Convexity of the triple helix of innovation game

Reference:
Mêgnigbêto Eustache.- Convexity of the triple helix of innovation game
International Journal of Innovation Science - ISSN 1757-2223 - 2024
Full text (Publisher's DOI): https://doi.org/10.1108/IJIS-03-2023-0071
To cite this reference: https://hdl.handle.net/10067/2024630151162165141

International Journal of Innovation Science

Convexity of the Triple Helix of innovation game

Journal:	*International Journal of Innovation Science*
Manuscript ID	IJIS-03-2023-0071.R3
Manuscript Type:	Original Research
Keywords:	Triple Helix, Innovation, game theory, innovation process, Innovation Index

Convexity of the Triple Helix of innovation game

Abstract

Purpose
The Triple Helix of university-industry-government relationships is a three-person cooperative game with transferable utility. Then, the core, the Shapley value and the nucleolus were used as indicators of the synergy within an innovation system. Whereas the Shapley value and the nucleolus always exist, the core may not. This paper determines the conditions for the core of the Triple Helix game to exist.

Design/methodology/approach
The core of a three-person cooperative game with transferable utility exists only if and only if the game is convex. The paper applies the convexity condition to the Triple Helix game.

Findings
The Triple Helix game is convex if and only if there is output within the system; it is strictly convex if and only if all the three bilateral and the trilateral relationships have an output.

Practical implications
Convex games are competitive situations in which there are strong incentives towards the formation of large coalitions; therefore, innovation actors must cooperate to maximise their interests. Furthermore, a Triple Helix game may be split into subgames for comprehensive analyses and several Triple Helix games may be combined for a global study.

Originality/value
This paper extends the meaning of the Shapley value and the nucleolus for Triple Helix innovation actors: the Shapley value indicates the quantity a player wins due to the coalitions he involves in and the nucleolus the return for solidarity of an innovation actor.

Keywords: Triple Helix; game theory, innovation, innovation actors, innovation process, innovation measurement

1. Introduction

The Triple Helix of university-industry-government relationships is a conceptual framework introduced by Etzkowitz and Leydesdorff (1995, 2000) as one of the variants of the nonlinear model of innovation (Etzkowitz et al., 2000; Leydesdorff, 2012; Meyer et al., 2014). The model postulates that the interactions between university, industry and government maintain a knowledge infrastructure; the knowledge it generates circulates among innovation actors and drives economic growth and social welfare (Leydesdorff & Etzkowitz, 2001). According to the theory, innovation is the outcome of the synergy resulting from the interactions between the three main actors of the system (Leydesdorff & Etzkowitz, 2001); the model has the functions of novelty production, wealth generation, and normative control (Leydesdorff, 2016; Leydesdorff & Park, 2014). The Triple Helix has gained attention since it was developed, as illustrated by an increasing number of papers, as recorded in bibliographic databases (Meyer et al., 2014; Schocair et al., 2023; Zakaria et al., 2023); various innovation systems throughout the world have been studied with the theory. The model has been investigated with techniques and tools from various theories or fields, e.g., informetrics, sociology, information theory and game theory.

In the Triple Helix theory, university designates both public and private higher education institutions (cf. OECD, 2015, p. 260) of which primary role is teaching and doing research; industry stands for both public and private enterprises (cf. OCDE & EUROSTAT, 2018, p. 60; and OECD, 2015, p. 201) of which primary role is research results transforming into commercialised products and services for social wellbeing; and, government is used for public research centres or administration. The term ‘government’ does not reduce to an executive institution (cf. OCDE & EUROSTAT, 2018, p. 60; and OECD, 2015, p. 234), but represents any public institution or power (Shinn, 2002b, 2002a); therefore, it should be understood as state, i.e., consisting of three distinct sets of powers, each with its assigned role: (i) one is the legislature, whose role is to make the law; (ii) the second is the executive (sometimes referred to as 'the government'), which is responsible for implementing the law; and the third is the judiciary, which is responsible for interpreting and applying the law (World Bank, 1997). Some researchers argued that the three actors are insufficient to explain the dynamics of innovation; they proposed the extension to other actors like civil society, financial institutions, innovation users, natural environment (Amaral et al., 2023), and spoke of Quadruple Helix (Carayannis & Campbell, 2009), or Quintuple Helix (Carayannis & Campbell, 2010) or N-tuple Helix (Leydesdorff, 2012). However, Leydesdorff (2021; see also Leydesdorff & Lawton Smith, 2022) explained that the triad of university, industry, and government actors (included their spheres) is adequate to explain the generation and utilisation of knowledge, as well as the resulting technological innovation and economic development.

Game theory was introduced into innovation studies by a certain number of publications; Baniak and Dubina (2012) reviewed some of them. Tol (2012) and Karpov (2014) resorted to Shapley values, the former for assessing research production and impact of schools and scholars, and the latter for allocating publication credit to co-authors; Hayes (2001, 2003) modelled decision-making in library cooperation with cooperative games theory; Schubert and Glänzel (2008) showed that ternary diagram − used to draw the core of cooperative game − could serve to study research collaboration and citations. Carayannis and Dubina (2014) on the one hand, and Dubina and Carayannis (2015) on the other hand, demonstrated that game theory could help in understanding the behaviour of innovation actors.

Some papers developed models. Dubina (2015a, 2015b) has just introduced and affirmed that it was an attempt to model innovation with game theory. Zheng and Li (2023) combined cooperative and noncooperative game approaches to study a supply chain, and Su et al. (2021) proposed an evolutionary game model to understand the dynamic evolution process of collaborative innovation behaviour with the existence of "incentives" from government. These two studies used a three-person cooperative game model; in the former, all the three players are enterprises, of which one supplier and two manufacturers; and, in the latter, they are university, industry and government. Zheng and Li (2023) proved that the cooperative game is convex, that the core is nonempty and concluded: "the more the players form a coalition, the more benefits the coalition will obtain" (p. 7). Even though Su et al. (2021) dealt with the actors of the Triple Helix, they modelled university and industry strategies based on the ones of the government. Therefore, the relationships among the players in these models are not engendered by research output but by funding and other incentives; besides, the papers did not intend to measure the synergy within the system they studied.

Mêgnigbêto (2017, 2018b) showed that the Triple Helix relationships constitute a three-person cooperative game with transferable utility; then, he proposed the core, the Shapley value and the nucleolus as indicators of the synergy within a Triple Helix innovation system. The core is a set of values; the Shapley value and the nucleolus are unique. The core indicates the existence and the extent of the synergy; the Shapley value determines the strength of an actor to create and lead to synergy and, the nucleolus the actor that shows solidarity to maintain the synergy. If the aforementioned papers resort to game theory to model the Triple Helix relation, only Zheng and Li (2023) found that the defined game is convex anyway; however, all the players are enterprises and the relations among them are not research output.

The Shapley value and the nucleolus of a cooperative game with transferable utility always exist, but the core may not. A coreless game is unstable (Serrano 2007, p. 7); in other words, in such a game, players may form and leave coalitions each time their interests are not ensured. Game theorists demonstrated that the conditions for the core exists – i.e., the core is non-empty – is that the game is convex (see e.g., Dehez, 2017; Gilles, 2010, p. 53; Pechersky, 2015; Shapley, 1965, p. 15, 1971).

