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INTERTEMPORAL CONSUMPTION WITH ANTICIPATING, REMEMBERING, AND EXPERIENCING SELVES*

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We study intertemporal choice through a novel and flexible framework that accounts for savoring of future consumption and memories of past consumption. The model uses standard intertemporal budget constraints but enriches preference structures with utility from anticipation, remembering, and experience. We also present an internal commitment mechanism that ensures dynamic consistency. We provide a revealed preference characterization of this model and apply it to quarterly consumption data from Spanish households. Utility from anticipation is important—and time inconsistency not strictly needed—to rationalize consumption patterns in the data.

1. INTRODUCTION

Many consumption decisions have an intertemporal aspect: When to go on a city trip? When to go to a fancy restaurant? When to open an expensive bottle of champagne? The workhorse model to analyze intertemporal choice is the exponential discounting (*ED*) framework.[1] Yet, there are interesting anomalies that *ED* fails to capture.

A first observation is the pleasurable deferral of desirable outcomes in behavioral experiments. Loewenstein (1987) asked respondents about their preferred timing to receive a kiss from their favorite movie star. Respondents stated their desire to postpone the kiss for a couple of days, to maximize the duration of "savoring." These behavioral patterns arise not only in hypothetical experiments but also in practice (Laajaj, 2017). [2] This is in clear opposition to the standard life-cycle model, which assumes positive devaluing (impatience).[3] A second observation is the disproportionate spending by young people on celebrations and ceremonies early in life. One explanation suggested by Gilboa et al. (2016) is that some goods generate happy memories long after consumption took place. The authors refer to these goods as "memorable" consumption. Hai et al. (2020) recently showed that households optimally

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[1] We refer to the seminal contributions of Samuelson (1937) and Koopmans (1960).

[2] Social gatherings (e.g., visits to cinema, restaurants, or bars) and holidays all induce a sense of savoring in the time leading up to the actual event. Furthermore, survey data collected by Loewenstein and Sicherman (1991) challenge the notion of discounted present-value maximization for wages; there is a widespread preference for *increasing* wage profiles.

[3] But see Guo (2020) for a recent exception to this positive devaluing.

choose a *nonsmooth* consumption profile of memorable goods. Decreasing streams of physical consumption not necessarily imply declining welfare.

Both behavioral patterns have the following feature in common: agents care not just about physical outcomes but also about the mental image of past and future outcomes. There is a temporal dissociation between physical consumption and utility benefits, which can be enjoyed *before*, *during*, and *after* consumption. In addition, the level of anticipatory emotions and pleasant memories will generally depend on the good under consideration.

What we do and preview of results To account for these behavioral patterns, we develop a model of intertemporal consumption where we enrich the structure of preferences, but preserve standard intertemporal budget constraints. Hence, our model is situated between the *ED* framework (Samuelson, 1937) and the theory of *total* utility (Kahneman et al., 1997). Like *ED*, it uses a linear budget constraint and can deal with multiple observations over time. However, consumers enjoy utility from anticipation of future consumption bundles, experience of a current bundle, and remembering of past bundles (Elster and Loewenstein, 1992). The underlying utility functions do not have to agree on the valuation of each bundle. This fits in a larger literature on multiple selves: individual decisions are driven by several rationales or selves. According to one stream of multiself models that focus on dynamic inconsistency, a "self" refers to the consumer's objective function at each decision moment (and these selves interact in noncooperative ways, O'Donoghue and Rabin, 1999; Kőszegi, 2009). This is slightly different from our setup, where the consumer's objective function at each decision moment is itself composed of three selves: an anticipating, a remembering, and an experiencing one. The preferences of these selves are aggregated into a collective decision (Chiappori, 1988, 1992), following the seminal paper of May (1954) and recent contributions by Ambrus and Rozen (2014) and Jackson and Yariv (2014).[4] The tools and techniques to fully characterize intertemporal consumption patterns generated by these preferences then depend on the dynamic consistency of the decisions.

Turning to a description of our results, we show in Proposition 1 that the model is generally not time-consistent. First, the decision power of the different selves can change over time, which is a form of limited commitment. Second, Caplin and Leahy (2001) provided a discussion of the complex relationship between anticipation and time consistency. In our setup, the duration of savoring decreases naturally as time moves forward. This creates incentives for agents to revise their anticipated consumption downward; so-called *reverse* time inconsistency. Reverse time inconsistency has hampered analyses of anticipation due to practical and theoretical reasons. From a practical perspective, Gilboa et al. (2016) explained that reverse time inconsistency invalidates the standard use of decision theory to study dynamic interactions with anticipating selves. More fundamentally, Loewenstein (1987) argued that systematic acts of time inconsistency raise concerns about self-credibility. If the agent is aware that she will postpone consumption indefinitely, how can she still enjoy benefits from anticipation at all? This suggests the existence of an internal mechanism to neutralize the reduced *duration* of anticipation over time. Proposition 2 introduces additional structure that ensures dynamic consistency. In particular, we posit that the decision weight of the anticipating self increases toward the end of the planning period. This structure is consistent with the view that in "shorter" planning periods the consumer's preferences for improvement are activated. Under this condition, one can resort to a decision-theoretic approach to intertemporal consumption.

The rest of the article then develops a time-consistent version of the theory, which has clear testable implications for finite data sets of price and quantity observations per consumer. We name this *ICARES*, short for *intertemporal consumption* with *anticipating, remembering*, and *experiencing selves*. We also discuss relevant polar cases: intertemporal

[4] Cherchye et al. (2020) recently represented food choices as the result of an efficient agreement between a healthy and an unhealthy self.

consumption with anticipating and experiencing selves (*ICAES*), remembering and experiencing selves (*ICRES*), and an experiencing self only (*ICES*). *ICAES*, *ICRES*, and *ICES* predict increasing, decreasing, and smooth consumption profiles, respectively. Consumption patterns generated by *ICARES* can moreover be U-shaped. We bring the models to the data by means of revealed preference theory. The revealed preference approach, following Afriat (1967) and Varian (1982), offers an elegant way of assessing the empirical content of intertemporal models. Proposition 3 summarizes the data restrictions imposed by *ICARES*. Although the empirical content of *ICARES* differs from that of static utility maximization (*GARP*, generalized axiom of revealed preferences), *ICARES* still nests the *ED* model. Moreover, *ICAES* and *ICRES* separately identify preferences for anticipation and preferences for recall. In particular, high levels of consumption combined with high prices at the beginning of a sequence of observations violate the characterizations of *ICES* and *ICAES* whereas high levels of consumption combined with high prices toward the end of a sequence violate the characterizations of *ICES* and *ICRES*. This feature distinguishes our framework from other nonparametric tests of intertemporal models. Crawford and Polisson (2014) demonstrated that the testable implications of rational habit formation are indistinguishable from the ones of rational anticipation.[5] *ICARES*, by contrast, starts from a formal description of the asymmetric roles of temporal selves. It also has separability properties that make identification possible.

We then apply the empirical characterization to budget survey data. This demonstrates that the framework is practically relevant outside tailor-made experimental settings. Budget survey data are still the basis of much empirical work in demand theory and consumer analysis. To be more precise, we apply our revealed preference tests to a widely studied quarterly data set from a Spanish panel of consumers (Encuesta Continua de Presupuestos Familiares; ECPF). For each respondent, we study expenditure on various nondurable commodities over the course of a year. We apply *ICARES*, *ICAES*, *ICRES*, and *ICES* separately to singles and couples. *ICARES* has a near-perfect in-sample fit but lacks power, due to its flexibility. Special case *ICAES*, by contrast, is sufficiently powerful and still rationalizes about two-thirds of the data. These results hold both for single consumers and for couples. We thus find strong evidence of savoring in these budget survey data. More generally, the results suggest that time *inconsistency* is not strictly needed to rationalize the consumption behavior of singles or couples. We then examine the "anticipatory" nature of the goods in our data. We measure this by the increase in expenditure on each good toward the end of the sequence, for the subset of consumers who pass *ICAES*. The degree of anticipation varies considerably across goods. It is more pronounced for restaurant expenditures, expenditure on recreation services and cinema, long-distance traveling, and a larger group of goods that are complementary to leisure activities and special celebrations. Overall, the results are intuitive and in line with earlier research into savoring, thus supporting the external validity of our approach.

Finally, we also discuss the limitations of our budget survey data. In practice, consumers sometimes face binding liquidity constraints or unexpected income shocks. Both these are unobserved to the econometrician. For the econometrician, there is additional uncertainty in the form of measurement error. We therefore test the sensitivity of our results to these empirical challenges. Our investigations highlight that, the data limitations notwithstanding, richer models of temporal utility flows are empirically supported. These models can be fruitfully applied to recover aspects of the consumer's, short-to-medium run, planning problem.

Overview of the article The rest of the article unfolds as follows: Section 2 introduces the theory and Section 3 presents a convenient revealed preference approach to implement the theory. Section 4 applies the model to quarterly consumption data and Section 5 conducts robustness tests. Section 6 presents related literature and Section 7 concludes.

[5] However, the authors' definition of anticipation is different from "savoring." In their setting, anticipation implies that the consumer's tastes in t are affected by her expected outcomes at future times $t+1, t+2, \ldots$.

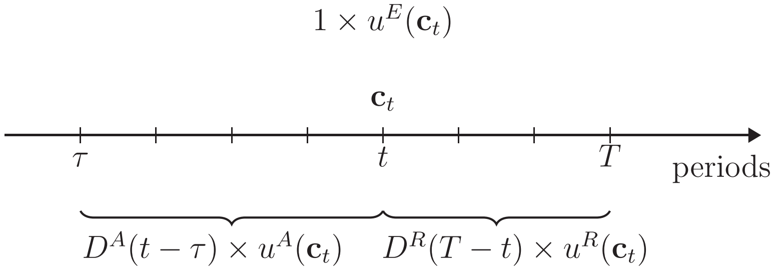

FIGURE 1

TIMELINE OF THE UTILITY FLOWS FROM ANTICIPATION, RECALL, AND EXPERIENCE OF $\mathbf{c}_t$.

2. A THEORY OF ANTICIPATING, REMEMBERING, AND EXPERIENCING SELVES

2.1. *Model Setup.* We start with an exposition of our notation. Agents choose a *consumption path* with expenditure on N commodities over the planning period $\mathcal{T} = (\tau_0, \ldots, T)$, where $T \in \mathbb{N}_0$ indicates the length of the period.[6] Let $(\mathbf{c}_t)_{t \geq \tau_0}$ denote a consumption path $(\mathbf{c}_{\tau_0}, \ldots, \mathbf{c}_T)$ where $\mathbf{c}_t \in \mathbb{R}^N_+$, and let $(\mathbf{p}_t)_{t \geq \tau_0}$ denote the corresponding price vectors $(\mathbf{p}_{\tau_0}, \ldots, \mathbf{p}_T)$ where $\mathbf{p}_t \in \mathbb{R}^N_{++}$. With r_t the applicable interest rate at time t, we can compute the relevant *discounted* prices as follows:

$$\boldsymbol{\rho}_t = \frac{\mathbf{p}_t}{\prod_{i=\tau_0}^{t-1} (1 + r_i)}.$$

In the empirical application, we use the average nominal interest rate on consumer loans. Furthermore, we use y to denote the *discounted* total resources of the consumer. For the moment, we will abstract from uncertainty regarding future prices and income. We thus adhere to perfect foresight, which is a restrictive yet common assumption in the revealed preference literature.[7] We will relax this assumption in Section 5. Feasible consumption plans are contained in the intertemporal budget set,[8]

$$(1) \qquad \mathcal{B}\big((\boldsymbol{\rho}_t)_{t \geq \tau_0}, y\big) = \left\{ (\mathbf{c}_t)_{t \geq \tau_0} \in \mathbb{R}^{N \times T}_+ \mid \sum_{t \geq \tau_0} \boldsymbol{\rho}_t \cdot \mathbf{c}_t \leq y \right\}.$$

A main feature of our study is that it allows a temporal dissociation between physical consumption and the utility gains from this consumption. To illustrate the utility flows in our framework, we first consider a single consumption bundle $\mathbf{c}_t$. Figure 1 situates this bundle graphically in the consumer's timeline, where the decision moment is given by $\tau \geq \tau_0$.

We let functions $u^i(\mathbf{c}_t)$ capture the instantaneous trade-offs between different goods in anticipation ($i = A$), experience ($i = E$), and recall ($i = R$). First, the mental image of future consumption $\mathbf{c}_t$ produces utility benefits in the form of savoring (Loewenstein, 1987). Agents enjoy benefits $u^A(\mathbf{c}_t)$ from anticipation *just before* consumption of $\mathbf{c}_t$. Furthermore, these psychological benefits carry over to other periods before t, and may even accumulate through time. The cumulative anticipated utility, seen from τ, is given by $D^A(t - \tau) \times u^A(\mathbf{c}_t)$. Map D^A

[6] Our theoretical framework can easily accommodate infinite horizons but we opt to present the finite-horizon version here, for ease of exposition and consistency with our empirical application. The finite-horizon version is clearly more in line with our emphasis on short-to-medium run planning of nondurable consumption. We refer to Appendix A.3 for details on the infinite-horizon extension.

[7] For instance, Demuynck and Verriest (2013), Adams et al. (2014) and Blow et al. (2021) also assume perfect foresight and apply their methods to the same data set as in the present article. We conduct an analysis of the predictability of prices in our data set in Appendix A.4.1.

[8] We use $\mathbf{x} \cdot \mathbf{z}$ to denote the dot product between the vectors $\mathbf{x}$ and $\mathbf{z}$.

is an increasing function of the remaining duration of savoring, $t - \tau$.[9] Cumulative anticipated utility from $\mathbf{c}_t$ is therefore explicitly time-dependent. Second, the mental image of past consumption produces utility gains in the form of happy memories (Gilboa et al., 2016). Agents enjoy benefits $u^R(\mathbf{c}_t)$ from remembering $\mathbf{c}_t$ *just after* its physical consumption. These psychological benefits carry over to other periods after t, and may also accumulate through time. The cumulative recollected utility from $\mathbf{c}_t$ is $D^R(T - t) \times u^R(\mathbf{c}_t)$. Map D^R is an increasing function of the total duration of remembering, $T - t$. The cumulative utility flow from recall of $\mathbf{c}_t$ is again explicitly time-dependent. It is worth to note that the time functions $D^A(t - \tau)$ and $D^R(T - t)$ capture the magnitude of the decoupling between a consumption event (taking place only once) and its utilities (enjoyed at multiple moments), and this magnitude is a function of time. Since there is no decoupling between physical consumption and its experience, agents simply derive value $u^E(\mathbf{c}_t)$ from physical consumption $\mathbf{c}_t$ at time t. We assume that mappings $u^i : \mathbb{R}_+^N \to \mathbb{R}$ are concave and monotonically increasing.[10]

We now generalize this logic from an environment with a single consumption bundle to an environment with repeated consumption events. Consider an agent who plans $\mathbf{c}_t$ for $t \in \{\tau, \ldots, T\}$. The total utility from anticipation of a consumption stream $(\mathbf{c}_t)_{t\geq\tau}$ is the sum of cumulative anticipated utilities:

$$U^A\left((\mathbf{c}_t)_{t\geq\tau}\right) = \sum_{t\geq\tau} D^A(t-\tau)u^A(\mathbf{c}_t). \tag{2}$$

Next, the total utility from recall is the sum of cumulative recollected utilities:

$$U^R\left((\mathbf{c}_t)_{t\geq\tau}\right) = \sum_{t\geq\tau} D^R(T-t)u^R(\mathbf{c}_t). \tag{3}$$

Finally, the total utility from experience integrates over all (standard) event utilities $u^E(\mathbf{c}_t)$, similar to the objective function of the life-cycle model:

$$U^E\left((\mathbf{c}_t)_{t\geq\tau}\right) = \sum_{t\geq\tau} u^E(\mathbf{c}_t). \tag{4}$$

The total utilities from anticipation (2), recall (3), and experience (4) differ in two main ways. The first distinction lies in the asymmetric weighting of consumption over time. Utility functions from anticipation, respectively, recall, attach more weight to consumption at the end (start) of the planning period. This asymmetry is reflected in the properties of the maps D^A (increasing in t) and D^R (decreasing in t). Functions $D^A(\cdot)$ and $D^R(\cdot)$ may also differ in form. The second distinction lies in the marginal rates of substitution between goods. Utility functions can differ in arguments and in shape. This accounts for heterogeneity in anticipatory emotions and enjoyable memories across commodities. The utility function from anticipation $u^A(\cdot)$ may, for instance, attach more weight to vacations and holidays and less weight to convenience goods from the supermarket. We do not a priori restrict commodities to be "anticipatory," "memorable," or "ordinary." In fact, in our setting, each commodity can have elements of all the temporal motives.

