variable input rate λ

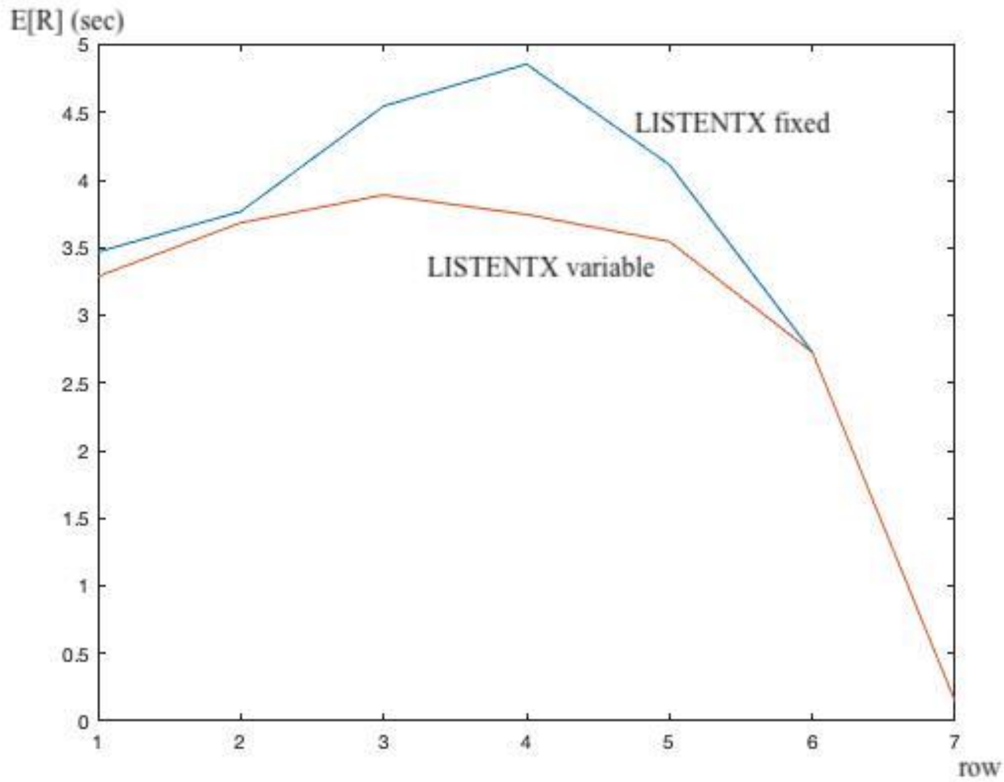

Figure 11: Average response time for fixed and variable values of LISTENTX

LISTENTX and the case when a different value is selected. For example, the gain for row 4 is more than 20%.
From this example it is clear that, in order to minimize the average response time, the system parameters should not be selected uniformly for each row.

6. Conclusions

In this paper we have proposed a finite capacity queue with repeated server vacations of three types as a model for an EH-WSN node that uses a receiver-initiated MAC protocol. The available energy is modeled by means of an extra variable. We obtain the distribution of the system occupancy at inspection instants and at arbitrary time instants together with the average response time. This single node model contains two parameters that allow to model the interaction with neighbor nodes in an EH-WSN. The determination of these parameters allows the evaluation of the average time a packet experiences to reach the sink of the EH-WSN.
Using this model, we show that when the external packet arrival rate is higher than a computable threshold, the beacon generation frequency drastically increases, leading to high delays. This threshold is the value of the external arrival rate such that the system capacity is used to receive these external packets whenever no packets are ready for transmission. Moreover, we have shown that this property is not due to the specific choice to the energy harvesting function. Furthermore, the selection of the length of the interval a node scans the medium for a beacon should depend on the external arrival rate. In particular, the external arrival rate should not exceed the above threshold. We also show that in a WSN, in order to minimize the end-to-end response-time, the parameters of a node should be determined depending on their location with respect to the sink.

References

[1] F. Afro zand R. Braun, Energy-efficient MAC protocols for wireless sensor networks: a survey (2020), *Int. J. Sensor Networks, 32(3),* pp.150-173.

[2] F. K. Shaikh and S. Zeadally (2016), Energy harvesting in wireless sensor networks: A comprehensive review*, Renewable and Sustainable Energy Reviews*, 55, 1041 – 1054.

[3] O.Kanoun (2018), Energy Harvesting for Wireless Sensor Networks: Technology, Components and System Design, *Berlin, Boston: De Gruyter Oldenbourg*, doi.org/10.1515/9783110445053

[4] K.S. Adu-Manu, N. Adam, C. Tapparello, H. Ayatolli, W. Heinzelman (2018), Energy-harvesting wireless sensor networks (EH-WSNs): A review, *ACM Transactions on Sensor Networks, 14(2),* pp. 1-50, doi.org/10.1145/3183338

[5] Z. Zheng, Z. Wang, Y. Peng, JI. Changqing, J. Qin (2020), Survey of MAC Protocols for Energy Harvesting Sensor Networks, *Computer Engineering and Applications, 56(15)*, pp. 24-29, doi.org/10.3778/j.issn.1002-8331.1911-034

[6] G. Famitafreshi, M. S. Afaqui, and Joan Melià-Seguí (2021), A Comprehensive Review on Energy Harvesting Integration in IoT Systems from MAC Layer Perspective: Challenges and Opportunities, *Sensors 21, no. 9: 3097*, https://doi.org/10.3390/s21093097