This paper seeks to determine the conditions (of distribution of values between Triple Helix actors) under which the Triple Helix of innovation game is convex, and consequently, has a non-empty core. It is structured as follows: in the next (second) section, the Triple Helix game is formally defined and the core expressed; the third section determines the conditions of the convexity of the Triple Helix game; the fourth section stress the interests of the convexity for the Triple Helix study; the fifth section illustrates and the last section summarises and concludes.

2. Formally define the Triple Helix game

A transferable utility cooperative game is defined by the set of its players and the characteristic function that attributes to each coalition of players its payoff. Let us consider a Triple Helix of innovation system, and let us denote university u, industry i and government g; the set of players is N = {u, i, g}; the coalitions they may form are the subsets of N, i.e., the set of the parties of N, that is, $P = \{\emptyset, \{u\}, \{i\}, \{g\}, \{u, i\}, \{u, g\}, \{i, g\}, \{u, i, g\}\}$. For simplification purpose, we will write, for instance, ui instead of {u, i} to designate the coalition formed by players u and i; therefore, $P = \{\emptyset, u, i, g, ui, ug, ig, uig\}$.

The characteristic function υ is a real-valued function defined from the set of parties of N (i.e., P) to the interval [0, 100], as interests are expressed as percentage share of output. If Figure 1 represents the basic configuration of the Triple Helix in term of percentage share of research output per sphere, the game is defined as follows (Mêgnigbêto, 2017, 2018b):

$\upsilon: P \rightarrow [0,100]$

$S \mapsto \upsilon(S)$ so that

$$\begin{cases} \upsilon(\emptyset) = 0 \\ \upsilon(u) = U \\ \upsilon(i) = I \\ \upsilon(g) = G \\ \upsilon(ui) = U + I + UI \\ \upsilon(ug) = U + G + UG \\ \upsilon(ig) = I + G + IG \\ \upsilon(uig) = U + I + G + UI + UG + IG + UIG \end{cases} \quad (1)$$

where U, I, G represent the output percentage share of university, industry and government on their own respectively; UI, UG, IG the output percentage shares of the joint collaboration between university and industry, university and government, industry and government (i.e., the bilateral collaboration outputs), and UIG the output percentage share of the joint collaboration of the three actors (i.e., the trilateral collaboration output). UI, UG and IG exclude UIG (Mêgnigbêto, 2017, 2018b).

Let us consider Z the total output within the considered innovation system. Z is required to be non-null; indeed, according to Mêgnigbêto (2018b, p. 1122), the utility is measured in terms of number of research output, neither in terms of use of research output nor in terms of transformation of produced knowledge. This means that if there is no research output within the system, there is no payoff to share and there is no utility to transfer; therefore, the definition of the game implicitly excludes the case where there is no research output within the system. Besides, U, I, G, UI, UG, IG and UIG are higher than or equal to 0.

$$Z = U + I + G + UI + UG + IG + UIG = 100 \quad (2)$$

In Figure 1, U_0, I_0 and G_0 represent the total output percentage shares of university, industry and government within the considered set of research output, including the ones throughout bi- or trilateral collaborations. So,

$$U_0 = U + UI + UG + UIG \quad (3)$$

$$I_0 = I + UI + IG + UIG \quad (4)$$

$$G_0 = G + UG + IG + UIG \quad (5)$$

The intersections of the sets in Figure 1 represent the output of bi- or trilateral collaborations (UI, UG, IG and UIG).

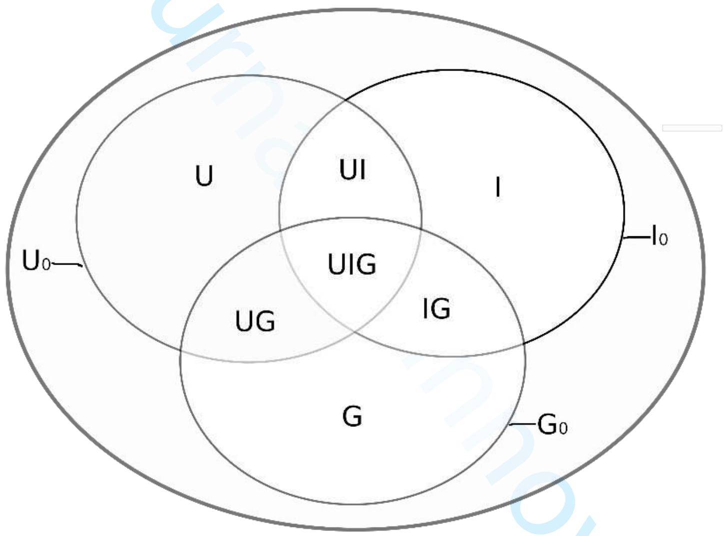

Figure 1. Triple Helix spheres' contributions to the Triple Helix relationships

Source : Mêgnigbêto (2017)

The core of the Triple Helix game is determined by the output percentage share of each actor on its own (the lower bound) and the total output percentage share the actor produced within the system, included in collaboration with other actors (the upper bound), under the condition that the three values add up to 100 (Mêgnigbêto, 2018b). In its analytic form, the core of a

Triple Helix cooperative game is the set of values x_u, x_i and x_g allocated to players u, i and g respectively, so that (Mêgnigbêto, 2018b):

$$\begin{cases} v(u) \leq x_u \leq v(uig) - v(ig) \\ v(i) \leq x_i \leq v(uig) - v(ug) \\ v(g) \leq x_g \leq v(uig) - v(ui) \\ x_u + x_i + x_g = 100 \end{cases} \quad (6)$$

The core of a cooperative game with transferable utility is non-empty if and only if the game is convex (see e.g. Dehez, 2017; Gilles, 2010, p. 53; Pechersky, 2015; Shapley, 1965, 1971).

3. Convexity and strict convexity

A cooperative game with transferable utility is convex if for any two coalitions S and T members of P, the following inequality holds (see e.g. Dehez, 2017; Gilles, 2010, p. 53; Pechersky, 2015; Shapley, 1965, 1971):

$$v(S) + v(T) \leq v(S \cup T) + v(S \cap T) \quad (7).$$

The game is strictly convex if the inequality in (7) is strict, i.e.,

$$v(S) + v(T) < v(S \cup T) + v(S \cap T) \quad (8).$$

In a convex game, a coalition earns likely more than the sum of the interests of its individual members (Karakaya, 2023; Maschler et al., 1967, p. 2); it means that any coalition of players earns at least the sum of the payoffs of its individual members. If the interests of any coalition are strictly higher than the sum of the interests of its individual members, the game is said to be strictly convex. Therefore, we formulate the two propositions below:

Proposition 1: A Triple Helix game is convex if and only if there is output within the system.

Proposition 2: A Triple Helix game is strictly convex if and only if all the three bilateral and the trilateral collaborations have output.