[9] In the special case where $D^A(t-\tau) = t - \tau + \tau_0$, the "cumulative" anticipated utility is simply the product of instant utility $u^A(\mathbf{c}_t)$ and the number of periods that precede it (Loewenstein and Prelec, 1993). Our generalization of this structure admits that anticipated utility changes *nonlinearly* with the time distance to consumption. A similar remark holds for recollected utility.

[10] Given our focus on savoring, we rule out aversive anticipatory emotions such as anxiety (Caplin and Leahy, 2001). Past consumption may also induce a sense of "loss." This is not taken into account by the baseline model, which focuses on "pleasant" memories. A small modification suffices to incorporate this. The function $-\tilde{u}^R(\mathbf{c}_t)$ can capture the negative impact of memories on utility. To keep the problem convex, assume that $\tilde{u}^R(\cdot)$ is subdifferentiable. The drawback is that u^A and this novel definition of $\tilde{u}^R$ have almost identical testable implications. Anticipatory emotions and "spiteful" memories push consumption in exactly the same direction (i.e., postpone consumption).

The natural next question is how consumers aggregate their preferences for anticipation, recall, and experience. In the presence of anticipation and given the conflicting temporal motives, such an aggregation might not be trivial. We will assume that consumption is the outcome of a bargaining process between an anticipating self (with preferences U^A), a remembering self (with preferences U^R), and an experiencing self (with preferences U^E) within the consumer. In that sense, at a given decision moment τ, the selves are assumed to behave cooperatively and select an efficient allocation of resources (Cherchye et al., 2020). We do admit that the respective decision weights of the selves depend on the time of decision $\tau \in \mathcal{T}$, thus allowing a form of limited commitment between the selves. The consumer's objective function in decision moment τ is then

$$V\big((\mathbf{c}_t)_{t\geq\tau}, \tau\big) = \omega^A(\tau)U^A\big((\mathbf{c}_t)_{t\geq\tau}\big) + \omega^R(\tau)U^R\big((\mathbf{c}_t)_{t\geq\tau}\big) + U^E\big((\mathbf{c}_t)_{t\geq\tau}\big), \tag{5}$$

where maps $\omega^i : \mathcal{T} \to \mathbb{R}$ determine the decision power of each self.

In sum, a consumer solves the following optimization problem in decision period τ:

$$\begin{gathered}\max\nolimits_{(\mathbf{c}_t)_{t\geq\tau}} V\big((\mathbf{c}_t)_{t\geq\tau}, \tau\big), \text{ subject to} \\ (\mathbf{c}_t)_{t\geq\tau} \in \mathcal{B}\big((\boldsymbol{\rho}_t)_{t\geq\tau}, y_\tau\big),\end{gathered} \tag{6}$$

where $y_\tau = y - \sum_{t=\tau_0}^{\tau-1} \boldsymbol{\rho}_t\mathbf{c}_t$ captures the consumer's residual income in period τ, conditional on past expenditures. The associated first-order conditions then read as follows:

$$\begin{gathered}\omega^A(\tau)D^A(t-\tau)\partial u^A(\mathbf{c}_t^*) + \omega^R(\tau)D^R(T-t)\partial u^R(\mathbf{c}_t^*) + \partial u^E(\mathbf{c}_t^*) = \lambda_\tau\boldsymbol{\rho}_t; \text{ and} \\ (\mathbf{c}_t^*)_{t\geq\tau} \in \mathcal{B}\big((\boldsymbol{\rho}_t)_{t\geq\tau}, y_\tau\big).\end{gathered} \tag{7}$$

We use notation $\partial u^i(\mathbf{c}_t)$ to denote the superdifferential of u^i $(i = A, R, E)$, following Rockafellar (1970). In case u^i is differentiable, the superdifferential equals the gradient of the function.

The left-hand side of (7) denotes the marginal utility of consumption at time t, seen from moment τ. This can be decomposed in three main effects: the marginal utility from savoring, memories, and experience. A necessary condition for $\mathbf{c}_t^*$ to lie on the optimal path is that these marginal utilities (in money terms) sum up to the market price. Parameter λ_τ captures the effect of relaxing the budget constraint, that is, the effect of a marginal change in the resources y_τ available for the remaining period $[\tau, T]$.

2.2. *Reverse Time Inconsistency.* The *ED* framework has strong mathematical appeal because its predicted choices are time-consistent. As time moves forward, the consumer has no incentives to diverge from her original plan (e.g., chosen in $\tau = \tau_0$). In this section, we show that time consistency generally does *not* hold for consumption with anticipating, remembering, and experiencing selves. In particular, compare the consumer's problem in decision moment τ, given by (6), with a similar problem when the consumer would make consumption choices at time $\tau' > \tau$:

$$\begin{gathered}\max\nolimits_{(\mathbf{c}_t)_{t\geq\tau'}} V\big((\mathbf{c}_t)_{t\geq\tau'}, \tau'\big), \text{ subject to} \\ (\mathbf{c}_t)_{t\geq\tau'} \in \mathcal{B}\big((\boldsymbol{\rho}_t)_{t\geq\tau'}, y_{\tau'}\big).\end{gathered} \tag{8}$$

We now first formalize the notion of dynamic consistency. In particular, let $(\mathbf{c}_t^*)_{t\geq\tau}$ be the solution to (6) and $(\hat{\mathbf{c}}_t)_{t\geq\tau'}$ the solution to (8). The consumer is then said to be dynamically consistent if $\mathbf{c}_t^* = \hat{\mathbf{c}}_t$ for all $t \in [\tau', T]$. Note that, if the consumer behaves in a time-consistent way, we can find consumption choices by solving (6) for $\tau = \tau_0$. There are however several differences between (6) and (8). The most important of these is that there is a change in the remaining duration of savoring associated with a consumption bundle in any future period. More specifically, $D^A(t-\tau') < D^A(t-\tau)$ because $\tau' > \tau$ and D^A is an increasing function. The consumer will enjoy *less* benefits from anticipation of consumption bundle $\mathbf{c}_t$ because the duration of savoring has become shorter. She will therefore have an incentive to reduce (or further postpone) this consumption. Loewenstein (1987) called this "*reverse time inconsistency*." Moreover, at the same time, there is a possible shift in the relative decision power of the anticipating self and the remembering self, $\omega^i(\tau') \neq \omega^i(\tau)$, for $i = A, R$. This can give rise to further time-inconsistency problems on the part of the consumer. Proposition 1 formalizes this (general) time-inconsistency property. We prove Proposition 1 by offering a counterexample to time consistency in Appendix A.1.

Proposition 1 (Time inconsistency). *Consider any $\tau, \tau' \in \mathcal{T}$, with $\tau < \tau'$. Let $(\mathbf{c}_t^*)_{t\geq\tau}$ be the solution to (6) and let $(\hat{\mathbf{c}}_t)_{t\geq\tau'}$ be the solution to (8), with $y_{\tau'} = y_\tau - \sum_{t=\tau}^{\tau'-1} \boldsymbol{\rho}_t \mathbf{c}_t^*$. Then, we do not necessarily have that $\hat{\mathbf{c}}_t = \mathbf{c}_t^*$ for all $t \in [\tau', T]$.*

2.3. *The Increasing Influence of the Anticipating Self.* Researchers interested in anticipation face issues of reverse time inconsistency. First, anticipating agents will generally, when the time leading up to the actual consumption shrinks, want to lower this consumption and reshuffle plans accordingly. This has blocked the use of standard decision-theoretic tools to analyze anticipation. Second, and more fundamentally, acts of reverse time inconsistency give rise to concerns about self-credibility. If agents delay consumption of a given commodity indefinitely—to maintain a sense of savoring—why would they still expect to consume this good at all? Loewenstein (1987) argued that acts of reverse time inconsistency ultimately come at a cost: they interfere with the agents' ability to savor *any* form of future consumption. These issues may be responsible for the lack of attention for savoring in the literature.

One way to circumvent the inconsistency is to assume *external* commitment devices that bind consumers to their plans. In laboratory experiments, for instance, subjects commit to chosen plans by design (Andreoni and Sprenger, 2012; Echenique et al., 2020). In a social context, consumption often requires organization and reservation (e.g., going out for drinks or dinner, buying tickets for the theater) thereby involving slightly more effort and commitment. Revising holiday plans may entail large cancellation costs. However, limiting anticipation and savoring to circumstances with clear reinforcement mechanisms seems overly restrictive (i.e., it would rule out anticipation for most types of nondurable consumption).

Anticipation is prevalent in everyday life. This suggests that other *internal* mechanisms mitigate acts of reverse time inconsistency and secure self-credibility.[11] We consider such internal mechanism to mitigate the reverse time inconsistency problem. To that end, when we study (5) again in detail, we can see that the anticipated utilities are weighted by two different factors: (i) the temporal shape of preferences, as represented by $D^A(t-\tau)$, and (ii) the decision weight $\omega^A(\tau)$ of the anticipating self at time τ. The interplay of both these factors determines the consumer's incentives to revise planned consumption. Proposition 2 presents the conditions of an internal mechanism that guarantees time-consistent behavior.

Proposition 2 (Time consistency). *Consider any $\tau, \tau' \in \mathcal{T}$, with $\tau < \tau'$. Let $(\mathbf{c}_t^*)_{t\geq\tau}$ be the solution to (6) and let $(\hat{\mathbf{c}}_t)_{t\geq\tau'}$ be the solution to (8), with $y_{\tau'} = y_\tau - \sum_{t=\tau}^{\tau'-1} \boldsymbol{\rho}_t \mathbf{c}_t^*$. Then, $\hat{\mathbf{c}}_t = \mathbf{c}_t^*$ for all $t \in [\tau', T]$ if*

$$\log \omega^A(\tau) = -\log D^A(t-\tau) + \alpha(t) \text{ for all } t \geq \tau, \text{ and } \tau \in \mathcal{T} \tag{9}$$

[11] In this respect, Benhabib and Bisin (2005) posit that the human brain consists of "automatic" and "controlled" processes. The latter can be used to exert internal commitment to earlier made consumption decisions, thereby overruling the temptation to alter consumption triggered by more automatic processes in the brain.

for some mapping $\alpha : \mathbb{R}_+ \rightarrow \mathbb{R}$, *and* $\frac{\partial \omega^R(\tau)}{\partial \tau} = 0$ *for all* $\tau \in \mathcal{T}$.

The conditions in Proposition 2 are sufficient for consumers to behave in a dynamically consistent way. When (9) holds, the effect of a decrease in the duration of savoring is exactly neutralized by an increase in the decision power of the anticipating self. This perfectly balances the two main forces that give rise to dynamic inconsistency issues in our setup: *reverse time inconsistency* associated with anticipation itself and *limited commitment* with respect to "bargaining" between the selves.

One qualitative restriction (9) imposes on the planning process is that the anticipating self's decision weight is increasing over time, that is, $\frac{\partial \omega^A(\tau)}{\partial \tau} > 0$. Such an increasing influence of anticipation on a consumer's decision making can be fruitfully compared to other, well-established behavioral patterns found and discussed in the literature. For example, notice that an increase in τ shortens the remaining planning period $[\tau, T]$, thereby drawing more attention to the "*sequential nature*" of the choice. This then activates the motives for anticipation (preferences for improvement), due to an increase in the perceived "integrity" aspects of outcome sequences (Loewenstein and Prelec, 1993). Such a negative correlation between the duration of the planning period and a preference for improvement (anticipation) has also been shown elsewhere. For example, Chapman (1996) conducted a study where subjects were asked about their preferences over different sequences of outcomes, varied by domain (health or money) and duration (lifetime or shorter, e.g., one year). The results suggested that both mattered for anticipatory preferences. In particular, though over the lifetime subjects seem to be relatively coherent with the notion of impatience, for shorter periods subjects had a more pronounced preference for improvement, indicating perhaps a stronger influence of anticipation. In a similar vein, Castillo et al. (2022) have shown that a larger fraction of subjects indicated a preference for improving sequences when the array size (the length of the sequence) decreased.

There are several tangential behavioral motives underpinning such anticipatory preferences *in sequences*. One of these motives is *loss aversion*, which implies that improving sequences or anticipatory behavior lead to a continual stimulation of positive benefits, in contrast with a declining sequence of outcomes in which a consumer experiences continuous losses. The loss aversion motive is also closely related to the so-called "contrast effect," where present outcomes are evaluated using a comparison with the past or future. Given that the strength of past comparisons is plausibly stronger (Prelec and Loewenstein, 1991), the contrast effect tends to stimulate preferences for anticipation (improvements over time). Another phenomenon underlying higher degrees of anticipation is the *recency effect*, in which evaluations of outcomes depend on the most recent experience. It is therefore beneficial to place desirable outcomes later in the planning period.

An important advantage of Proposition 2 is that it allows us to study consumption behavior by solving (6) for $\tau = \tau_0$. In fact, given condition (9), the consumer's overall utility (5) can also be written as:

$$\tilde{V}\left((\mathbf{c}_t)_{t \geq \tau_0}\right) = V\left((\mathbf{c}_t)_{t \geq \tau_0}, \tau_0\right) = \sum_{t \geq \tau_0} \left[a(t) u^A(\mathbf{c}_t) + b(t) u^R(\mathbf{c}_t) + u^E(\mathbf{c}_t)\right], \tag{10}$$

where $a(t) = \omega^A(\tau) D^A(t - \tau)$, and $b(t) = \omega^R(\tau) D^R(T - t)$. Given that D^A is increasing in the residual time $t - \tau$ between the consumption event and the decision moment, we have that a is increasing in t. Furthermore, given that D^R is increasing in the time distance $T - t$ between the end of the planning period and the consumption event, we have that b is decreasing in t.

An interesting feature of the framework is that, in contrast to the *ED* model, the overall utility of the consumer is nonstationary, as evidenced from the form in (10). This highlights the fact that stationarity of the objective function is not necessary for dynamic consistency.

This point has also been made recently in the contribution by Drouhin (2020), who characterized all felicity functions that yield dynamically consistent choices. The form of $\tilde{V}$ is consistent with this class of utility functions.[12]

A special case of our framework is formally similar to (nonstationary) intertemporal models with discount functions $D(t)$. To see this, one can impose that all subutility functions $i = A, R, E$ are the same: $u^A(\cdot) = u^R(\cdot) = u^E(\cdot) = u(\cdot)$. Then, the discount function $D(t)$ can be written as a reduced form of the decision weights ω^i and time functions D^i associated with anticipation, remembering, and experience. More specifically, $D(t) = a(t) + b(t) + 1$. Given that $a(t)$ is nondecreasing in t whereas $b(t)$ is nonincreasing, $D(t)$ can be increasing, decreasing, or smooth.