[7] P. Kaur, B.S. Sohi, P. Singh (2019), Recent advances in MAC protocols for the energy harvesting based WSN: a comprehensize review, *Wireless Personal Communications 104,* 423-440

[8] H. H. RR. Sherazi, L. A. Grieco, G. Boggia (2018), A comprehensive review on energy harvesting MAC protocols in WSNs: Challenges and tradeoffs, *Ad Hoc Networks,* 71, 117-134

[9] X. Fafoutis, N. Dragoni (2011), ODMAC: An On-demand MAC protocol for energy harvesting – wireless sensor networks, *ACM Symp. Performance Evaluation Wireless Ad Hoc, Sensor, Ubiquitous Netw., Miami, FL, USA,* pp. 49-56

[10] Z. A. Eu and H. Tan (2012), Probabilistic polling for multi-hop energy harvesting wireless sensor networks, *2012 IEEE International Conference on Communications (ICC)*, Ottawa, ON, pp. 271-275

[11] H. Liu, W. He and W. K. Seah (2014), "LEB-MAC: Load and energy balancing MAC protocol for energy harvesting powered wireless sensor networks," *2014 20th IEEE International Conference on Parallel and Distributed Systems (ICPADS)*, Hsinchu, pp. 584-591

[12] K. Nguyen, V.-H. Nguyen, D.-D. Le, Y. Li, D.A. Duc, S. Yamada (2014), ERI-MAC: An energy-harvested receiver-initiated MAC protocol for wireless sensor networks, *Int. J. of Distributed Sensor Networks, 10,* 1-8

[13] S.C. Kim, J.H. Jeon, H.J. Park (2012), QoS-Aware Energy-Efficient (QAEE) MAC protocol for energy harvesting wireless sensor networks, *Convergence Hybrid Inform. Technol.*, Daejeon, Rep. of Korea, 41-48

[14] J. Varghese and S. V. Rao (2014), Energy efficient exponential decision MAC for energy harvesting-wireless sensor networks, *2014 International Conference on Advances in Green Energy (ICAGE)*, Thiruvananthapuram, 239-244

[15] Z.A. Eu, H.P. Tan, W.K.G. Seah (2011), Design and performance analysis of MAC schemes for WSN powered by ambient energy harvesting, *Ad Hoc Netw., 9(3),* 300-323

[16] F. Shahzad and T. R. Sheltami (2015), An efficient MAC scheme in wireless sensor network with energy harvesting (EHWSN) for cloud-based applications. *In Proceedings of the IEEE 40th Local Computer Networks Conference Workshops,* 783-788

[17] X. Fafoutis, N. Dragoni (2012), Analytical Comparison of MAC Schemes for Energy Harvesting - Wireless Sensor Networks*, Ninth International Conference on Networked Sensing Systems (INSS)*

[18] H. Wang and R. Simon (2016), Modelling wireless sensor networks with energy harvesting: A stochastic calculus approach, *14th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt)*, 1-8

[19] K. Wu, Y. Jiang, and D. Marinakis (2012), A stochastic calculus for network systems with renewable energy sources, *Proceedings of the 2012 IEEE Conference on Computer Communications Workshops (INFOCOM WKSHPS'12)*, 109– 114

[20] W. Jiao, G. Liu and H. Wu (2018), Queue Performance of Energy Harvesting Cognitive Radio Sensor Networks with Cooperative Spectrum Sharing, *IEEE Access, 6,* 73548-73560

[21] W.M. Kempa (2019), Analytical model of a wireless sensor network (WSN) node operation with a modified threshold-type energy saving mechanism, *Sensors, 19(14) : 3114*

[22] Y-H. Chen, B. Ng, W.K.G. Seah, A-C. Pang (2016), Modeling and analysis: energy harvesting in the Internet of Things, *Proc. 19th International Conference on Modeling, Analysis and Simulation of Wireless and Mobile Systems,* 156-165

[23] E. Cuypere, K. De Turck, D. Fiems (2018), A queueing model of an energy harvesting sensor node with data buffering, *Telecommunication Systems, 67,* 281-295

[24] W. Liu, X. Zhou, S. Durrani, H. Mehrpouyan and S. D. Blostein (2016), Energy Harvesting Wireless Sensor Networks: Delay Analysis Considering Energy Costs of Sensing and Transmission, *IEEE Transactions on Wireless Communications*, vol. 15, no. 7, pp. 4635-4650, doi: 10.1109/TWC.2016.2543216

[25] J.S. Jornet, I.F. Akyildiz (2012), Joint Energy Harvesting and Communication Analysis for Perpetual Wireless Nanosensor Networks in the Terahertz Band, *IEEE Transactions on nanotechnology 11*, 570–580.

[26] S. Tang, L. Tan, T. Liu (2021), Modeling and performance analysis of energy harvesting wireless communication systems with reliable energy backup, *Int. Journal of Communications Systems, 34 (3),* e4698

[27] C. Blondia (2021), A queueing model for a wireless sensor node using energy harvesting, *Telecommunication Systems, 77(2),* 335-349

[28] C. Blondia (2021), An analytical model for a class of receiver-initiated MAC protocols for energy-harvesting wireless sensor networks, *10th IFIP Int. Conf. on Performance Evaluation and Modeling in Wireless and Wired Networks (PEMWN),* 6 p.

[29] C. Delgado, J.M. Sanz, C. Blondia, J. Famaey (2021), Battery-Less LoRaWAN Communications using Energy Harvesting: Modeling and Characterization, *IEEE Internet of Things Journal, 8(4), 2694-2711*