Proofs. (See details in Appendix 1)

Inequality (7) is equivalent to

$$v(S \cup T) \geq v(S) + v(T) - v(S \cap T) \qquad (9).$$

Let us apply inequality (9) to the coalitions in the Triple Helix innovation system. We get the system of inequalities below:

$$\begin{cases} \upsilon(ui) \geq v(u) + v(i) - v(\emptyset) \\ \upsilon(ug) \geq v(u) + v(g) - v(\emptyset) \\ \upsilon(ig) \geq v(i) + v(g) - v(\emptyset) \\ \upsilon(uig) \geq v(ui) + v(g) - v(\emptyset) \\ \upsilon(uig) \geq v(ug) + v(i) - v(\emptyset) \\ \upsilon(uig) \geq v(ig) + v(u) - v(\emptyset) \end{cases} \qquad (10).$$

The system of inequalities (10) gives

$$\begin{cases} U + I + UI \geq U + I \\ U + G + UG \geq U + G \\ I + G + IG \geq I + G \\ U + I + G + UI + UG + IG + UIG \geq (U + I + UI) + G \\ U + I + G + UI + UG + IG + UIG \geq (U + G + UG) + I \\ U + I + G + UI + UG + IG + UIG \geq (I + G + IG) + U \end{cases} \qquad (11)$$

which in turn gives

$$\begin{cases} UI \geq 0 \\ UG \geq 0 \\ IG \geq 0 \\ UIG \geq 0 \end{cases} \qquad (12)$$

The system of inequalities (12) suggests that the Triple Helix game is convex whether all the three bilateral and the trilateral collaborations produce output or not. Conversely, when UI, UG, IG, and UIG are greater than or equal to 0, coalitions earn likely more than the sum of the payoffs of their individual members.

With the same logical reasoning, a Triple Helix game is strictly convex if UI, UG, IG and UIG all are strictly higher than 0. Let us notice that a strictly convex game is convex, but the reverse is not true.

4. Convexity and Triple Helix of innovation game

"Convex games have nice properties" (Shapley, 1971, p. 6) that paves the way for an in-depth study of the Triple Helix. i) the core always exists, ii) the game has incentives for coalitions forming, iii) the Shapley value belongs to the core, iv) the game may be decomposed, and v) the game may sum up with other convex games.

"The core of a game may be interpreted as the set of sociologically stable outcomes, in that no coalition can upset any one of them. In a game with an empty core, at least one set of players must fail to realise its full potential, no matter how the winnings are divided" (Shapley, 1965, p. 15). The convexity also determines the form of the core; indeed, if n is the number of players, the dimension of the core is at most $n - 1$ if the game is convex and exactly $n - 1$ if the game is strictly convex; if the game is strictly convex, the core has exactly $2^n - 2$ polyhedral faces of dimension $n - 2$ (Shapley, 1965, 1971). We conclude that the core of a Triple Helix of innovation game is a polygon, a segment, or a point when the game is convex, but a polygon with 6 faces − and hence 6 vertices − i.e., a hexagon, if the game is strictly convex. Bloch and de Clippel (2010, p. 5) confirmed while speaking of convex games in general: "if the core has the shape of a polygon, it is an hexagon characterised by six extreme points". Therefore, the surface area of the core of a Triple Helix of innovation game can be used as measurement of the synergy in a Triple Helix innovation system as Mêgnigbêto (2018b, 2018a, 2019) did.

Due to the convexity of the Triple Helix game, coalitions in the Triple Helix game ensure players get at least their individual payoff. “Convex games are competitive situations in which there are strong incentives towards formation of large coalitions” (Shapley, 1965, p. iii); therefore, the conditions and the rules of the game prompt players to build coalitions in order to maximise their interests. The convexity “expresses a sort of increasing marginal utility for coalition membership” (Shapley, 1965, p. 2); it has increasing returns with respect to the coalition size: the greater the number of members of a coalition, the larger the interest of the coalition and the individual member’s interests. In such games, “the grand coalition is the most efficient organisation of agents” (Alonso-Meijide et al., 2022). Consequently, the Shapley value of any player belongs to the core, it is higher than or equal to the player’s payoff (Shapley, 1965, p. 17); else, a player may engage in a game or coalition, and could get a payoff lower than what he would get if he works out of that coalition.

An innovation system is a complex system (Katz, 2006, 2016), indeed; so is the Triple Helix innovation system (Leydesdorff, 2003). The convexity of the Triple Helix game is a tool that may help in decomposing a game into parts (subgames) that could then be studied separately to portray a “local view” of the system, and understand the behaviours of innovators for the purpose of comparison at “global level”. Therefore, constraints on actors at local or global level that explain the players’ behaviours could be assessed. Let note, however, that a convex game is decomposable (therefore, may split into its finest components), and that a strictly convex game is indecomposable (González-Díaz & Sánchez-Rodríguez, 2008; see also Shapley, 1965, and 1971). Lastly, convex games may be summed up or combined; thus, for example, in an innovation system, one may study the subsystem engendered by publications on the one hand, and the one by patents on the other hand, or an innovation system per year, then combined them and derived the resulting game characteristics (core, Shapley value and Nucleolus).

The properties of convex games help in interpreting indicators like the core, the Shapley value, and the Nucleolus. Mathematically, the Shapley value is the expected marginal contribution of a player (Roth, 1988a, p. 6) ; it can be interpreted as the "expected utility of playing a game" (Roth, 1988b). Within the framework of the Triple Helix, the Shapley value was interpreted as the strength of a player in leading to and creating synergy (Mêgnigbêto, 2018b, 2018a). In other words, if an actor engages in a game, the Shapley value ensures him at least its own utility, and after the game ends, he may earn more. This lightning brought by the findings of this paper enables us to extend the meaning of the Shapley value: the Shapley value indicates the quantity a player wins due to the coalitions he engages in. Indeed, on its own, he would expect less and with collaboration he expects or realises the Shapley value. Because the Shapley value is within the core (the game in convex) and is always higher than the payoff, the difference between these two values is what the player expects to win due to the synergy he creates or leads to. We can also conclude that for any Triple Helix game, all players lead to and create synergy at different extent. So, the convexity of the game limits antagonisms and conflicts between innovation actors; conversely, it encourages cooperation and collaboration; therefore, the synergy may be maximal and the core wider. Similarly, the Nucleolus was interpreted as the degree of the solidarity shown by an actor. Any member who accepts to show solidarity will have a utility larger than expected if he worked alone, because the nucleolus belongs to the core; he will not have the opportunity to break the coalition and render the game unstable.

5. Illustration

In this section, we consider the USA publications data for the year 2000 (cf. Table 1 "Case 1. Real data") collected from the CD-ROM version of the Science Citation Index by Leyderdorff (2003) in his seminal publication on the mutual information as an indicator of the Triple Helix relationships. The other hypothetical cases (derived from the USA real data case) admit successively that i) university and industry collaboration yields no output (Case 2. UI = 0), ii) university and industry collaboration yields no output, so does the collaboration between industry and government (Case 3. UI = IG = 0), iii) none of the three bilateral collaborations yields output (Case 4. UI = IG = UG = 0) and iv) the trilateral collaboration has not output; other values are kept. Table 2 and Table 3 respectively present the characteristic functions, the cores, the nucleoli and the Shapley values of the derived Triple Helix games. We used the R statistical software (R Development Core Team, 2023) and its packages "Game Theory" (Cano-Berlanga et al., 2017) to compute the Shapley values and the nucleoli, "CoopGame" (Staudacher & Anwander, 2019) to determine the vertices of the cores (with their ternary coordinates), "Ternary" (Smith et al., 2023) to convert ternary coordinates to Cartesian coordinates in order to draw the cores on a plan and compute their surface areas.