3. TESTABLE IMPLICATIONS

3.1. *Empirical Characterization.* We now turn to an empirical characterization of the model. We have access to a (finite) data set of prices, interest rates, and consumption choices across multiple commodities; $\mathcal{D} = \{\mathbf{p}_t, r_t, \mathbf{c}_t\}_{t \in \mathcal{T}}$. The goal is to test whether the observed choices, together with the prices and interest rates, are consistent with the implications of the model. The characterization is based on revealed preference (RP) theory. The revealed preference approach has two main advantages. First, it allows us to test the intertemporal behavior of each consumer separately, thus maximally accounting for preference heterogeneity. Second, it imposes no parametric specification on utility functions $u^i(\cdot)$ of the different selves. We can run our tests without specifying, a priori, which goods are partly anticipatory or partly memorable in nature.

In principle, beyond the restriction in Proposition 2, the time functions D^A and D^R can also remain unspecified. It is however common in the revealed preference approach to choose a functional form for time discounting. A functional specification is useful for practical purposes (i.e., to reduce the curse of dimensionality) and because it improves empirical tractability. We opt for an exponential form: the accumulation of anticipated utilities from $\mathbf{c}_t$ over time is given by $D^A(t-\tau) = (\beta^A)^{t-\tau+1}$. Similarly, the compounding of recollected utilities from $\mathbf{c}_t$ is captured by $D^R(T-t) = (\beta^R)^{T-t+1}$. Thus, parameter $\beta^A \geq 1$ (respectively, $\beta^R \geq 1$) determines the extent to which instantaneous utilities from anticipation (respectively, remembering) accumulate and carry over to period τ (respectively, period T). Note that, in the limiting case where $\beta^i = 1$, the position of $\mathbf{c}_t$ in the sequence no longer matters. To satisfy condition (9) (needed for dynamic consistency) our parameterization of compounding psychological utility implies that the decision weight of the anticipating self takes the form $\omega^A(\tau) = \omega_0^A (\beta^A)^{\tau-\tau_0}$, whereas the decision weight of the remembering self is fixed at $\omega^R(\tau) = \omega_0^R$. Notice that ω_0^A and ω_0^R reflect the "initial" decision weights at time τ_0.

Consumption choices are made as a solution to maximizing (10), subject to the intertemporal budget constraint $(\mathbf{c}_t^*)_{t\geq\tau_0} \in \mathcal{B}\left((\boldsymbol{\rho}_t)_{t\geq\tau_0}, y\right)$. The associated first-order conditions are then:

$$\omega_0^A (\beta^A)^{t-\tau_0+1} \partial u^A(\mathbf{c}_t^*) + \omega_0^R (\beta^R)^{T-t+1} \partial u^R(\mathbf{c}_t^*) + \partial u^E(\mathbf{c}_t^*) = \lambda \boldsymbol{\rho}_t; \text{ and}$$
$$(\mathbf{c}_t^*)_{t\geq\tau_0} \in \mathcal{B}\left((\boldsymbol{\rho}_t)_{t\geq\tau_0}, y\right). \tag{11}$$

Clearly, (11) are very similar to (7), with the difference that now we have specified D^A, D^R (and $\omega^A(\tau), \omega^R(\tau)$). Since the chosen specification satisfies dynamic consistency condition (9) by construction, any consumption plan $(\mathbf{c}_t^*)_{t\geq\tau_0}$ that solves system (11) will also solve subsequent optimization problems for decision periods $\tau > \tau_0$.

To ease reading, we simplify the notion of "intertemporal consumption with anticipating, remembering, and experiencing selves, and exponential time factors satisfying (9)" to "ratio-

[12] Drouhin uses a continuous time, single consumption good, representative agent framework with a capital accumulation constraint. The class of time-consistent utility functions is of the form $u(c, \tau, t) = d(\tau) u(c, t) + \gamma(t, \tau)$.

nalizability by *ICARES*." Definition 1 formalizes our notion of rationalizability by *ICARES*. We also define three polar cases with stronger testable implications: *ICAES*, *ICRES*, and *ICES*.

Definition 1 (Rationalizability). Consider the data set $\mathcal{D} = \{\mathbf{p}_t, r_t, \mathbf{c}_t\}_{t\in\mathcal{T}}$.

- A consumption plan $(\mathbf{c}_t)_{t\geq\tau_0}$ is rationalizable by ICARES if there exist monotone and concave functions u^A, u^R, u^E, decision weights ω_0^A, ω_0^R, and time factors $\beta^A, \beta^R \geq 1$ so that condition (11) holds.
- A consumption plan $(\mathbf{c}_t)_{t\geq\tau_0}$ is rationalizable by ICAES if there exist monotone and concave functions u^A, u^E, a decision weight ω_0^A, and a time factor $\beta^A \geq 1$ so that condition (11) holds with $\omega_0^R = 0$.
- A consumption plan $(\mathbf{c}_t)_{t\geq\tau_0}$ is rationalizable by ICRES if there exist monotone and concave functions u^R, u^E, a decision weight ω_0^R, and a time factor $\beta^R \geq 1$ so that condition (11) holds with $\omega_0^A = 0$.
- A consumption plan $(\mathbf{c}_t)_{t\geq\tau_0}$ is rationalizable by ICES if there exists a monotone and concave function u^E so that condition (11) holds with $\omega_0^A = \omega_0^R = 0$.

A few remarks are in order. First, following Browning (1989), Crawford (2010), Demuynck and Verriest (2013), and Adams et al. (2014), we define rationalizability in terms of consistency with the first-order conditions of the associated model. Second, the revealed preference approach allows researchers to test a model for each consumer separately. This is equally reflected in Definition 1, where the data $\mathcal{D} = \{\mathbf{p}_t, r_t, \mathbf{c}_t\}_{t\in\mathcal{T}}$ are a time series for one consumer. In principle, each consumer is characterized by a unique combination of selves (ω_0^A, ω_0^R) and a unique set of utility functions associated with experience, anticipation, and/or remembering.

We define shadow price vectors $\tilde{\mathbf{p}}_t^i = \omega_0^i/\lambda \times \partial u^i(\mathbf{c}_t)$ for $i = A, R, E$. Using these prices, we formulate our main result. The proofs are in Appendix A.1.

Proposition 3. *Consider the data set $\mathcal{D} = \{\mathbf{p}_t, r_t, \mathbf{c}_t\}_{t\in\mathcal{T}}$. A consumption plan $(\mathbf{c}_t)_{t\in\mathcal{T}}$ is rationalizable by ICARES if and only if there exist numbers u_t^A, u_t^R, u_t^E, nonnegative shadow prices $\tilde{\mathbf{p}}_t^A, \tilde{\mathbf{p}}_t^R, \tilde{\mathbf{p}}_t^E$, and time factors $\beta^A, \beta^R \geq 1$ such that for all $s, t \in \mathcal{T}$:*

(12)
$$u_s^A - u_t^A \leq \tilde{\mathbf{p}}_t^A \cdot (\mathbf{c}_s - \mathbf{c}_t);$$

(13)
$$u_s^R - u_t^R \leq \tilde{\mathbf{p}}_t^R \cdot (\mathbf{c}_s - \mathbf{c}_t);$$

(14)
$$u_s^E - u_t^E \leq \tilde{\mathbf{p}}_t^E \cdot (\mathbf{c}_s - \mathbf{c}_t);$$

(15)
$$\left(\beta^A\right)^{t-\tau_0+1} \times \tilde{\mathbf{p}}_t^A + \left(\beta^R\right)^{T-t+1} \times \tilde{\mathbf{p}}_t^R + \tilde{\mathbf{p}}_t^E = \boldsymbol{\rho}_t.$$

In addition, ICAES, respectively, ICRES, requires that $\tilde{\mathbf{p}}_t^R = 0$ $(\tilde{\mathbf{p}}_t^A = 0)$. Finally, ICES requires that $\tilde{\mathbf{p}}_t^A = \tilde{\mathbf{p}}_t^R = 0$.

Proposition 3 shows that our definitions of rationalizability have equivalent representations in terms of data consistency with systems of (in)equalities. Conditions (12)–(14) are Afriat-style inequalities and stem from the properties of the utility functions (in particular concavity). Conditions (15) reflect the first-order conditions (11). It is worth to note that these conditions still hold in the case of corner solutions: zero consumption of some goods n.[13] The additional structure imposed by *ICAES*, *ICRES*, and *ICES* comes in the form of straightforward restrictions on the shadow price vectors $\tilde{\mathbf{p}}_t^A$ and $\tilde{\mathbf{p}}_t^R$. Conditional on values for β^A and β^R, the system of inequalities (12)–(15) is linear in the unknowns (the utility numbers and shadow prices). This makes testing for rationalizability computationally manageable. In practice, we will run a grid search on β^A and β^R.

Let us now study the conditions of Proposition 3 in more detail. First, perhaps surprisingly, *ICARES* does not nest repeated static utility maximization. The empirical content of the utility maximization hypothesis is exhausted, in a revealed preference sense, by the *GARP*. More formally,

COROLLARY 1. *ICARES and (repeated) static utility maximization have independent testable implications.*

Second, the RP conditions in Proposition 3 resemble the RP characterization for additive separability by Varian (1983). Varian (1983) derived necessary and sufficient conditions for rationalizability of the data by a sum of utility functions. Our characterization has three distinguishing features: the marginal utility of wealth λ is constant (and hence absorbed in utilities u_t^i and shadow prices $\tilde{\mathbf{p}}_t^i$), we have a *weighted* sum of utilities with *time-dependent* weights $\left(\beta^A\right)^{t-\tau_0+1}$ and $\left(\beta^R\right)^{T-t+1}$, and the same bundle $\mathbf{c}_t$ can enter as an argument in all three subutility functions. The latter property demonstrates that our framework does not impose weak (or even latent, Blundell and Robin, 2000) separability between anticipatory, memorable, and ordinary consumption goods. The additive structure of *ICARES* stems from two modeling choices: (i) we "decompose" the consumption problem by means of subutility functions from anticipation, recall, and experience, and (ii) the total subutility from each temporal motive is the sum of the corresponding discounted utility flows across consumption events. Whereas the summation in (ii) may seem restrictive, we show that our model is in fact a strict generalization of the *ED* model. The revealed preference conditions of *ICARES* (and, more specifically, *ICRES*) are a subset of the revealed preference conditions of the *ED* model with uniform discount factors β. The test of *ICRES* collapses to the one of *ED* when all $u_s^E, u_t^E, \tilde{p}_{n,t}^E = 0$ and $\beta^R = 1/\beta$.

COROLLARY 2. *ED is a special case of ICRES.*

Finally, an attractive feature of our setup is that preferences for anticipation and preferences for recall have separate testable implications. This is in sharp contrast to the more general "nonseparable" preference structures discussed in Crawford and Polisson (2014). The asymmetric roles of temporal selves, combined with the separability properties of *ICARES,* enable us to empirically distinguish anticipated from recollected utilities. Corollary 3 formalizes the independence of *ICAES* and *ICRES*.

COROLLARY 3. *ICAES and ICRES have independent testable implications.*

3.2. *Implications for Consumption Patterns: Simulations.* In this section, we first illustrate the empirical predictions of each of our models. To this end, we choose a parametric form for the utility functions and use this to generate simulated intertemporal consumption. We

[13] At corners where $c_{n,t} = 0$, it is in fact possible that

$$\left(\beta^A\right)^{t-\tau_0+1} \times \tilde{p}_{n,t}^A + \left(\beta^R\right)^{T-t+1} \times \tilde{p}_{n,t}^R + \tilde{p}_{n,t}^E \leq \rho_{n,t}.$$

However, if this is the case and the Afriat inequalities hold at $c_{n,t} = 0$, one can always find (larger) values of $\tilde{p}_{n,t}^A$, $\tilde{p}_{n,t}^R$, or $\tilde{p}_{n,t}^E$ so that the condition holds with equality and without violating the Afriat inequalities. The revealed preference approach is therefore robust to corner solutions.

Table 1
SIMULATED CONSUMPTION BEHAVIOR UNDER SCENARIO I.

Data Generated by *ICES*	Data Generated by *ICAES*	Data Generated by *ICRES*	Data Generated by *ICARES*
$\mathbf{c}_t$	$\mathbf{c}_t$	$\mathbf{c}_t$	$\mathbf{c}_t$
(0.3902, 1.6)	(1.0465, 0.6129)	(1.6995, 0.6129)	(1.6058, 0.3791)
(0.4, 1.6)	(1.2565, 0.6129)	(1.4772, 0.6129)	(1.5959, 0.3791)
(0.4, 1.6)	(1.4772, 0.6129)	(1.2565, 0.6129)	(1.5959, 0.3791)
(0.3902, 1.6)	(1.6995, 0.6129)	(1.0465, 0.6129)	(1.6058, 0.3791)

demonstrate that *ICAES*, *ICRES*, and *ICES* produce increasing, decreasing, and smooth consumption profiles, respectively. Consumption generated with *ICARES* can moreover be U-shaped with reduced levels of consumption in the middle of the sequence.

Next, we discuss how the simulated consumption patterns allow us to discriminate between different models. We highlight the specific data patterns that lead to a rejection of a model. A common feature of underlying violations is that there are observations in the data with both high prices and high levels of consumption. We show in scenario I that the distinction between *ICRES* and *ICAES* lies in the position (early or late, respectively) of such observations in the sequence. We then present in scenario II a data set that is incompatible even with the most flexible characterization (*ICARES*). This data set has high levels of consumption, at relatively high prices, in the middle of the sequence. The data set in scenario III still passes *ICARES*/*ICRES* albeit only with moderate levels of remembering. This scenario illustrates that the time factors (in case β^R) can be bounded from above in certain situations.

In all our simulations, we set $[\tau_0, T] = [1, 4]$ and we restrict attention to $N = 2$ commodities for simplicity. We use utility functions of the (logarithmic) Cobb–Douglas form:

$$u^i(c_{1,t}, c_{2,t}) = \alpha^i \log(c_{1,t}) + \left(1 - \alpha^i\right) \log(c_{2,t}), \text{ where } i = A, R, E. \tag{16}$$

Finally, the intertemporal budget constraint is

$$\sum_{t=1}^{4} \left[\rho_{1,t} c_{1,t} + \rho_{2,t} c_{2,t}\right] = 8.$$

Scenario I In this scenario, we set $\alpha^E = 0.2$ and $\alpha^A = \alpha^R = 1$. In words, good 1 produces little utility from experience, whereas good 2 yields no utility from anticipation or recall. The extent to which the anticipated (recollected) utilities of consumption accumulate through time and carry over to earlier (later) periods is given by a time factor $\beta^A = \beta^R = 1.2$. The consumer faces the following prices, where each row corresponds to a time period t:

$$\begin{pmatrix} \mathbf{p}_t \\ (1.025, 1) \\ (1, 1) \\ (1, 1) \\ (1.025, 1) \end{pmatrix}.$$

We then use these prices and the overall resources available, $y = 8$, to simulate consumption behavior according to *ICES* ($\omega_0^A = \omega_0^R = 0$), *ICAES* ($\omega_0^A = 1, \omega_0^R = 0$), *ICRES* ($\omega_0^A = 0, \omega_0^R = 1$), and *ICARES* ($\omega_0^A = \omega_0^R = 1$). Table 1 reports the consumption patterns for good 1 (first column) and good 2 (second column) across different time periods (rows). Under *ICES*, consumption is more or less smoothed over time. By contrast, *ICAES*, respectively, *ICRES*, predicts increasing (decreasing) consumption of good 1. *ICARES* finally produces a small increase in consumption of good 1 at the start *and* at the end of the sequence.