Game theorists represent the core of a n-person transferable utility game on a ternary diagram. A ternary diagram allows for the representation of an "object" with three components on an equilateral triangle, each side of the triangle serving as scale for one component. In this paper, we resorted to TernaryPlot (Blom, 2023) to plot the cores (Figure 2 to Figure 5).~~Specialised packages above provide tools to plot the core on a ternary diagram. The surface area of the core (as percentage of the surface of the equilateral triangle) is low (cf. Table 3), making the core "invisible" on the supporting +triangle. So, to better visualise their form, we present below a "zoomed version" of the cores (Figure 2 to Figure 5) plotted with Geogebra (GeoGebra GmbH, 2023; Hohenwarter, 2002; Hohenwarter et al., 2018)~~.

In Case 1 (real data), all the three bilateral and the trilateral collaborations yield publications, so the game is strictly convex, and the core has six faces (Figure 2). In the other theoretical cases, each time a bilateral collaboration is dropped, the core loses one face, and its surface area diminishes. In Figure 3, the collaboration between university and industry is null (Case2, UI = 0) and the core is a pentagon with 0.694 as surface area. Figure 4 presents a tetragon and results from the absence of both the university-industry and the industry-government collaborations (Case 3, UI = IG = 0). Figure 5 shows the form of the core when all the three bilateral collaborations (Case 4, UI = IG = UG = 0) disappear; the core loses three faces and reduces to a triangle. However, the absence of the trilateral collaboration only has no effect on the form of the core (Case 5, UIG = 0), but on its surface area. One may notice that for any actor, the Shapley value and the nucleolus (Table 3) fall within the interval of the "interests" of the actors as determined in the core's analytical form, conforming to the convexity of the considered games.

Table 1. Simulation of Triple Helix spheres contribution based of the USA publications system in Science Citation Index in 2000

		U	I	G	UI	UG	IG	UIG	Total
Case 1. Real data	#	152,449	6,506	24,134	7,200	37,834	1,782	2,666	232,571
	%	65.55	2.80	10.38	3.10	16.27	0.77	1.15	100
Case 2. UI=0	#	152,449	6,506	24,134	0	37,834	1,782	2,666	225,371
	%	67.64	2.89	10.71	0	16.79	0.79	1.18	100
Case 3. UI=IG=0	#	152,449	6,506	24,134	0	37,834	0	2,666	223,589
	%	68.18	2.91	10.79	0	16.92	0	1.19	100
Case 4.	#	152,449	6,506	24,134	0	0	0	2,666	185,755

UI=IG=UG=0	%	82.07	3.50	12.99	0	0	0	1.44	100
Case 5. UIG=0	#	152,449	6,506	24,134	7,200	37,834	1,782	0	185,755
	%	66.31	2.83	10.50	3.13	16.46	0.78	0	100

Table 2. Characteristic functions of the derived Triple Helix games

	u	i	g	ui	ug	ig	uig
Case 1. Real data	65.55	2.80	10.38	71.44	92.19	13.94	100
Case 2. UI=0	67.64	2.89	10.71	70.53	95.14	14.39	100
Case 3. UI=IG=0	68.18	2.91	10.79	71.09	95.90	13.70	100
Case 4. UI=IG=UG=0	82.07	3.50	12.99	85.57	95.06	16.49	100
Case 5. UIG=0	66.31	2.83	10.50	72.27	93.26	14.10	100

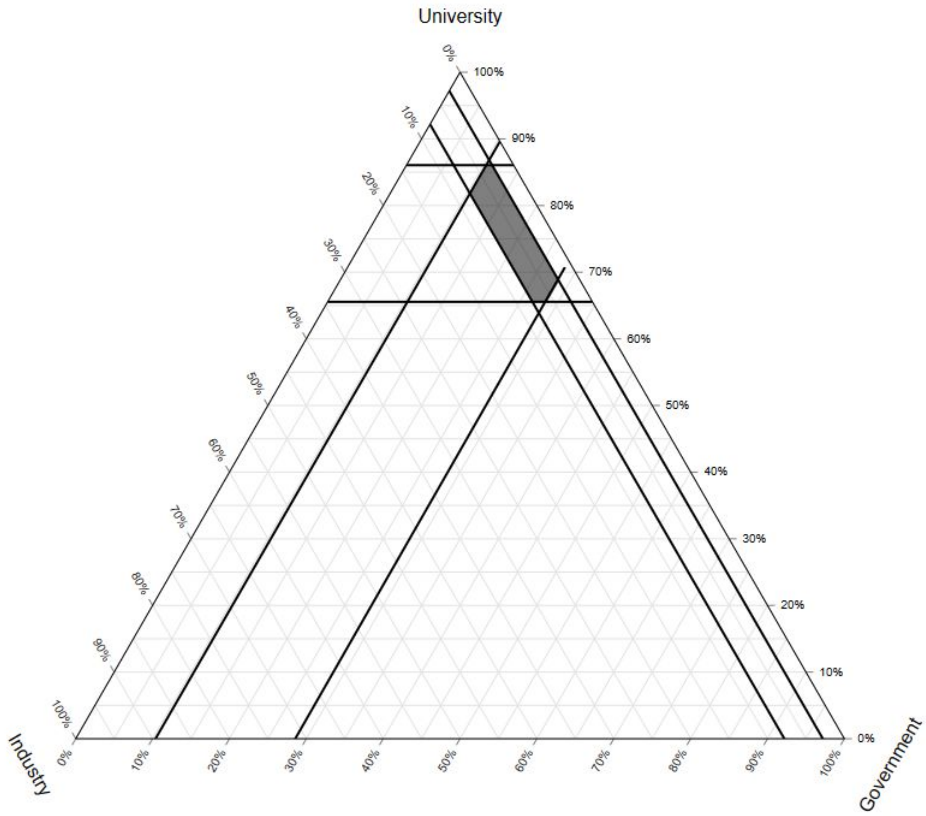

Figure 2. Core of the USA Triple Helix game

The game is strictly convex, and the core (the coloured part of the surface area of the supporting triangle) is a hexagon.

Table 3. Cores, Nucleoli and Shapley values of the considered Triple Helix games

Case	Core			Nucleolus			Shapley value		
	Analytic form	Surface area	Number of faces	u	i	g	u	i	g
Case 1. Real data	$\begin{cases} 65.55 \le x_u \le 86.06 \\ 2.80 \le x_i \le 7.81 \\ 10.38 \le x_g \le 28.56 \\ x_u + x_i + x_g = 100 \end{cases}$	1.778	6	81.810	5.305	12.885	75.612	5.112	19.277
Case 2. UI = 0	$\begin{cases} 67.64 \le x_u \le 85.61 \\ 2.89 \le x_i \le 4.86 \\ 10.71 \le x_g \le 29.47 \\ x_u + x_i + x_g = 100 \end{cases}$	0.694	5	84.430	3.875	11.695	76.428	3.678	19.893
Case 3. UI = IG = 0	$\begin{cases} 68.18 \le x_u \le 86.30 \\ 2.91 \le x_i \le 4.10 \\ 10.79 \le x_g \le 28.91 \\ x_u + x_i + x_g = 100 \end{cases}$	0.417	4	85.110	3.505	11.385	77.042	3.307	19.652
Case 4. UI = IG = UG = 0	$\begin{cases} 82.07 \le x_u \le 83.51 \\ 3.50 \le x_i \le 4.94 \\ 12.99 \le x_g \le 14.43 \\ x_u + x_i + x_g = 100 \end{cases}$	0.021	3	82.550	3.980	13.470	82.550	3.980	13.470
Case 5. UIG = 0	$\begin{cases} 66.31 \le x_u \le 8590 \\ 2.83 \le x_i \le 6.74 \\ 10.50 \le x_g \le 27.73 \\ x_u + x_i + x_g = 100 \end{cases}$	1.335	6	82.760	4.785	12.445	76.103	4.783	19.113

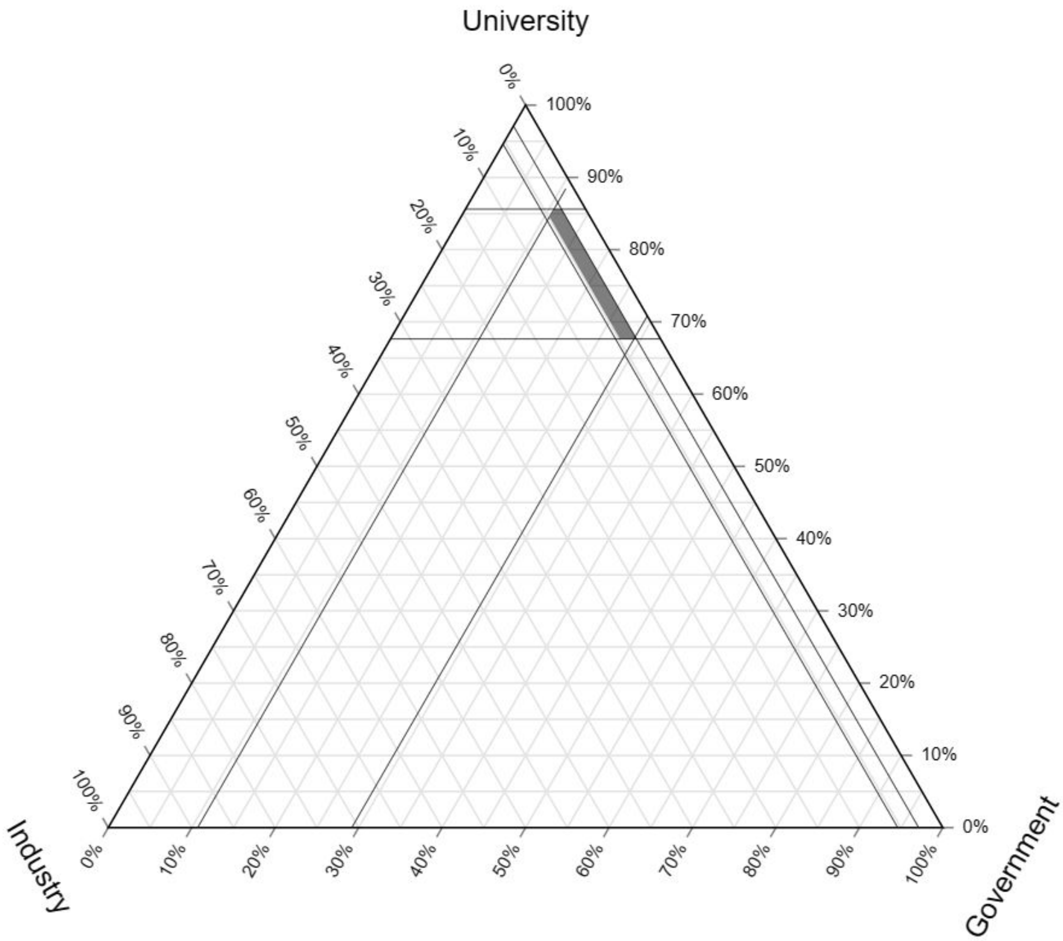

Figure 3. Core of the hypothetic USA Triple Helix game (UI = 0)

The game is convex, and the core (the coloured part of the surface area of the supporting triangle) is a pentagon.

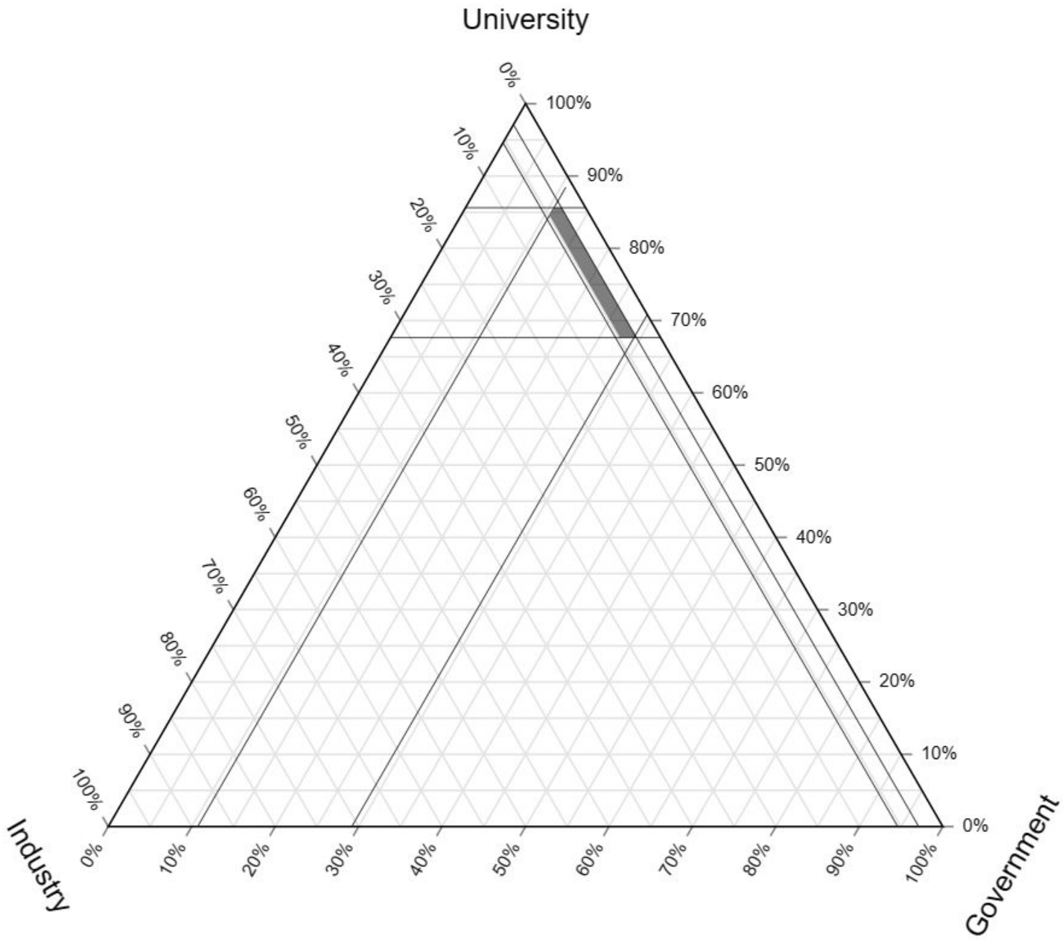

Figure 4. Core of the hypothetic USA Triple Helix game (UI = 0 and IG = 0)