We now apply the tests of Proposition 3 to the simulations of Table 1.

The smoothed consumption profile (generated with $\omega_0^A = \omega_0^R = 0$) satisfies the revealed preference characterizations of *ICES*, *ICAES*, *ICRES*, and *ICARES*. This must necessarily hold because the data were generated by *ICES*, and *ICES* is a special case of all other models. This data set satisfies the conditions of *ICES* because the consumer buys more of a commodity at times when the commodity is cheap (e.g., consider the consumption of good $n = 1$ at times $t = 2, 3$).

Next, the increasing consumption pattern ($\omega_0^A = 1, \omega_0^R = 0$) satisfies the conditions of *ICAES* but violates the conditions of *ICRES*, whereas the decreasing pattern ($\omega_0^A = 0, \omega_0^R = 1$) satisfies *ICRES* but violates *ICAES*. This shows that *ICAES* and *ICRES* have separate testable implications, thereby confirming Corollary 3. The data set generated with $\omega_0^A = 1, \omega_0^R = 0$ violates *ICRES* because condition (15) imposes that $\left(\beta^R\right) \tilde{p}_{1,4}^R + \tilde{p}_{1,4}^E = 1.025$ and $\left(\beta^R\right)^2 \tilde{p}_{1,3}^R + \tilde{p}_{1,3}^E = 1$, whereas conditions (13)–(14) impose $\tilde{p}_{1,4}^R \leq \tilde{p}_{1,3}^R$ and $\tilde{p}_{1,4}^E \leq \tilde{p}_{1,3}^E$. No set of shadow prices $\tilde{p}_{1,3}^R, \tilde{p}_{1,4}^R, \tilde{p}_{1,3}^E, \tilde{p}_{1,4}^E \geq 0$ can meet these specific requirements. High levels of consumption combined with high prices *toward the end of a sequence* violate the characterizations of *ICES* and *ICRES*. Analogously, the data set generated with $\omega_0^A = 0, \omega_0^R = 1$ violates *ICAES* because condition (15) imposes that $\left(\beta^A\right) \tilde{p}_{1,1}^A + \tilde{p}_{1,1}^E = 1.025$ and $\left(\beta^A\right)^2 \tilde{p}_{1,2}^A + \tilde{p}_{1,2}^E = 1$, whereas conditions (13)–(14) impose $\tilde{p}_{1,1}^A \leq \tilde{p}_{1,2}^A$ and $\tilde{p}_{1,1}^E \leq \tilde{p}_{1,2}^E$. No set of shadow prices $\tilde{p}_{1,1}^A, \tilde{p}_{1,2}^A, \tilde{p}_{1,1}^E, \tilde{p}_{1,2}^E \geq 0$ can meet all these requirements. High levels of consumption combined with high prices *at the beginning of a sequence* violate the characterizations of *ICES* and *ICAES*.

The final data set (generated with $\omega_0^A = \omega_0^R = 1$) *starts* with a similar pattern of decreasing prices and quantities as the third data set and *ends* with a similar pattern of increasing prices and quantities as the second data set. By the above-mentioned reasoning, these data violate both the conditions of *ICAES* and the conditions of *ICRES*. *ICARES*, by contrast, rationalizes this because it generates sufficient anticipation of consumption at time $t = 4$ to rationalize the *final increase* in good 1, whereas at the same time generating sufficient remembering of consumption at time $t = 1$ to rationalize the *initial decrease* in good 1. Only *ICARES* can rationalize the consumption profile that peaks in the beginning *and* in the end of the sequence—the periods with the highest relative prices.

Scenario II We also use simulations to investigate the differences between *ICARES* and two other classes of consumption models: static models of utility maximization and dynamic models with *hyperbolic* discounting. Hyperbolic discounting offers a time-inconsistent description of intertemporal consumption. The static model is a useful benchmark because it nests many intertemporal models, such as exponential (and hyperbolic) discounting.[14]

Under the current scenario, we adjust the utility specification as follows: In particular, for the *ICARES* simulations, we will now assume that all anticipation comes from good 2 ($\alpha^A = 0$) whereas for the other models we assume a simple logarithmic form

$$u(c_1) = \log(c_1).$$

[14] As originally shown by Browning (1989), testing the life-cycle model under perfect foresight (and in the absence of borrowing constraints) is equivalent to testing a condition called cyclical monotonicity (CM), which is stronger than *GARP*. For quasi-hyperbolic discounting, it has been shown that its RP characterization implies intra-period consistency with *GARP* (see Blow et al., 2021).

TABLE 2
SIMULATED CONSUMPTION BEHAVIOR UNDER SCENARIO II.

Data Generated by *ICARES*	Data Generated by Hyperbolic Model	Data Generated by Static Utility Maximization
$\begin{pmatrix} \mathbf{c}_t \\ (1.1938, 0.8067) \\ (1.0110, 0.9450) \\ (0.8586, 1.0963) \\ (0.7203, 1.2782) \end{pmatrix}$	$\begin{pmatrix} \mathbf{c}_t \\ (1.9961, 0) \\ (1.9983, 0) \\ (1.9943, 0) \\ (1.9345, 0) \end{pmatrix}$	$\begin{pmatrix} \mathbf{c}_t \\ (0.9756, 0) \\ (2.9603, 0) \\ (1, 0) \\ (3, 0) \end{pmatrix}$

We again assume that total resources over the entire time horizon are given by $y = 8$, and for the static utility maximization case we consider a sequence of incomes (1,3,1,3). Prices are given by:

$$\begin{pmatrix} \mathbf{p}_t \\ (1.025, 1) \\ \left(\sqrt{1.025} + 0.001, 1\right) \\ \left(1, \sqrt{1.025} + 0.001\right) \\ (1, 1.025) \end{pmatrix}.$$

We then simulate consumption over $T = 4$ periods for *ICARES*, static utility maximization, and the quasi-hyperbolic discounting model. With regards to the latter, we assume that the hyperbolic planner is sophisticated. She is aware that her future self's preferences over consumption will be different; nonetheless, she cannot commit to any future plans. Consequently, a sophisticated quasi-hyperbolic discounter still behaves in a time-inconsistent manner. A full revealed preference analysis of such time-inconsistent model has been studied in Blow et al. (2021). We select a discount factor of $\delta = 0.97$ for this consumer. For *ICARES*, we assume a strong weighting of both anticipated and recollected utilities: we set $\omega_0^A = \omega_0^R = 4$. Table 2 reports the simulated consumption patterns for good 1 (first column) and good 2 (second column) under scenario II. *ICARES* predicts that consumption of the first—memorable—good decreases systematically whereas consumption of the second—anticipatory—good increases continuously. The hyperbolic model predicts that consumption of the first good peaks in period 2. Finally, according to the static model, consumption peaks in periods 2 and 4. This is simply because the consumer is more wealthy in these periods.

We now apply revealed preference tests to these simulated data. We first note that the data generated by *ICARES* violate *GARP* (and thereby immediately violate the RP characterization of quasi-hyperbolic discounting). The main reason for this violation is that, in the second and third periods, the consumer buys more of a commodity when it is relatively more expensive.

We further note that the data generated by the static model and the hyperbolic model violate the revealed preference characterization of *ICARES*. The violation is situated in the fact that consumption of good 1 increases between observations 1 and 2, and then decreases between observations 2 and 3. Conditions (12)–(14) imply $\tilde{p}^i_{1,2} \leq \tilde{p}^i_{1,3}$ and $\tilde{p}^i_{1,2} \leq \tilde{p}^i_{1,1}$ for all $i = A, R, E$. This is not compatible with conditions (15) associated with the shadow prices of good 1 at times $t = 1, 2, 3$. We refer the interested reader to detailed calculations in Appendix A.2.1. In words, between $t = 2$ and $t = 3$, consumption of good 1 falls despite the corresponding price decrease. We need strong remembering motives to explain this. However, strong remembering is inconsistent with the fact that consumption is higher in $t = 2$ compared to $t = 1$.[15] *ICARES* cannot explain why the consumer buys more in period 2—in the middle of a sequence—when consumption is expensive.

[15] The price decrease at the start of the sequence is too modest to reconcile the low initial consumption with remembering.

TABLE 3
SIMULATED CONSUMPTION BEHAVIOR UNDER SCENARIO III.

Data Generated by *ICRES*

$$\begin{pmatrix} \mathbf{c}_t \\ (1.2444, 0.7029) \\ (1.2455, 0.7029) \\ (1.2389, 0.7029) \\ (1.1422, 0.7029) \end{pmatrix}$$

For completeness, we also verify that the final increase in consumption in period 4, generated by the static model, violates the revealed preference conditions of the hyperbolic model.

Scenario III In this scenario, we study the effect of changes in the factors β^A and β^R. It can be easily observed that higher values of β^A and β^R allow heavier shifts in the first-order conditions from one period to the next. Intuitively, this broadens the range of data patterns that can be rationalized. In this exercise, however, we show that higher values for β^A and β^R *not necessarily* explain the data better. We again select the parametric form specification as in scenario I. Furthermore, we will assume a new time series for prices,

$$\begin{pmatrix} \mathbf{p}_t \\ (1.175, 1) \\ (1.08, 1) \\ (1, 1) \\ (1, 1) \end{pmatrix}.$$

We now simulate the *ICRES* model with a parameter value $\beta^R = 1.1$. The other preference parameters are fixed to the values of scenario I. Table 3 shows the results of the simulation. The consumption of good 1 first increases, reaches its peak in period 2, and then decreases until period 4.

The data set is similar to the one in scenario II, but this time the price of consumption in $t = 1$ is sufficiently high. The data set is consistent with *ICRES* with moderate (but not high) levels of β^R, producing an upper bound on β^R. We first verify that the data indeed satisfy the revealed preference characterization of *ICRES* with $\beta^R = 1.1$. As in scenario II, some remembering is required to explain the drop in consumption of good 1 between $t = 2$ and $t = 3$. In this case, however, the high price of consumption in $t = 1$ reconciles remembering with the fact that consumption is also higher in $t = 2$ compared to $t = 1$. Still, the data violate the revealed preference conditions with $\beta^R = 1.2$. In particular, conditions (13) and (14) imply $\tilde{p}^i_{1,2} \leq \tilde{p}^i_{1,3}$ and $\tilde{p}^i_{1,2} \leq \tilde{p}^i_{1,1}$ for $i = R, E$. Very high levels of β^R attach so much weight to consumption in $t = 1$ that the high price of good 1 can no longer fully rationalize its low quantity. Detailed calculations are in Appendix A.2.2. Intuitively, the fact that the consumption of good 1 peaks in period 2 (and not in period 1) can only be rationalized by *moderate* levels of remembering.

4. EMPIRICAL APPLICATION

In this section, we apply our models of *ICARES* to budget survey data. We start with a discussion of the data from the ECPF. We then bring the time-consistent versions *ICAES*, *ICRES*, and *ICARES* to the data by means of our revealed preference characterizations. We find strong empirical support for *ICAES*. This shows that dynamic inconsistency, as in Adams et al. (2014) or Blow et al. (2021), is not the only possible mechanism that can explain intertemporal consumption data. It suffices to allow for flexible forms of anticipation and remembering. We finally identify which goods have a strong anticipatory element. We classify goods on the basis of their anticipatory nature.

4.1. *Data Description.* For our application, we use data from the ECPF collected by the Spanish Statistics Office (INE). We selected these data for two main reasons. First, consumption data are collected in every quarter. We focus on respondents who reported their expenditure in four *consecutive* quarters between 1985 and 1996. [16] The quarterly nature of the data is important. After all, our objective is to describe consumption planning decisions in the short-to-medium run. The consumer's entire life cycle can be composed of several of these planning periods (i.e., preferences can change with age). The quarterly dimension also limits our assumptions of stationary utility u^i and perfect foresight to a relatively narrow time interval (i.e., one year). We study a relaxation of the perfect foresight assumption in Section 5. Second, the ECPF is used frequently for nonparametric tests of intertemporal consumption models, see, for instance, Crawford (2010), Demuynck and Verriest (2013), Adams et al. (2014), and Blow et al. (2021). This allows us to benchmark the empirical performance of *ICARES* against the performance of other intertemporal models.

We implement the sample selection criteria discussed in Adams et al. (2014). In particular, we only keep consumers who completed all four interviews. Furthermore, the sample selection only keeps households that have a fixed number of children over the four interviews, to exclude "big shocks" such as child birth. We also select households in which individuals have a stable employment status over the interviews, again excluding shocks such as job loss. This alleviates issues of nonseparability between time use and consumption. We only keep consumers with positive total expenditures over the four interviews. Seen together, the selection criteria give us a sample of 2,052 consumers (1,880 couples plus 172 singles). This strikes a balance between a sufficiently sizable data set on one hand and a long enough panel to conduct revealed preference tests on the other hand.[17]

We consider eight nondurable commodities: (i) food and drinks, (ii) clothing and footwear,(iii) household services including heating, water, and furniture repair, (iv) transport, (v) petrol, (vi) leisure including cinema, theatre, and sports,(vii) personal services, (viii) restaurants and bars. At this point, we also would like to add that the level of aggregation of goods in our empirical application mirrors real-life situations where consumption events are inherently conglomerate in nature, thereby adding to the point that our framework can be useful in real-life data sets. However, we revisit the expenditure on specific subgroups of goods in Subsection 4.3. Table 4 reports summary statistics of the expenditure shares of our eight commodities. This shows that 46% of the budget is spent on food, 16% on clothing, and 13% on restaurant costs.

The averages in Table 4 shed little light on the degree of improvement over the observed sequence. To help us distinguish special cases of *ICARES*, we now study the temporal profiles of consumption in the data. Expenditures vary *between consumers* and *over time*. Let $w^n_{ht} = p^n_{ht} c^n_{ht}$ denote expenditure on commodity n by consumer h at time t. We regress w^n_{ht} on total intertemporal budget y_h. We also add interactions between y_h and time dummies $(t = 2)$, $(t = 3)$, and $(t = 4)$. We further include 44 dummies to absorb effects of quarterly variation in economic conditions between 1985 and 1996 (Browning and Collado, 2007).[18] We thus exploit

[16] The ECPF follows a quarterly rotation design, with around 12.5% of respondents being replaced each quarter. This implies that respondents can be followed for a maximum consecutive block of two years.

[17] In our final sample, there are 1,514 unique households for which we have at least one "block" of four consecutive observations. For a nonnegligible subset of households (i.e., almost 30%), we actually observe several "blocks" of four observations. However, in most of these cases, the blocks of observations are disjoint; thus there are interruptions after every four observations. Moreover, these gaps can be quite substantial, with an average gap of about three years (12 quarters). Only 23 of the 1,514 households have at least eight *successive* consumption observations. No household has more than eight successive consumption observations. We therefore analyze each set of four observations separately, resulting in a total of 2,052 consumption data sets. This mitigates issues of imperfect foresight, preference shifts, and updating of plans in series of observations separated by large time intervals, while still maximizing the size of our data set.

[18] Although our data set spans 12 years (1985–96), we only include 44 quarterly dummies. The reason is that price data are missing for the first three quarters of 1985. The fourth quarter of 1985 is also left out because it forms the reference group.

TABLE 4
DESCRIPTIVE STATISTICS OF EXPENDITURE SHARES (IN PERCENT) ON EIGHT NONDURABLE COMMODITIES.

	Sample Mean	Sample Stdev
Allfood	45.72	19.02
Clothing	16.39	13.46
Hhserv	4.58	6.61
Transport	4.62	7.60
Petrol	7.13	7.79
Leisure	5.67	7.38
Pserv	2.52	4.40
Foodout	13.38	13.09

TABLE 5
REGRESSIONS OF CONSUMPTION EXPENDITURE ON TOTAL INTERTEMPORAL BUDGET, INTERACTIONS WITH TIME DUMMIES, AND QUARTERLY CONTROLS.

	Allfood	Clothing	Hhserv	Transport	Petrol	Leisure	Pserv	Foodout
allexp	0.079***	0.045***	0.018***	0.014***	0.017***	0.020***	0.009***	0.042***
	(0.003)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.003)
$t = 2 \times$ allexp	−0.002	0.000	0.001	0.001	0.000	0.001	0.000	0.003
	(0.002)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)
$t = 3 \times$ allexp	0.001	−0.001	0.001	0.002	0.001	0.001	0.001	0.002
	(0.002)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)
$t = 4 \times$ allexp	−0.000	0.000	0.001	0.002	0.001*	0.001	−0.000	0.004**
	(0.002)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)
Observations	8208	8208	8208	8208	8208	8208	8208	8208

NOTE: The sample consists of all consumers. Estimates of quarterly controls are suppressed for compactness.

independent variation between the calendar date of the data point (i.e., 44 quarterly dummies) and the position of the data point in the sequence of observations (i.e., three positional dummies $t = 2, 3, 4$). We finally include a constant. We can then estimate

$$w^n_{ht} = \eta^n + \gamma^n \times y_h + \theta^n_t \times y_h + \sum_{k=5}^{48} \delta^n_k \times (\textit{Calender time}_{ht} = k) + \varepsilon^n_{ht}. \tag{17}$$

Parameter γ^n captures income effects when $t = 1$ whereas coefficients δ^n_k capture the effects of yearly and seasonal variations. The main parameters of interest, θ^n_t, reflect the effect of the data point's position in the sequence of observations for that consumer. Each regression uses 8,208 consumer-observations. We cluster standard errors by consumer. Table 5 presents our estimates.

The positive coefficients of total expenditure ($\gamma^n > 0$) show that all goods are normal. The coefficients θ^n_t associated with $t = 2, 3, 4$ are mostly nonnegative, reflecting a small increase in expenditure toward the end of the observed sequence. However, the differences in consumption are not statistically significant. There are two exceptions: the increase in fuel in $t = 4$ is significant at the 10% level and the increase in food expenditure away from home in $t = 4$ is significant at the 5% level. The latter suggests that restaurant visits exhibit a strong anticipatory motive. But generally speaking, the consumption profiles are relatively smooth. One possible explanation is that the results in Table 5 are averages among consumers with different preferences types. It aggregates the temporal expenditure profiles of consumers with strong anticipating selves and the profiles of consumers with strong remembering selves. Increasing and decreasing profiles may cancel each other out. We therefore turn to our revealed preference method to identify different consumer types.

4.2. *Revealed Preference Tests.* We now assess the empirical performance of *ICARES* and its special cases *ICAES*, *ICRES*, and *ICES* through a revealed preference analysis. An important feature of the revealed preference approach is that it allows us to verify data consistency for each consumer separately. This accounts for general unobserved preference heterogeneity between consumers.

For each consumer, we observe the prices and quantities over four successive quarters. However, unlike specially tailored lab experiments, budget survey data typically do not contain information on the true start τ_0 or end T of a consumer's planning period. We first discuss the implications of this lack of information, before presenting the results of the revealed preference tests.

Implementation with unobserved τ_0 and T The *ICARES* model of Definition 1 is time-consistent. The trade-offs between consumption events are not affected by the decision time. This in itself is not sufficient to implement the model with standard budget survey data. After all, the consumption events we observe lie in a restricted time interval $[\underline{t}, \bar{t}]$ where $\underline{t}$ denotes the time of our first observation for that household and $\bar{t}$ the time of our last observation. The household's true planning period $[\tau_0, T]$ will typically be longer, where $\tau_0 \leq \underline{t}$ and $\bar{t} \leq T$.[19] This difference between observation periods $[\underline{t}, \bar{t}]$ and planning periods $[\tau_0, T]$ poses two empirical challenges.

First, we cannot control for consumption events within $[\tau_0, T]$ that take place before $\underline{t}$ or after $\bar{t}$. This is where *the additive structure* of our model comes in. The contribution to overall welfare of a given consumption event (including all its utility flows) is independent of the consumption events before or after that. Thus the results will be robust to other consumption events that may fall within the planning period but outside our window of observations.[20]

Second, the *ICARES* model of Definition 1 is nonstationary. Postponing all consumption events with the same amount of time can change the trade-offs between the events. More specifically, the trade-offs between consumption events in principle depend on the time distance of the events to the start or end of the planning period. This dependency on τ_0 or T is reflected in the accumulation process of anticipated or recollected utilities, $(\beta^A)^{t-\tau_0+1}$ or $(\beta^R)^{T-t+1}$. In practice, the econometrician rarely observes τ_0 and T,[21] and this complicates the analysis. However, in our setup, the combination of *nonparametric utilities* and *multiplicative weights* β^i makes it so that the revealed preference results are independent of the total duration of the consumption plan. After all, the factors $(\beta^A)^{-\tau_0}$ and $(\beta^R)^T$ are ultimately absorbed by the nonparametric utilities u^A and u^R, respectively. In other words, one can consider an equivalent characterization with $\tilde{u}^A = (\beta^A)^{-\tau_0} \times u^A$ and $\tilde{u}^R = (\beta^R)^T \times u^R$ that suppresses the unknowns τ_0 and T. So, although the outer bounds of the planning period can change the utility trade-offs between events in the *ICARES* model, ceteris paribus, the revealed preference characterizations associated with different values of τ_0 or T all coincide. The nonparametric results are therefore robust to (unobserved) parameters τ_0 and T.[22] This also implies that we cannot identify the outer bounds of the planning period with nonparametric utilities and multiplicative factors β^i.

Revealed preference results For each consumer, we test whether their choices satisfy the revealed preference conditions of Proposition 3. The test has as inputs the prices and quantities

[19] By contrast, if planning periods were very short, the sequence of observations could come from different planning windows. One could then test data consistency with *ICARES* for subsets of the observed sequence.

[20] This is different from Hai et al. (2020), for instance, where the willingness to pay for consumption can also depend on the stock of memory accumulated in earlier periods.

[21] After all, we do not impose $\tau_0 = 1$ and $T \to \infty$. In other words, the consumer's whole life can comprise several (successive) planning periods. This reinforces the interpretation of our framework as a short-to-medium-run consumption model.

[22] It is worth to reiterate that the revealed preference results *do* depend on β^A and β^R as well as on the positioning of consumption in the timeline. Even with nonparametric utilities, parameters β^A and β^R determine the *changes* in anticipated and recollected utility flows between consumption observations at different points in time.

TABLE 6
PASS RATES, POWER, AND PREDICTIVE SUCCESS FOR DIFFERENT SPECIFICATIONS OF *ICARES*.

	Singles				Couples			
	ICES	*ICAES*	*ICRES*	*ICARES*	*ICES*	*ICAES*	*ICRES*	*ICARES*
Pass rates	0.093	0.494	0.820	0.948	0.035	0.455	0.795	0.964
Power	0.955	0.770	0.301	0.121	0.953	0.773	0.300	0.121
Selten	0.048	0.265	0.121	0.068	−0.012	0.228	0.095	0.085
Selten lb	0.005	0.190	0.061	0.033	−0.020	0.205	0.077	0.077
Selten ub	0.092	0.339	0.180	0.104	−0.004	0.251	0.113	0.094

of eight commodities over some observed *subset* of the planning period. As discussed above, the revealed preference results are robust to the time distance between the actual start of the planning period and our first observation, and robust to the time distance between the last observation in our data and the end of the planning period. We specify a range of parameter values $\beta^A, \beta^R \in [1, 1.2]$, with step size 0.05, for our grid search. This allows $(\beta^A)^4$ and $(\beta^R)^4$ to accumulate to at most 2, representing a doubling of anticipated/recollected utilities for consumption at the very start/end of the sequence. Each test takes the form of a linear programming problem. If the program has a solution for *any* combination of β^A, β^R (e.g., $\beta^A = \beta^R = 1.2$)[23] then we assign a value of one ("pass").

We first compute goodness of fit. Averaging over all consumers gives the mean in-sample fit of the model with the data ("pass rate"). Table 6 contains the pass rates of *ICES*, *ICAES*, *ICRES*, and *ICARES*. We report pass rates separately for singles and couples because household composition may also affect the decision-making structure of the consumer. Only 9% of the singles and 4% of the couples can be rationalized by *ICES*. This is not surprising: *ICES* is equivalent to the nonparametric test of the life-cycle model without discounting. Browning (1989) showed that this imposes strong restrictions on observed behavior. On the contrary, up to 49% of the singles and 46% of the couples can be rationalized by *ICAES*. This already supports our notion of intertemporal consumption with anticipating and experiencing selves. Finally, *ICRES* fits between 80% (couples) and 82% (singles) of the data whereas *ICARES* fits 95% (singles) to 96% (couples). We also explore the probability that consumers pass *ICAES* but not *ICRES*, and vice versa. We find that 8% of the data satisfy *ICAES* (not *ICRES*) whereas almost 42% satisfy *ICRES* (not *ICAES*). Corollary 3 is not just a theoretical curiosum: our method separates preferences for anticipation from preferences for recall on the basis of widely available data from budget surveys. In the next subsection, we will return to the subset of consumers consistent with *ICAES* to learn more about the (anticipatory) nature of commodities.

A limitation of assessing the empirical quality of a test merely by its goodness of fit is that the latter does not control for the fact that flexible models such as *ICAES*, *ICRES*, and especially *ICARES* more easily rationalize any kind of behavior. The revealed preference literature offers an elegant method to quantify the empirical power of nonparametric tests. Discriminatory power is the probability that a test rejects random behavior. In practice, it is one minus the pass rate of *simulated* random data sets. We follow the majority of papers in this literature and implement Bronars (1987)'s method of power measurement, adapted to an intertemporal setting. In particular, at each iteration b and for each consumer h, we simulate random behavior by drawing 4×8 random budget shares from a uniform distribution on the unit simplex. Using the actual (observed) prices, we can then compute consumption vectors $\left(\mathbf{c}_t^{h,b}\right)_{t \in \mathcal{T}}$ that exhaust the intertemporal budget constraint. This vector, together with prices and interest rates, then serves as a (simulated) data set $\{\mathbf{p}_t, r_t, \mathbf{c}_t^{h,b}\}_{t \in \mathcal{T}}$. We repeat this

[23] Higher values of β^i generally improve goodness of fit, and the main gains in empirical fit are realized between $\beta^i = 1$ and $\beta^i = 1.2$.

procedure 205,200 times.[24] We then apply each model to the *same* set of random quantities. Table 6 reports our power estimates. As expected, *ICES* is the most powerful model. It rejects consistency for about 95% of the simulated data. *ICAES* is also still powerful, rejecting consistency for 77% of the simulated data. Power drops dramatically for *ICRES* (30%) and *ICARES* (12%).[25]

Given the trade-off between in-sample fit (i.e., psychological realism) and discriminatory power (i.e., empirical tractability), we combine both measures into a single metric of "predictive success." We follow Selten (1991) and Beatty and Crawford (2011) and compute predictive success as the sum of pass rates and power minus one. Predictive success is always situated between −1 and 1. Higher scores indicate better empirical performance. A predictive success of −1 represents the worst possible outcome: none of the observed behavior but all of the random data can be rationalized. A predictive success of 1 is the best possible result: all of the observed behavior but none of the random data can be rationalized. Finally, a predictive success of 0 suggests that the empirical performance of the model is similar to the performance of a "model" of completely random behavior. The predictive success of the most restrictive model (*ICES*) is small overall, although it is strictly positive (5%) for singles. The most flexible model (*ICARES*) performs a bit better but predictive success is still limited to 7% for singles and 9% for couples. *ICRES* improves further on this empirical performance to 12% for singles and 10% for couples. We find the strongest empirical support for *ICAES*, with predictive success scores that exceed 20% for both singles and couples. This improves further on Selten's index found for hyperbolic discounting models (around 7% in Blow et al., 2021) and models in which habits form as durables (around 15% in Demuynck and Verriest, 2013).[26] We can also compare the predictive success of *ICARES* with that of *ED*. We consider different values for the uniform discount factor, ranging from 1/1.2 to 1. About 26% of the singles and 11% of the couples pass the conditions of *ED*. Predictive success is on average 13% and −3%, respectively. *ED* performs better for singles than couples, but the predictive success of *ICAES* is higher overall.

The last lines of Table 6 report 95% confidence intervals around the mean predictive success. To construct these intervals, we follow an econometric approach put forward by Demuynck (2015). The procedure uses as inputs the variance of observed "pass" results among consumers, the variance of simulated power results among consumers, and finally the covariance of the consumers' pass results and power estimates. For couples, the lower bound on predictive success of *ICAES* (20.5%) exceeds the upper bounds on predictive success of the other characterizations (maximum 11.3%). For singles, the lower bound on predictive success of *ICAES* (19%) is lower due to the small number of singles in the sample. Still, this exceeds the upper bounds on predictive success in all other models (maximum 18%).

To summarize, 49% of the singles and 46% of the couples behave exactly like predicted by *ICAES*. The natural next question is what distinguishes these *ICAES* consumer types from the rest. We conducted a probit regression of consistency with *ICAES* on the basis of household type (single or couple, with or without children), age of the household head, and the household's total expenditure over the period of observation. Age data are available in broad intervals: younger than 26, 26–35 years old, 36–45 years old, 46–55 years old, and older than 55. The regression uses a total of 2,052 households as data points. Neither age nor relationship status help to explain consistency with *ICAES*. We do find that the likelihood of *ICAES*

[24] We simulate $B = 100$ random data sets *per consumer*. We draw expenditure shares from a uniform distribution on the unit simplex.

[25] Given a decreasing path for discounted prices (Figure A.3) the most likely candidate to obtain RP violations is a decreasing consumption path. The latter cannot be rationalized by *ICAES* either, which is mainly responsible for generating increasing consumption paths. However, simulated decreasing consumption paths can easily be rationalized by *ICRES*, and this explains why *ICRES* has less discriminatory power (when prices decrease) than *ICAES*. Furthermore, given that the data set mostly contains increasing consumption profiles, the pass rate of actual data with *ICAES* remains relatively high.

[26] Note that, similar to Demuynck and Verriest (2013) and Blow et al. (2021), we focus on "sharp" rationalizability tests; however, we can easily accommodate for alternative measures such as the Afriat index.

Table 7
DESCRIPTIVE STATISTICS OF EXPENDITURE SHARES (IN PERCENT) ON EIGHT NONDURABLE COMMODITIES, FOR CONSUMERS CONSISTENT WITH *ICAES*.

	Sample Mean	Sample Stdev
Allfood	43.90	18.59
Clothing	16.45	13.14
Hhserv	4.82	6.55
Transport	4.81	7.88
Petrol	7.47	7.93
Leisure	5.89	7.19
Pserv	2.66	4.56
Foodout	14.01	13.34

consistency declines with the number of (young) children in the household. These households may have experienced a recent fertility event, and this may distort their planning activities in the short-to-medium run. We refer to Appendix A.4.2 for the complete regression outputs.