The game is convex, and the core (the coloured part of the surface area of the supporting triangle) is a tetragon.

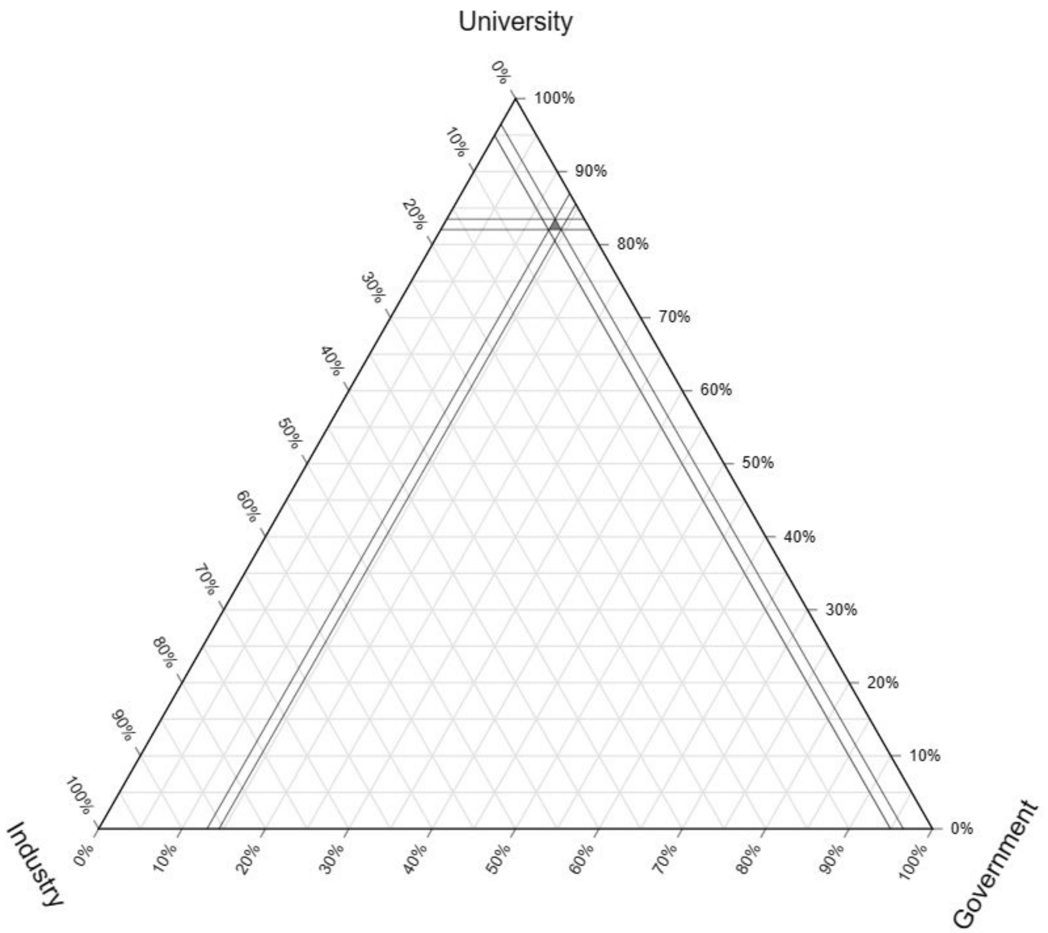

Figure 5. Core of the hypothetic USA Triple Helix game (UI = IG = UG = 0)

The game is convex, and the core (the coloured part of the surface area of the supporting triangle) is a triangle.

6. Discussion and conclusion

The Triple Helix of innovation game is convex anyway, so the core always exists. The game is strictly convex if and only if all the three bilateral and the trilateral collaboration output; then, the core has the form of a polygon with six faces. Starting with the real case of a strictly convex game, the publication data of the USA in 2003, we derived hypothetical cases where successively, one, two and three bilateral collaborations are null. Each time a bilateral relationship is missed, the core loses a face and the level of synergy decreases. If only the trilateral collaboration does not yield output, the game is still convex (but no longer strictly convex), the core still has six faces, but its surface area diminishes. If the trilateral collaboration is null along with the three bilateral collaborations, only one-player coalitions have output, and the core reduces to a point. These results illustrate that the interactions between innovations actors (by the means of collaboration) are the source of the synergy within a Triple Helix innovation system as the theory formulates (see Etzkowitz & Leydesdorff, 1995, 2000; Leydesdorff & Etzkowitz, 2001). On the other hand, if one actor, for example industry, does not output within the system, neither individually nor in collaboration, he will appear like a dummy player and the game will reduce to a two-player one; the core is then a segment.

The results above suggest that the core has its maximal surface area and faces when the game is strictly convex. As the core indicates the existence of synergy, and its surface area the extent of the synergy, this finding suggests that the synergy is maximal when the game is strictly convex, i.e., all the three bilateral and the grand coalition forms. It is also an illustration that actors get the maximum of their interests when the game is strictly convex, in other words, when they collaborate with each other, both bilaterally and trilaterally. The finding reinforces the use of the existence of the core as an indicator of synergy, and its surface area (expressed as percentage of the surface area of the ternary triangle) as the level of synergy within a Triple Helix innovation system.

The convexity of the Triple Helix game requires all the possible coalitions to form. It also means that innovators are free to choose their partners, but they must collaborate with each another. Forming the grand coalition should be the objective of any innovator; it requires negotiations with both partners individually (bilateral collaboration) and collectively (trilateral collaboration). It explains why when a bilateral collaboration is missed, the core changes its form and its surface area decreases.

The study has one limitation: it proves that the Triple Helix game may be split into subgames or summed up with other Triple Helix games, but it did neither deal with the manner nor give examples; indeed, these goals go beyond the scope. The results may, however, be applied to real cases to draw accurate conclusions.

Appendix 1. Proof of propositions 1 and 2

Let us recall inequality (9): $v(S \cup T) \geq v(S) + v(T) - v(S \cap T)$

1) Let us consider the empty set $\emptyset$ and S, any other coalition. $S \cup \emptyset = S\ et\ S \cap \emptyset = \emptyset$.