4.3. *Identification of "Anticipatory" Goods.* The empirical support for *ICAES* is consistent with behavioral patterns observed in lab experiments (Loewenstein, 1987). However, *ICAES* opens the door for more comprehensive analyses of anticipation based on budget survey data. These analyses can yield insight in the (anticipatory) nature of a wider range of commodities. It can also deal with nonseparabilities in anticipation across goods. Common temporal profiles may indicate a degree of complementarity between anticipatory consumption goods. Practical considerations typically limit the range of commodities that can be used in lab experiments. Moreover, the majority of experiments focus on one commodity in isolation, in a tightly controlled decision-making environment. We now want to further illustrate how standard budget survey data with expenditure information, on a range of goods and services, can be fruitfully combined with our revealed preference analysis to shed more light on the relative importance of anticipated utilities across these different commodities.

As a first step, in Table 7, we replicate Table 4 specifically for the subset of consumers who satisfy *ICAES*. The mean expenditure shares are similar overall, but the share of food at home decreases from 46% to 44% whereas the share of food away increases from 13% to 14%. Anticipating types spend more on restaurants and less on food and drinking at home. However, the commodities used in this first comparison are aggregates of a wide range of different goods. For instance, transport includes long-distance transportation but also standard public transport. Leisure is a combination of books, newspapers, and magazines but also cinema, theater, football, and other entertainment services. Moreover, the results in Table 7 shed little light on the *temporal* consumption profile of these goods.

As a second step, we regress expenditure associated with separate subgroups (included in the eight commodities) on total expenditure, interactions of total expenditure and the observation's position in the sequence, and quarterly dummies to absorb seasonal and yearly effects. We estimate specification (17) again but this time we limit the sample to all consumers consistent with *ICAES*. In this way, we can identify which goods enter as arguments in the utility function of an anticipating self. To recall, the anticipating self values consumption more when it lies in the future. So, increasing parameter estimates $\theta_4^n > \theta_3^n > \theta_2^n > 0$ associated with the expenditure on good n suggest that n is an argument of the anticipatory utility function. Table 8 presents the results. The first two columns contain the names of subgroups (goods) and the commodities to which they belong. Columns 3 and 4 show the mean and standard deviation of expenditure shares of each subgroup. The next column gives the estimates of income effects divided by standard error. The final three columns present the estimates of θ_2^n, θ_3^n, θ_4^n, again deflated by the respective standard errors. We rank goods from high θ_4^n to low θ_4^n.

Table 8
MEAN AND SPREAD OF BUDGET SHARES (IN PERCENT) AND ESTIMATES OF INCOME EFFECTS γ AND SEQUENCE EFFECTS θ (COEFFICIENT DIVIDED BY STANDARD ERROR; B/SE), PER SUBGROUP OF GOODS.

Item	Commodity	Mean	Sd	γ	θ_2	θ_3	θ_4
restaurants	Foodout	14.005	13.345	14.152	3.261	3.931	5.728
childfoot	Clothing	1.462	2.601	7.652	1.549	2.934	4.438
mensfoot	Clothing	0.959	2.230	3.996	2.291	1.969	3.956
motfuel	Petrol	7.474	7.929	13.707	3.227	2.937	3.872
womensunder	Clothing	0.581	1.572	5.391	1.088	2.653	3.681
other_trans	Transport	3.272	6.415	7.966	1.666	3.158	3.447
mensouter	Clothing	3.277	6.161	6.191	1.060	0.941	3.404
beer	Allfood	0.540	1.421	5.303	2.248	2.266	3.312
childouter	Clothing	2.294	4.800	7.546	−0.549	1.638	3.229
recservs	Leisure	1.469	3.375	8.971	2.060	1.622	3.162
pcareservs	Pserv	1.100	2.836	6.301	2.617	1.983	3.155
cinema	Leisure	0.655	2.361	4.591	1.687	2.359	2.773
pcarendur	Pserv	1.561	3.474	5.725	1.726	2.352	2.684
nuts	Allfood	0.566	1.051	6.749	−0.117	2.018	2.682
accessories	Clothing	0.619	1.877	5.806	1.257	2.336	2.674
cleaning	Hhserv	2.164	2.811	10.351	0.028	1.462	2.479
longdistance	Transport	0.835	4.110	2.676	2.525	2.174	2.450
pastry	Allfood	2.035	2.469	8.817	1.937	1.929	2.372
processed_meat	Allfood	0.751	1.533	7.665	0.124	2.035	2.297
lamb	Allfood	1.074	2.717	4.758	0.537	2.235	2.216
deli_meat	Allfood	3.234	3.966	8.639	0.838	1.961	2.212
cheese	Allfood	2.194	2.400	12.520	0.336	0.920	2.202
wine	Allfood	0.566	1.619	3.761	0.188	1.494	2.147
sugar	Allfood	0.249	0.601	2.367	1.655	2.161	2.055
womensouter	Clothing	4.057	7.108	8.608	1.731	1.181	2.031
cookoil	Allfood	1.338	2.803	5.529	0.739	1.945	1.978
prime_meat	Allfood	2.229	3.442	8.905	0.704	1.426	1.901
other_alc	Allfood	0.190	1.141	4.104	0.214	1.197	1.887
mensunder	Clothing	0.458	1.572	4.848	−0.334	1.325	1.873
recgoods	Leisure	1.816	4.317	6.213	0.481	2.246	1.824
fruit	Allfood	2.818	2.626	12.587	1.520	1.602	1.751
fooddrink_remain	Allfood	0.663	4.304	3.512	1.246	0.937	1.566
fresh_fish	Allfood	2.688	3.391	10.262	−0.168	1.990	1.550
domservs	Hhserv	1.931	5.843	6.087	0.750	1.110	1.523
chocolate	Allfood	0.523	1.064	6.437	−0.692	1.464	1.466
processed_fish	Allfood	0.861	1.583	6.709	0.556	0.422	1.338
hhservs	Hhserv	0.210	1.341	3.009	1.159	1.206	1.293
processed_veg	Allfood	0.528	1.122	6.111	1.631	1.466	1.269
pubtrans	Transport	0.703	2.075	5.634	2.429	1.864	1.269
spirits	Allfood	0.298	1.202	3.702	−0.364	1.010	1.249
foot_remain	Clothing	0.016	0.316	−2.064	1.064	1.383	1.235
newsbook	Leisure	1.947	3.221	9.616	1.312	1.717	1.131
other_meat	Allfood	0.559	1.512	4.499	−1.087	0.238	1.044
nonalcbev	Allfood	0.865	1.382	8.670	0.234	2.044	1.039
potatoes	Allfood	0.721	1.243	5.999	0.821	2.230	1.020
eggs	Allfood	0.769	0.949	7.708	0.155	0.304	0.979
fresh_veg	Allfood	1.645	1.767	9.766	−0.766	0.564	0.928
molluscs	Allfood	1.049	2.415	4.257	−1.131	0.442	0.847
rice	Allfood	0.183	0.410	3.049	0.496	1.576	0.836
nondur_article	Hhserv	0.511	1.091	5.855	−0.681	0.193	0.829
cereals	Allfood	0.065	0.258	2.949	0.576	1.091	0.778
dried_veg	Allfood	0.352	0.848	2.757	−0.081	0.798	0.690
bread	Allfood	2.467	2.107	11.017	1.071	1.414	0.670
other_food	Allfood	0.963	1.781	6.417	−0.521	0.442	0.633
womensfoot	Clothing	1.118	2.420	6.624	0.575	1.206	0.422
pork	Allfood	1.449	2.616	4.888	−0.825	−0.009	0.355
milk	Allfood	2.734	2.628	5.944	−1.183	0.169	0.297
tobacco	Allfood	2.935	4.014	7.072	−1.555	−0.667	0.239

(*Continued*)

TABLE 8
(CONTINUED)

Item	Commodity	Mean	Sd	γ	θ_2	θ_3	θ_4
cloth_remain	Clothing	0.158	1.659	2.255	0.699	−0.113	0.218
beef	Allfood	0.447	1.830	1.952	0.904	0.073	0.198
pasta	Allfood	0.665	1.493	4.369	0.236	1.870	0.167
poultry	Allfood	1.657	2.304	7.021	0.706	0.233	0.010
childunder	Clothing	1.388	3.105	7.426	−0.694	0.039	−0.743
butter	Allfood	0.125	0.332	5.435	0.200	1.084	−0.801
coffee	Allfood	0.588	1.301	5.718	−1.221	−0.105	−0.939
footrepair	Clothing	0.058	0.262	3.652	−0.766	−2.836	−2.228
preserved_milk	Allfood	0.318	1.518	4.434	0.638	−0.983	−2.707

NOTE: The sample is restricted to consumers consistent with *ICAES*.

Most estimates of "sequence" effects θ_t^n are nonnegative. This is not surprising: *ICAES* typically generates increasing consumption profiles. Yet, we find considerable heterogeneity in θ^n between goods. In the first set of goods (*restaurants* to *womensouter*), consumption is *much higher* in the last observation of the sequence. This is statistically significant at the 5% level. In the second set (*cookoil* to *fruit*), the effect is still there but only significant at the 10% level. We do not find significant temporal profiles for goods between *fooddrink_remain* and *poultry*. A small set of goods at the bottom of Table 8 is characterized by declining profiles.

Restaurant expenditure is the consumption category that increases the most over the sequence of observations. Interestingly, this appears to validate the use of restaurant visits in hypothetical choice experiments related to anticipation. Loewenstein and Prelec (1993) reported preferences for improvement when respondents could choose between sequences of restaurant visits. We must also note that the expenditure share of *restaurants* is high. Food at home (and clothing) subgroups have diverse temporal profiles so they appear in all parts of the table. Some foods and drinks are characterized by strongly increasing patterns: beer and wine, nuts, pastry and sugar, cheese, processed and delicacy meat and lamb. Other foods, including pork, milk, tobacco, beef, pasta, poultry, butter, coffee, and preserved milk have constant or declining profiles. The first group of goods appears to be complementary to leisure activities and special celebrations, whereas the second group reflects more habit purchases. Intuitively, it makes sense that the utility from psychological consumption—such as savoring—is associated with the *less* frequently purchased commodities. Hai et al. (2020) analogously used frequent zero purchases, and lumpy expenditure spikes, to operationalize memorable consumption goods. In the leisure commodity, the increase in expenditure on recreation services and cinema (theater) is more outspoken than the increase in recreation goods and books purchases. In the transport commodity, long-distance traveling (and other transportation) increases more than public transportation toward the end of the sequence. Overall, the ranking of goods in Table 8 is not inconsistent with the notion of "anticipatory" goods. This validates our interpretation of *ICAES* in terms of utility flows from savoring.

5. ROBUSTNESS

In this section, we address the robustness of our findings to the implementation of liquidity constraints, uncertainty, measurement error in prices and quantities, and the window of observations.

Liquidity constraints First, the presence of (binding) borrowing constraints impedes on the consumers' possibilities to smooth consumption and may thus lead to an overrejection of standard *ED* models (Dean and Sautmann, 2021). It is convenient in this context to reformulate the optimization problem associated with *ICARES* using spot prices. In particular, consider the following:

$$\max_{(\mathbf{c}_t)_{t\geq\tau_0}} \sum_{t=\tau_0}^{T} \left[\omega_0^A \left(\beta^A\right)^{t-\tau_0+1} u^A(\mathbf{c}_t) + \omega_0^R \left(\beta^R\right)^{T-t+1} u^R(\mathbf{c}_t) + u^E(\mathbf{c}_t)\right], \text{ subject to}$$

$$s_t \geq -b_t, \text{ and}$$

$$\mathbf{p}_t \cdot \mathbf{c}_t + s_t = \tilde{y}_t + (1 + r_{t-1})s_{t-1}, \text{ for all } t \in \mathcal{T}. \tag{18}$$

Problem (18) is similar to the baseline *ICARES* optimization problem, although there are a few key differences. A first difference is that the budget constraint is now expressed in sequential form, where s_t refers to *savings*. Second, the presence of borrowing constraints $s_t \geq -b_t$ puts a limit on the amount of debt consumers can incur. Finally, $(\tilde{y}_t)_{t\in\mathcal{T}}$ is the sequence of income levels. The first-order conditions are as follows:

$$\omega_0^A \left(\beta^A\right)^{t-\tau_0+1} \partial u^A(\mathbf{c}_t) + \omega_0^R \left(\beta^R\right)^{T-t+1} \partial u^R(\mathbf{c}_t) + \partial u^E(\mathbf{c}_t) = \tilde{\lambda}_t \mathbf{p}_t, \tag{19}$$

$$\tilde{\lambda}_t = (1 + r_t)\tilde{\lambda}_{t+1} + \tilde{\mu}_t. \tag{20}$$

These conditions are relatively standard. Condition (19) is clearly similar to the first-order optimality conditions of the *ICARES* model *without* liquidity constraints. The main difference between (19) and (11) is the fact that $\tilde{\lambda}_t$ is now the Lagrange multiplier of the sequential budget constraint, and (19) refers to spot prices instead of discounted prices. The condition in (20) is a consumption Euler equation which takes into account the presence of the liquidity constraint with Lagrange multiplier $\tilde{\mu}_t$. In case the borrowing constraint is not binding, we have $\tilde{\mu}_t = 0$ and (20) collapses to the standard consumption Euler equation. If the liquidity constraint does bind, $\tilde{\mu}_t > 0$. We can rewrite (20) more succinctly as follows:

$$\tilde{\lambda}_t \geq (1 + r_t)\tilde{\lambda}_{t+1}. \tag{21}$$

This admits that the marginal utility from wealth in period t *exceeds* the marginal utility from wealth (multiplied by $1 + r_t$) in period $t + 1$.

We adjust our definition of shadow prices slightly: $\tilde{\mathbf{p}}_t^i = \omega_0^i \times \partial u^i(\mathbf{c}_t)$ with $i = A, R, E$ and $t \in \mathcal{T}$. The RP characterization of problem (18) is very similar to the RP conditions in Proposition 3, but the Lagrange multipliers $\tilde{\lambda}_t$ associated with the budget constraints will now explicitly enter the conditions to be verified for rationalizability. In particular, the sequence $\left(\tilde{\lambda}_t\right)_{t\in\mathcal{T}}$ will have to satisfy the monotonicity condition in (21). Since the interest rates are observable, the RP characterization remains computationally tractable in the sense that, conditional on the β's, the system of RP conditions is linear in its unknowns.

The second row of Table 9 presents the new predictive success results. These results are very similar to the baseline findings without liquidity constraints. The predictive success of *ICAES* does not change much. At the same time, the empirical performance of *ICRES* and *ICARES* goes down. Overall, the predictive success of *ICAES* clearly exceeds that of other characterizations. We report the corresponding pass rates in Appendix A.4.3. Table A.2 shows that liquidity constraints have only a small effect on pass rates. Even with the extension of liquidity constraints, *ICES* explains less than 20% of the choices in the sample.

Uncertainty Second, the strong yet common assumption of perfect foresight with respect to the economic environment (prices, interest rates, and income levels) may not hold in practice. Without this assumption, or further stronger assumptions about the expectational process of agents, RP tests will lack empirical content. Specifically, the smoothing of marginal utilities from wealth ($\tilde{\lambda}_t = (1 + r_t)\,\tilde{\lambda}_{t+1}$) may no longer hold after a series of large unexpected shocks

TABLE 9
PREDICTIVE SUCCESS FOR DIFFERENT SPECIFICATIONS OF *ICARES*, WITH LIQUIDITY CONSTRAINTS, UNCERTAINTY, MEASUREMENT ERROR, AND FOR SUBSAMPLES THAT START IN THE FIRST QUARTER.