Therefore, inequality (9) holds because $v(S) \geq v(S) - v(\emptyset)$.

2) Let us consider the three one-member coalitions i, u and g only. The intersection of any two of them is the empty set and the union of any two of them returns in a two-member coalition ui, ug or ig. By definition, $v(ui) = U + I + IG = v(u) + v(i) + IG$. Because IG is greater than or equal to 0, we can write $v(ui) \geq v(u) + v(i)$ and then

$v(ui) \geq v(u) + v(i) - v(\emptyset)$ (13).

With the same logical reasoning we find

$v(ug) \geq v(u) + v(g) - v(\emptyset)$ (14)

and

$v(ig) \geq v(i) + v(g) - v(\emptyset)$ (15).

3) Let us consider S as any one-member coalition and T any two-member coalition. S may be included in T. If $S \cap T = \emptyset\ then$ S ∪ T = N and inequality (9) becomes

$v(N) \geq v(S) + v(T)$.

The application gives

$v(uig) \geq v(i) + v(ug)$ (16)

$v(uig) \geq v(u) + v(ig)$ (17)

$v(uig) \geq v(g) + v(ui)$ (18)

If $S \cap T = S\ then$ S ∪ T = T and inequality (9) becomes

$v(T) \geq v(S) + v(T) - v(S)$ which always holds.

4) S is any one-member or a two-member coalition and T is the grand coalition N.

$S \cap N = S$ *and* $S \cup T = N$, therefore, inequality (9) becomes $v(N) \geq v(S)$ which always holds.

5) S and T are any two-member coalitions. The intersection of S and T gives the empty set or a one-member coalition and their union the grand coalition. So, inequality (9) becomes

$v(N) \geq v(S) + v(T)$

or

$v(N) \geq v(S) + v(T) - v(S \cap T)$

The development of the last inequality gives the three inequalities below:

$$v(uig) \geq v(ui) + v(ug) - v(u) \quad (19)$$

$$v(uig) \geq v(ui) + v(ig) - v(i) \quad (20)$$

$$v(uig) \geq v(ug) + v(ig) - v(g) \quad (21)$$

Inequalities (16) to (21) result in system (10) of whose solution gives the condition for the Triple Helix game to be a convex or strictly convex.

Acknowledgements

The author is grateful to the Associate Editor and the three anonymous referees for their valuable and helpful comments that improved the original submission and for their encouragement to continue working on the Triple Helix theory.

References

Alonso-Meijide, J. M., Álvarez-Mozos, M., Fiestras-Janeiro, M. G., & Jiménez-Losada, A. (2022). On convexity in cooperative games with externalities. *Economic Theory*, *74*, 265–292. https://doi.org/10.1007/s00199-021-01371-8

Amaral, M., Cai, Y., Rocha Perazzo, A., Rapetti, C., & Piqué, J. (2023). The legacy of Loet Leydesdorff to the Triple Helix as a theory of innovation and economic development. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.4484329

Baniak, A., & Dubina, I. (2012). Innovation analysis and game theory: A review. *Innovation: Management, Policy & Practice*, *14*(2), 178–191. https://doi.org/10.5172/impp.2012.14.2.178

Bloch, F., & de Clippel, G. (2010). Cores of combined games. *Journal of Economic Theory*, *145*(6), 2424–2434. https://doi.org/10.1016/j.jet.2009.04.004

Blom, J. (2023). *TernaryPlot.com: A zero-setup ternary diagram generator* [Computer software]. https://www.ternaryplot.com/

Cano-Berlanga, S., Giménez-Gómez, J.-M., & Vilella, C. (2017). Enjoying cooperative games: The R package GameTheory. *Applied Mathematics and Computation*, *305*, 381–393. https://doi.org/10.1016/j.amc.2017.02.010

Carayannis, E. G., & Campbell, D. F. (2010). Triple Helix, Quadruple Helix and Quintuple Helix and how do knowledge, innovation and the environment relate to each other?: A proposed framework for a trans-disciplinary analysis of sustainable development and social ecology. *International Journal of Social Ecology and Sustainable Development*, *1*(1), 41–69.

Carayannis, E. G., & Campbell, D. F. J. (2009). ‘Mode 3’ and ‘Quadruple Helix’: Toward a 21st century fractal innovation ecosystem. *International Journal of Technology Management*, *46*(3/4), 201. https://doi.org/10.1504/IJTM.2009.023374

Carayannis, E. G., & Dubina, I. N. (2014). Thinking beyond the box: Game theoretic and living lab approaches to innovation policy and practice improvement. *Journal of Knowledge Economy*, *5*(3), 427-439.

Dehez, P. (2017). On Harsanyi dividends and asymmetric values. *International Game Theory Review*, *19*(3). https://doi.org/10.1142/S0219198917500128

Dubina, I., & Carayannis, E. (2015). *Promoting innovation in emerging economies: Game theory as a tool for policy and decision making*. U.S. Russia Foundation for Economic Advancement and Rule of Law - International Research & Exchanges Board.

Dubina, I. N. (2015a). A basic formalization of the interaction of the key stakeholders of an innovation ecosystem. *Mathematical Economics*, *11*(18), 33–42.

Dubina, I. N. (2015b). *A game theoretic formalization of the Triple Helix innovation conception*. 40–45.

Etzkowitz, H., & Leydesdorff, L. (1995). The Triple Helix—University-Industry-Government relations: A laboratory for knowledge-based economic development. *EEASST Review*, *14*(1), 14–19.

Etzkowitz, H., & Leydesdorff, L. (2000). The dynamics of innovation: From National Systems and '"Mode 2"' to a Triple Helix of university–industry–government relations. *Research Policy*, *29*(2), 109–123.

Etzkowitz, H., Webster, A., Gebhardt, C., & Cantisano Terra, B. R. (2000). The future of the university and the university of the future: Evolution of ivory tower to entrepreneurial paradigm. *Research Policy*, *29*(2000), 313–330.

Gilles, R. P. (2010). *The cooperative game theory of networks and hierarchies*. Springer.

González-Díaz, J., & Sánchez-Rodríguez, E. (2008). Cores of convex and strictly convex games. *Games and Economic Behavior*, *62*(1), 100–105. https://doi.org/10.1016/j.geb.2007.03.003

Hayes, R. M. (2001). *Models for library management, decision-making and planning*. Academic Press.

Hayes, R. M. (2003). Cooperative game theoretic models for decision-making in contexts of library cooperation. *Library Trends*, *51*(3), 441–461.

Karakaya, M. (2023). Transferable utility games and the existence of core allocations. *International Academic Research and Reviews in Social, Human and Administrative Sciences*, 183–198.

Karpov, A. (2014). Equal weights coauthorship sharing and the Shapley value are equivalent. *Journal of Informetrics*, *8*(1), 71–76. https://doi.org/10.1016/j.joi.2013.10.008

Katz, J. S. (2006). Indicators for complex innovation systems. *Research Policy*, *35*(7), 893–909.

Katz, J. S. (2016). What is a complex innovation system? *PloS One*, *11*(6). https://doi.org/10.1371/journal.pone.0156150

Leydesdorff, L. (2003). The mutual information of university-industry-government relations: An indicator of the Triple Helix dynamics. *Scientometrics*, *58*(2), 445–467.

Leydesdorff, L. (2012). The Triple Helix, Quadruple Helix, …, and an N-tuple of Helices: Explanatory models for analyzing the knowledge-based economy? *Journal of the Knowledge Economy*, *3*(1), 25–35.

Leydesdorff, L. (2016). Triple Helix models of innovation: Are synergies generated at national or regional levels? *Hélice*, *5*(1), 19–20.

Leydesdorff, L. (2021). *The evolutionary dynamics of discursive knowledge: Communication-theoretical perspectives on an empirical philosophy of science*. Springer International Publishing.

Leydesdorff, L., & Etzkowitz, H. (2001). The transformation of university-industry-government relations. *Electronic Journal of Sociology*, *5*(4). http://www.sociology.org/content/vol005.004/th.html

Leydesdorff, L., & Lawton Smith, H. (2022). Triple, Quadruple, and Higher-Order Helices: Historical Phenomena and (Neo-)Evolutionary Models. *Triple Helix*, *9*(1), 6–31. https://doi.org/10.1163/21971927-bja10022

Leydesdorff, L., & Park, H. (2014). Can synergy in Triple Helix relations be quantified? A review of the development of the Triple Helix indicator. *Triple Helix: A Journal of University-Industry-*

Government Innovation and Entrepreneurship, 1(1), 1–18. https://doi.org/10.1186/s40604-014-0004-z

Maschler, M., Peleg, B., & Shapley, L. S. (1967). *The Kernel and bargaining set for convex games* (Research Memorendum 1/25; p. 16). City University of New York - Hebrew University of Jerusalem.

Mêgnigbêto, E. (2017). Modelling the Triple Helix relationships with game theory: The rules of the game. *ISSI Newsletter*, *13*(3), 48–55.

Mêgnigbêto, E. (2018a). Measuring synergy within a Triple Helix innovation system using game theory: Cases of some developed and emerging countries. *Triple Helix: A Journal of University-Industry-Government Innovation and Entrepreneurship*, *5*(1), 1–22. https://doi.org/10.1186/s40604-018-0054-8

Mêgnigbêto, E. (2018b). Modelling the Triple Helix of university-industry-government relationships with game theory: Core, Shapley value and nucleolus as indicators of synergy within an innovation system. *Journal of Informetrics*, *12*(4), 1118–1132. https://doi.org/10.1016/j.joi.2018.09.005

Mêgnigbêto, E. (2019). Synergy within the West African Triple Helix innovation systems as measured with game theory. *Journal of Industry - University Collaboration*, *1*(2), 96–114. https://doi.org/10.1108/JIUC-03-2019-0008

Meyer, M., Grant, K., Morlacchi, P., & Weckowska, D. (2014). Triple Helix indicators as an emergent area of enquiry: A bibliometric perspective. *Scientometrics*, *99*(1), 151–174. https://doi.org/10.1007/s11192-013-1103-8

OCDE, & EUROSTAT. (2018). *Oslo Manual: Guidelines for collecting, reporting and using data on innovation* (4th edition). OECD Publishing; EUROSTAT. https://doi.org/10.1787/9789264304604-en

OECD. (2015). *The measurement of scientific, technological and innovation activities: Frascati manual: Guidelines for collecting and reporting data on research and experimental* *development* (7th ed). OECD Publishing. https://ec.europa.eu/eurostat/cache/metadata/Annexes/isoc_se_esms_an2.pdf

Pechersky, S. (2015). A note on external angles of the core of convex TU games, marginal worth vectors and the Weber set. *International Journal of Game Theory*, *44*, 487–498. https://doi.org/10.1007/s00182-014-0441-y

R Development Core Team. (2023). *R: a language and environment for statistical computing* [Computer software]. R Foundation for Statistical Computing. http://www.r-project.org

Roth, A. E. (1988a). Introduction to the Shapley value. In A. E. Roth (Ed.), *The Shapley value: Essays in honor of Lloyd S. Shapley* (pp. 1–27). Cambridge University Press.

Roth, A. E. (1988b). The expected utility of playing a game. In A. E. Roth (Ed.), *The Shapley value: Essays in honor of Lloyd S. Shapley* (pp. 51–70). Cambridge University Press.

Schocair, M. M., Dias, A. A., Galina, S. V. R., & Amaral, M. (2023). The Evolution of the Triple Helix Thematic: A Social Networks Analysis. *Triple Helix*, *9*(3), 325–368. https://doi.org/10.1163/21971927-bja10037

Schubert, A., & Glänzel, W. (2008). Ternary plots of science in a tripolar world. *ISSI Newsletter*, *4*(3), 51–52.

Shapley, L. S. (1965). *Notes on n-person games. VI : the core of convex games* (Memorandum for the United States Air Force RAND Project RM-4571-PR; p. 24). The RAND Corporation.

Shapley, L. S. (1971). Cores of convex games. *International Journal of Game Theory*, *1*(1), 11–26. https://doi.org/10.1007/BF01753431

Shinn, T. (2002a). Nouvelle production du savoir et Triple Hélice: Tendances du prêt-à-penser les sciences. *Actes de La Recherche En Sciences Sociales*, *141–142*, 21–30. https://doi.org/10.3406/arss.2002.2815

Shinn, T. (2002b). The triple helix and new production of knowledge prepackaged thinking on science and technology. *Social Studies of Science*, *32*(4), 599–614. https://doi.org/10.1177/0306312702032004004

Smith, M. R., Actions-User, Sanselme, L., & Badger, T. G. (2023). *Ternary: An R Package for Creating Ternary Plots* (2.2.1) [Computer software]. Zenodo. https://zenodo.org/record/1068996

Staudacher, J., & Anwander, J. (2019). *Using the R package CoopGame for the analysis, solution and visualization of cooperative games with transferable utility*. https://cran.r-project.org/web/packages/CoopGame/CoopGame.pdf

Su, N., Shi, Z., Zhu, X., & Xin, Y. (2021). An Evolutionary Game Model of Collaborative Innovation Between Enterprises and Colleges Under Government Participation of China. *SAGE Open*, *11*(1), 215824402199485. https://doi.org/10.1177/2158244021994854

Tol, R. S. J. (2012). Shapley values for assessing research production and impact of schools and scholars. *Scientometrics*, *90*(3), 763–780. https://doi.org/10.1007/s11192-011-0555-y

World Bank. (1997). *World development report: The state in a changing world*. Oxford University Press.

Zakaria, H., Kamarudin, D., Fauzi, M. A., & Wider, W. (2023). Mapping the helix model of innovation influence on education: A bibliometric review. *Frontiers in Education*, *8*, 1142502. https://doi.org/10.3389/feduc.2023.1142502

Zheng, X.-X., & Li, D.-F. (2023). A new biform game-based investment incentive mechanism for eco-efficient innovation in supply chain. *International Journal of Production Economics*, *258*, 108795. https://doi.org/10.1016/j.ijpe.2023.108795