Selten	Singles				Couples			
	ICES	*ICAES*	*ICRES*	*ICARES*	*ICES*	*ICAES*	*ICRES*	*ICARES*
Baseline	0.048	0.265	0.121	0.068	−0.012	0.228	0.095	0.085
Liq Constr	0.092	0.276	−0.008	−0.004	0.003	0.234	−0.009	−0.004
Uncertainty								
$\alpha = 0.99$	0.040	0.261	0.056	0.019	−0.024	0.257	0.040	0.036
$\alpha = 0.975$	−0.003	0.196	0.035	−0.003	−0.041	0.218	0.002	0.009
$\alpha = 0.95$	−0.054	0.023	−0.001	−0.011	−0.133	0.031	−0.015	−0.002
Price error								
$\sigma_p = 0.005$	0.041	0.247	0.100	0.069	−0.008	0.227	0.089	0.089
$\sigma_p = 0.01$	0.035	0.247	0.111	0.078	−0.006	0.219	0.076	0.089
$\sigma_p = 0.05$	−0.010	0.160	0.044	0.043	−0.012	0.111	−0.023	0.023
Quantity error								
$\sigma_c = 0.01$	0.047	0.265	0.122	0.074	−0.011	0.224	0.095	0.086
$\sigma_c = 0.05$	0.047	0.247	0.139	0.080	−0.007	0.223	0.091	0.087
$\sigma_c = 0.10$	0.042	0.230	0.133	0.080	−0.002	0.226	0.096	0.087
First quarter	0.071	0.271	0.138	0.048	−0.004	0.26	0.08	0.091

to the sequence of income levels $(\tilde{y}_t)_{t\in\mathcal{T}}$. Ex ante, the conditions could be adjusted in the following way:

$$\tilde{\lambda}_t = \mathbb{E}\big[(1 + r_t)\tilde{\lambda}_{t+1}\big], \tag{22}$$

but ex post, there may be strong variation of the marginal utilities of wealth $\tilde{\lambda}_t, \tilde{\lambda}_{t+1}$. We therefore adapt our conditions as follows:

$$\alpha(1 + r_t)\tilde{\lambda}_{t+1} \leq \tilde{\lambda}_t \leq \frac{1 + r_t}{\alpha}\tilde{\lambda}_{t+1}, \tag{23}$$

where $\alpha \in (0, 1]$. Note that the test becomes weaker in case $\alpha < 1$, which captures the possibility that $\tilde{\lambda}_t = (1 + r_t)\,\tilde{\lambda}_{t+1}$ is violated due to unobserved randomness (e.g., in income flows $\tilde{y}_t$, $t \in \mathcal{T}$). In practice, we will implement several tests of rationalizability for given values of α on a grid, thus varying the degree of uncertainty.

Rows 3–5 of Table 9 present the new predictive success results for various levels of α. The results indicate that larger deviations (i.e., smaller α) systematically reduce predictive success for all specifications under consideration. At $\alpha = 0.975$, the predictive success of *ICAES* is still about 20% whereas that of other characterizations is less than 5%. At $\alpha = 0.95$, *ICAES* is the only specification with a positive predictive success rate for both singles and couples.

Measurement error Third, the data may suffer from measurement error. The latter can affect the empirical performance of economic models in RP analyses. For example, Aguiar and Kashaev (2021) have shown that there is a tendency to overreject the hypothesis of static utility maximization (*GARP*) when one ignores mismeasurement.

To be more precise, let $\mathbf{c}_t^\dagger$ for $t \in \mathcal{T}$ denote the true consumption levels. Then suppose we quantify errors $\varepsilon_{n,t}$, for each good $n \in \{1, \ldots, N\}$ and for each period $t \in \mathcal{T}$, in the classical (multiplicative) form as follows:

$$c_{n,t} = \varepsilon_{n,t} c_{n,t}^\dagger.$$

Assume in addition that errors are lognormal and the variance is uniform across commodities, $\log \varepsilon_{n,t} \sim N\left(-\frac{\sigma_c^2}{2}, \sigma_c^2\right)$. Drawing $\varepsilon_{n,t}$ from this distribution, where $\mathbb{E}\left[\varepsilon_{n,t}\right] = 1$, we can estimate true consumption levels via $\tilde{c}_{n,t} = \frac{c_{n,t}}{\varepsilon_{n,t}}$. We then test rationalizability for *ICARES*, *ICAES*, and *ICRES* by applying the conditions in Proposition 3 to the adjusted data sets $\{\rho_t, \tilde{\mathbf{c}}_t\}_{t\in\mathcal{T}}$. We consider three scenarios with small ($\sigma_c = 0.01$), medium ($\sigma_c = 0.05$), and large ($\sigma_c = 0.10$) errors.[27]

Price data could also be subject to measurement error. Prices in the application are not measured at the level of each consumer, as they come from the INE. These general price indices are proxies for the consumers' true prices (which in reality may vary between consumers). We thus repeat our analyses with the data adjusted for price errors. We consider three scenarios with small ($\sigma_p = 0.005$), medium ($\sigma_p = 0.01$), and large ($\sigma_p = 0.05$) errors. [28]

Rows 6–8 and 9–11 of Table 9 present the new predictive success results with price and quantity error, respectively. The results are robust to substantial amounts of consumption error. The results also hold with small to moderate amounts of price error. Very large price error seems to put downward pressure on the predictive success of all RP models. However, even if $\sigma_p = 0.05$, the predictive success of *ICAES* (16% for singles and 11% for couples) still clearly exceeds that of the other models under consideration.

Observation periods starting in the first quarter Finally, in our main sample, observation periods start in different quarters of the year. Some differences in temporal consumption patterns between consumers may be due to seasonal effects. Although we control for seasonal variation (i.e., via quarterly dummies) in Subsection 4.3, seasonality may still affect the revealed preference tests. In addition, some sequences of observations may be more likely to coincide with a planning period than others.

In this final exercise, we further separate the effect of a consumption event's "position in the observed sequence" from the "calendar date of the observation." We repeat the analyses for the subgroup of consumers for whom the first (final) observation coincides with the first (final) quarter of the year. If yearly consumption plans were formed at the beginning of each new year, for instance, then the integrity of the sequence of observations should be strong in this subgroup. The restricted subsample consists of 609 consumers: 51 singles and 558 couples.

The results are in the bottom row of Table 9. The empirical performance of the models is very similar to the baseline sample overall, but predictive success of *ICAES* among couples increases further from 22.8% to 26%.

6. RELATED LITERATURE

We now position the present article in the larger literature. First, there have been other studies describing the complex psychological features pertaining to consumption events. Morewedge (2015), for instance, asked a sample of Americans to describe the contribution of anticipation, remembering, and experience to the total pleasure derived from various activities. The relative contribution of anticipation varied from 15% (exercise) to 24% (vacation); the relative contribution of memory from 13% (dinner) to 30% (wedding). The contributions in Loewenstein (1987) and Loewenstein and Sicherman (1991) have focused on the feature of anticipated utility, whereas Gilboa et al. (2016) and Hai et al. (2020) presented models of remembering and memorable goods. Gilboa et al. (2016) emphasized the complexity, due to re-

[27] Blundell et al. (2008) showed that, under the additional assumption that consumption is a martingale process with drift, σ_c^2 can be estimated via the (negative) covariance of the growth of stochastic log consumption. Our results are extremely robust to the values of σ_c^2. In particular, we also used a grid based on the results in Casado (2011), who estimated the (annual) variance of consumption measurement error in the ECPF for the years under consideration between 0.06 and 0.08.

[28] We make a similar assumption as for consumption, namely, that log prices are subject to classical measurement error with variance σ_p^2. This is similar to the assumption in Varian (1985) and Beatty and Crawford (2011). Discounted prices change by 0.005 to 0.011 index points from one quarter to the next; so $\sigma_p = 0.01$ already adds substantial amounts of noise to the data. In Adams et al. (2014), price measurement error with standard deviation 0.1 is sufficient to make all data consistent with the *ED* model.

verse time inconsistency, of allowing for anticipation without additional assumptions on anticipatory preferences. In that regard, our article provides such additional structure, which makes the use of a decision-theoretic framework possible.

To the best of our knowledge, Baucells and Bellezza (2017) is the only other paper that models the temporal profile of "instant" utilities from anticipation, remembering, and experience. Beside physical and psychological consumption, the authors also consider reference points. A distinguishing feature of their framework is that the carrier of utility is *effective* consumption: the difference between consumption and some reference point. The reference point is endogenous: savoring increases the target against which future consumption is valued (adaptation). The model of Baucells and Bellezza (2017) incorporates a wide range of insights from psychology. The main difference with respect to *ICARES* is that Baucells and Bellezza (2017) study the utility flows from a *single* consumption event. *ICARES*, by contrast, considers an environment with more than one good, and with consumption observations at multiple points in time. Given the wider range of commodities, *ICARES* also allows the utility functions before, during, and after events to differ in arguments and in shape. Finally, we set our framework in discrete time, to tailor it to revealed preference testing and identification. Indeed, as we have shown, *ICARES* and its restricted versions produce straightforward testable implications outside specially tailored lab settings.

Second, our article uses tools from the revealed preference literature to analyze the empirical content of *ICARES* and its special cases. Revealed preference theory was introduced early by Samuelson (1938) and Houthakker (1950). The seminal contributions by Afriat (1967) and Varian (1982) made revealed preference analysis operational and applicable in survey data on expenditures. By now, the technique has been used in a wide range of applications: household consumption choices, choices from nonlinear budget sets, analyzing stable matching patterns, etc. We refer to Crawford and De Rock (2014) for more applications of (empirical) revealed preference methods. The revealed preference approach has several advantages. First, it is intrinsically nonparametric, and thus completely independent of the specific functional form of utility. Next, it allows econometricians to analyze each consumer separately, thus incorporating a large degree of individual heterogeneity. Revealed preference methods have already been applied fruitfully to the analysis of intertemporal models; notable examples are the study of rational habit formation (Crawford, 2010), rational addiction (Demuynck and Verriest, 2013), intertemporal collective choice (Adams et al., 2014), discounted utility models (Dziewulski, 2018), the exponentially discounted utility model (Echenique et al., 2020), and models of quasi-hyperbolic discounting (Blow et al., 2021).

Finally, our article adds to the large literature on behavioral deviations from (exponential) discounting; the *ED* framework. The *ED* model has excellent empirical tractability but imposes strong assumptions. We focus on two of these assumptions and refer to Frederick et al. (2002) for a comprehensive overview. *ED* typically assumes *positive devaluing* —with constant discount factors $\beta \leq 1$—and *independence of discounting from consumption*. Utility from anticipation violates the first assumption; the dependency of anticipatory emotions and pleasant memories on the type of consumption good violates the second. Capturing these behavioral phenomena within the context of *ED* requires a flexible definition of discount factors β. A first amendment is to let discount factors vary over time: $\beta(t)$. This also permits negative devaluing (i.e., $\beta(t)$ increasing in t). A second amendment is to let discount factors vary between goods: $\beta(n)$. This addresses differences in temporal profiles between anticipatory, memorable, and ordinary consumption goods. However, even with these extensions, *ED* still has important limitations. First, *ED* with $\beta(t, n)$ suffers from a curse of dimensionality. The number of parameters grows multiplicatively with the number of observations *and* the number of goods. Second, the literature shows a large dispersion of discount factors.[29] Estimates change dramatically from one experiment to the next, and this does not shed much light on the behavioral mechanisms underlying intertemporal consumption. Finally, Manzini et al. (2010) stud-

[29] Frederick et al. (2002) list a range of estimates from experimental and survey data in table 1 and figure 2.

ied choices between time sequences of monetary outcomes. The authors found that standard models based on discounting could not explain the data, no matter how much variability in discount factors was allowed.

7. CONCLUSION

This article is motivated by two observations of intertemporal behavior that violate the predictions of the life-cycle model. First, consumers sometimes postpone desirable outcomes in order to extend the duration of "savoring" (Baucells and Bellezza, 2017). Second, consumers sometimes spend disproportionate amounts of income on holidays, celebrations, and ceremonies early in life to maximize the duration of "memories" generated by this consumption (Gilboa et al., 2016). Both observations have one feature in common: a dissociation between physical consumption and the utility flows from this consumption.

We propose and test a new model in which consumers enjoy utility from savoring of future events, experience of a current event, and remembering of past events. We thus represent consumption as the outcome of a bargaining process between three temporal selves: an anticipating self, a remembering self, and an experiencing self. The selves can have different valuations of the same commodity.

The choices produced by our model are generally not time-consistent. First, the bargaining between selves may suffer from commitment issues. This complicates the aggregation of different temporal motives. Second, and more fundamentally, the duration of savoring decreases naturally as time moves forward. This induces acts of *reverse* time inconsistency and undermines the very notion of anticipation through loss of self-credibility. We put forward an internal mechanism that can mitigate acts of reverse time inconsistency: the decreasing duration of savoring is offset by an increasing decision weight of the anticipating self. This imposes the qualitative condition that the decision weight of the anticipating self increases toward the end of each planning period. Such condition is in line with the view that shorter planning periods "activate" preferences for improvement.

To bring the theory to the data, we specify time factors that satisfy the conditions of the internal commitment mechanism. We leave the utility functions and the initial decision weights unspecified. The corresponding model, *ICARES*, is a time-consistent version of intertemporal consumption with anticipating, remembering, and experiencing selves. *ICARES* nests a number of interesting polar cases: *ICAES*, *ICRES*, and *ICES*. *ICARES* and its special cases have straightforward testable implications even outside specially tailored lab experiments. We derive the corresponding revealed preference characterization.

We apply this characterization to a panel data set of quarterly consumption by Spanish households (ECPF). *ICARES* rationalizes almost all observations, but lacks discriminatory power. The most successful specification is *ICAES*: it rationalizes close to two-thirds of the data, and is still fairly powerful. We then investigate heterogeneity in the "anticipatory" nature of consumption goods. For *ICAES* consumers, we find that restaurant expenditure, leisure services, and food expenditure complementary to these leisure activities increase more sharply over the planning period compared to other expenditure items. In line with experimental findings of Loewenstein (1987), our evidence from budget survey data confirms that anticipation matters for understanding consumption patterns. More generally, this is one of the first papers to provide a successful rationalization of consumption patterns for the full data set (i.e., both singles and couples) with a model that also satisfies time consistency.

A large literature has studied deviations from the discounted utility framework, but most of this work focused on violations of time consistency (present bias or myopia). Anticipatory emotions and memorable consumption have received less attention. Especially remarkable is the lack of evidence from budget survey data. The intertemporal framework proposed in this article is situated between the theory of total utility (Kahneman et al., 1997) and the discounted utility model (Samuelson, 1937). The former enhances psychological realism by incorporating *all* the utility flows from savoring and memories; the latter maintains empirical tractability for consumption choices from standard intertemporal budget constraints.

DATA AVAILABILITY STATEMENT The data that support the findings are based on the ECPF data used in Adams, Abi, Cherchye, Laurens, De Rock, Bram, and Verriest, Ewout. Replication data for: Consume Now or Later? Time Inconsistency, Collective Choice, and Revealed Preference, and available in openICPSR at https://doi.org/10.3886/E112718V1. In addition, the Supplemental material provides additional details for replication of the main revealed preference analysis in this article.

A.1. *Proofs.*

A.1.1. *Proof of Proposition 1.* We prove Proposition 1 by offering a counterexample to dynamic consistency. Comparing the optimal solutions to (A.2) and (A.3) in the numerical example below shows that in general (i.e., without additional structure on the dynamics of intra-selves bargaining and/or the time functions D^A and D^R) our framework does not satisfy time consistency.

A numerical example We start from a parametric specification of utility functions u^A, u^R, u^E, decision weight functions ω^A, ω^R, and an intertemporal budget constraint. We simulate a consumption plan $(\mathbf{c}_t^*)_{t\geq 1}$ at the start of $\tau = 1$. We then show that the optimal plan $(\hat{\mathbf{c}}_t)_{t\geq 2}$ changes at the start of $\tau = 2$. That is, $\hat{\mathbf{c}}_t \neq \mathbf{c}_t^*$ for some $t \geq 2$.

We restrict this exercise to $N = 2$ goods, so $\mathbf{c}_t = (c_{1,t}, c_{2,t})$. We choose the following parametric specification for the utility functions:

$$u^i(c_{1,t}, c_{2,t}) = \alpha^i \log(c_{1,t}) + \left(1 - \alpha^i\right) \log(c_{2,t}), \text{ where } i = A, R, E. \tag{A.1}$$

We let $\alpha^A = \alpha^R = 1$: the utility from anticipation and recall comes exclusively from good 1. For simplicity, we assume that $\omega^R(\tau) = \omega_0^R$ and $\omega^A(\tau) = \omega_0^A$, where $\omega_0^A > \omega_0^R$. In words, the anticipating self has relatively more influence over the decision-making process than the remembering self. With regards to the time functions D^A and D^R, we assume a simple exponential form: $D^A(t - \tau) = \left(\beta^A\right)^{t-\tau+1}$ and $D^R(T - t) = \left(\beta^R\right)^{T-t+1}$, where $\beta^A = \beta^R > 1$. We normalize all prices to unity. The optimization problem at $\tau = 1$ reads as follows:

$$\max_{(\mathbf{c}_t)_{t\geq 1}} \sum_{t=1}^{T} \left[\left(\omega_0^A \left(\beta^A\right)^t + \omega_0^R \left(\beta^R\right)^{T-t+1} + \alpha^E\right) \log(c_{1,t}) + \left(1 - \alpha^E\right) \log(c_{2,t})\right], \text{ subject to}$$

$$\sum_{t=1}^{T} (c_{1,t} + c_{2,t}) = y. \tag{A.2}$$

Solving the associated system of first-order conditions then yields the following solutions for consumption levels:

$$c_{1,t}^* = \frac{C(t)y}{\sum_{t=1}^{T}[1 + C(t)]}, \text{ and}$$
$$c_{2,t}^* = \frac{y}{\sum_{t=1}^{T}[1 + C(t)]}, \text{ for all } t \geq 1,$$

where $C(t) = \left(\omega_0^A \left(\beta^A\right)^t + \omega_0^R \left(\beta^R\right)^{T-t+1} + \alpha^E\right)(1 - \alpha^E)^{-1}$. Notice that the profile for consumption of the first good is increasing, given $\omega_0^A > \omega_0^R$ and $\beta^A = \beta^R > 1$. This is in line with the assumption in this example that the anticipating self is more influential in the decision process than the remembering self. Also noteworthy is the consumption smoothing with respect to good 2. This good produces no utility from anticipation or remembering.

Next, we study the consumption choices made by this consumer, but starting from the decision period $\tau = 2$. The equivalent of (A.2) can then be formulated as follows:

$$\max_{(\mathbf{c}_t)_{t\geq 2}} \sum_{t=2}^{T} \left[\left(\omega_0^A \left(\beta^A\right)^{t-1} + \omega_0^R \left(\beta^R\right)^{T-t+1} + \alpha^E \right) \log\left(c_{1,t}\right) + \left(1-\alpha^E\right) \log\left(c_{2,t}\right) \right], \text{ subject to}$$

$$\sum_{t=2}^{T} \left(c_{1,t} + c_{2,t}\right) = \hat{y}, \tag{A.3}$$

where $\hat{y} = y - c_{1,1} - c_{2,1}$ are the available resources for expenditures over the horizon $t \in (2, \ldots, T)$. By again solving the associated system of first-order conditions, we obtain the optimal consumption choices:

$$\hat{c}_{1,t} = \frac{\hat{C}(t)\hat{y}}{\sum_{t=2}^{T}\left[1 + \hat{C}(t)\right]}, \text{ and}$$

$$\hat{c}_{2,t} = \frac{\hat{y}}{\sum_{t=2}^{T}\left[1 + \hat{C}(t)\right]}, \text{ for all } t \geq 2,$$

with $\hat{C}(t) = \left(\omega_0^A \left(\beta^A\right)^{t-1} + \omega_0^R \left(\beta^R\right)^{T-t+1} + \alpha^E\right)\left(1-\alpha^E\right)^{-1}$. It can now be shown that, given $\omega_0^A \neq 0$, the optimal consumption choices for $t \geq 2$ are such that $\hat{\mathbf{c}}_t \neq \mathbf{c}_t^*$. This simple parametric example shows how reverse time inconsistency, driven by the anticipating self, can produce dynamically inconsistent behavior on the part of consumers.

A.1.2. *Proof of Proposition 2.* Consider any $\tau, \tau' \in \mathcal{T}$, with $\tau < \tau'$. Let $(\mathbf{c}_t^*)_{t\geq\tau}$ denote the solution to (6). Now suppose, toward a contradiction, that $(\mathbf{c}_t^*)_{t\geq\tau'}$ is *not* the solution to (8). Then, there must exist a sequence $(\hat{\mathbf{c}}_t)_{t\geq\tau'} \in \mathcal{B}\left((\boldsymbol{\rho}_t)_{t\geq\tau'}, y_{\tau'}\right)$ such that

$$\begin{aligned} &\sum_{t\geq\tau'} \left[\omega^A(\tau')D^A(t-\tau')u^A(\hat{\mathbf{c}}_t) + \omega^R(\tau')D^R(T-t)u^R(\hat{\mathbf{c}}_t) + u^E(\hat{\mathbf{c}}_t)\right] \\ &> \sum_{t\geq\tau'} \left[\omega^A(\tau')D^A(t-\tau')u^A(\mathbf{c}_t^*) + \omega^R(\tau')D^R(T-t)u^R(\mathbf{c}_t^*) + u^E(\mathbf{c}_t^*)\right]. \end{aligned} \tag{A.4}$$

1. Assume (9) holds. We thus have that, for all $\tau \in \mathcal{T}$ and $t \geq \tau$:

$$\omega^A(\tau)D^A(t-\tau) = \exp(\alpha(t)) \equiv a(t). \tag{A.5}$$

Taking derivatives on both sides yields the condition:

$$\frac{\partial\left(\omega^A(\tau)D^A(t-\tau)\right)}{\partial\tau} = 0. \tag{A.6}$$

This implies independence of function $\omega^A(\tau)D^A(t-\tau) = a(t)$ from the decision moment. Similarly, $\frac{\partial\omega^R(\tau)}{\partial\tau} = 0$ immediately imposes independence of $b(t) = \omega^R(\tau)D^R(T-t)$ from the decision moment. Using this information, we can rewrite (A.4) as follows:

$$\begin{aligned} &\sum_{t\geq\tau'} \left[a(t)u^A(\hat{\mathbf{c}}_t) + b(t)u^R(\hat{\mathbf{c}}_t) + u^E(\hat{\mathbf{c}}_t)\right] + \sum_{t=\tau}^{\tau'-1} \left[a(t)u^A(\mathbf{c}_t^*) + b(t)u^R(\mathbf{c}_t^*) + u^E(\mathbf{c}_t^*)\right] \\ &> \sum_{t\geq\tau} \left[a(t)u^A(\mathbf{c}_t^*) + b(t)u^R(\mathbf{c}_t^*) + u^E(\mathbf{c}_t^*)\right]. \end{aligned} \tag{A.7}$$

2. We now show that $\left(\mathbf{c}_\tau^*, \mathbf{c}_{\tau+1}^*, \ldots, \hat{\mathbf{c}}_{\tau'}, \ldots, \hat{\mathbf{c}}_T\right)$ was also feasible at $t = \tau$. To that end, note that $(\hat{\mathbf{c}}_t)_{t \geq \tau'} \in \mathcal{B}(\boldsymbol{\rho}_t, y_{\tau'})$ implies:

$$\text{(A.8)} \qquad \sum_{t \geq \tau'} \frac{\mathbf{p}_t}{\prod_{i=\tau_0}^{t-1}(1+r_i)} \cdot \hat{\mathbf{c}}_t \leq y_{\tau'} = (y - \sum_{t \leq \tau'-1} \frac{\mathbf{p}_t}{\prod_{i=\tau_0}^{t-1}(1+r_i)} \cdot \mathbf{c}_t^*).$$

Rearranging terms in (A.8), we obtain:

$$\sum_{t \geq \tau'} \frac{\mathbf{p}_t}{\prod_{i=\tau_0}^{t-1}(1+r_i)} \cdot \hat{\mathbf{c}}_t + \sum_{t \geq \tau}^{\tau'-1} \frac{\mathbf{p}_t}{\prod_{i=\tau_0}^{t-1}(1+r_i)} \cdot \mathbf{c}_t^* \leq (y - \sum_{t \leq \tau-1} \frac{\mathbf{p}_t}{\prod_{i=\tau_0}^{t-1}(1+r_i)} \cdot \mathbf{c}_t^*)$$

$$\text{(A.9)} \qquad \sum_{t \geq \tau}^{\tau'-1} \frac{\mathbf{p}_t}{\prod_{i=\tau_0}^{t-1}(1+r_i)} \cdot \mathbf{c}_t^* + \sum_{t \geq \tau'} \frac{\mathbf{p}_t}{\prod_{i=\tau_0}^{t-1}(1+r_i)} \cdot \hat{\mathbf{c}}_t \leq y_\tau.$$

3. We now put results (1) and (2) together. Condition (A.9) shows that $\left(\mathbf{c}_\tau^*, \mathbf{c}_{\tau+1}^*, \ldots, \hat{\mathbf{c}}_{\tau'}, \ldots, \hat{\mathbf{c}}_T\right) \in \mathcal{B}(\boldsymbol{\rho}_t, y_\tau)$ was feasible at decision period $t = \tau$. Condition (A.7) implies that $\left(\mathbf{c}_\tau^*, \mathbf{c}_{\tau+1}^*, \ldots, \hat{\mathbf{c}}_{\tau'}, \ldots, \hat{\mathbf{c}}_T\right)$ would have also yielded higher overall utility than $(\mathbf{c}_t^*)_{t \geq \tau}$. This contradicts our opening statement that $(\mathbf{c}_t^*)_{t \geq \tau}$ solves (6). We conclude that $(\mathbf{c}_t^*)_{t \geq \tau'}$ must solve (8), thereby confirming dynamic consistency of the consumer under condition (9).

A.1.3. *Proof of Proposition 3.* The proof consists of a necessary and a sufficient part.

Necessity: We first prove that the intertemporal consumption with anticipating, remembering, and experiencing selves (*ICARES*) implies the system of conditions in Proposition 3. From concavity of u^A, u^R, and u^E, we know that for all $s, t \in \mathcal{T}$:

$$u^A(\mathbf{c}_s) - u^A(\mathbf{c}_t) \leq \partial u^A(\mathbf{c}_t) \cdot (\mathbf{c}_s - \mathbf{c}_t);$$
$$u^R(\mathbf{c}_s) - u^R(\mathbf{c}_t) \leq \partial u^R(\mathbf{c}_t) \cdot (\mathbf{c}_s - \mathbf{c}_t);$$
$$u^E(\mathbf{c}_s) - u^E(\mathbf{c}_t) \leq \partial u^E(\mathbf{c}_t) \cdot (\mathbf{c}_s - \mathbf{c}_t).$$

Define utilities $u_t^A = \omega_0^A/\lambda \times u^A(\mathbf{c}_t)$ and marginal utilities $\tilde{\mathbf{p}}_t^A = \omega_0^A/\lambda \times \partial u^A(\mathbf{c}_t)$ in line with Section 3, and similarly for u_t^R and u_t^E, and $\tilde{\mathbf{p}}_t^R$ and $\tilde{\mathbf{p}}_t^E$. This produces conditions (12)– (14). Conditions (15) are a direct translation of the first-order conditions.

Sufficiency: We subsequently prove that the system of conditions in Proposition 3 implies existence of utility functions u^A, u^R, and u^E, weights ω_0^A and ω_0^R, and parameters β^A and β^R so that (11) holds. Consider a subset of observations $\tilde{\mathcal{T}} \subseteq \mathcal{T}$ and sum conditions (14) over this subset. We obtain:

$$\text{(A.10)} \qquad 0 \leq \sum_{s,t \in \tilde{\mathcal{T}}} \tilde{\mathbf{p}}_t^E \cdot (\mathbf{c}_s - \mathbf{c}_t).$$

Condition (A.10) is referred to as *cyclical monotonicity* (Rockafellar, 1970) and implies existence of a concave utility function u^E so that

$$\partial u^E(\mathbf{c}_t) = \tilde{\mathbf{p}}_t^E.$$

We can repeat this argument and sum conditions (13) over a subset of observations in $\tilde{\mathcal{T}}$. We obtain:

$$0 \leq \sum_{s,t \in \tilde{\mathcal{T}}} \tilde{\mathbf{p}}_t^R \cdot (\mathbf{c}_s - \mathbf{c}_t) \tag{A.11}$$

and this implies existence of a concave map u^R such that:

$$\partial u^R(\mathbf{c}_t) = \tilde{\mathbf{p}}_t^R.$$

Likewise, summing conditions (12) over a subset of observations $\tilde{\mathcal{T}}$, we obtain:

$$0 \leq \sum_{s,t \in \tilde{\mathcal{T}}} \tilde{\mathbf{p}}_t^A \cdot \left(\mathbf{c}_s^A - \mathbf{c}_t^A\right). \tag{A.12}$$

Thus there exists a concave function u^A so that:

$$\partial u^A(\mathbf{c}_t) = \tilde{\mathbf{p}}_t^A.$$

Finally, we take $\omega_0^A = \omega_0^R = \lambda = 1$ without losing generality. Then (15), with $\partial u^i(\mathbf{c}_t) = \tilde{\mathbf{p}}_t^i$, yields first-order conditions (11) for consistency with *ICARES*.

A.1.4. *Proof of Corollary 1.* It is sufficient to provide two data sets, one which satisfies *ICARES* but violates the generalized axiom of revealed preferences (*GARP*) and conversely. To that end, consider Table 2 of our simulation exercise. The data set generated by *ICARES* in the simulation exercise violates *GARP*. On the other hand, the data set consistent with *GARP* violates the RP characterization of *ICARES*. This shows that the empirical content of *ICARES* is independent from the content of static utility maximization.

A.1.5. *Proof of Corollary 2.* The revealed preference conditions of the exponential discounting (*ED*) are sufficient (but not necessary) for consistency with *ICARES*. Consider a data set that passes the conditions of *ED*. We show that it also passes the conditions of intertemporal consumption with remembering and experiencing selves (*ICRES*). Consistency of $\mathcal{D} = \{\boldsymbol{\rho}_t, \mathbf{c}_t\}_{t \in \mathcal{T}}$ with *ED* implies $u_s - u_t \leq \frac{\boldsymbol{\rho}_t}{\beta^t} \cdot (\mathbf{c}_s - \mathbf{c}_t)$ for all $s, t \in \mathcal{T}$. This is equivalent to $u_s - u_t \leq (\beta^R)^t \boldsymbol{\rho}_t \cdot (\mathbf{c}_s - \mathbf{c}_t)$ with $\beta^R = 1/\beta$.

Then, one can redefine $\tilde{u}_s = u_s/(\beta^R)^{T+1}$ and $\tilde{u}_t = u_t/(\beta^R)^{T+1}$ to obtain $\tilde{u}_s - \tilde{u}_t \leq (\beta^R)^{t-T-1} \boldsymbol{\rho}_t \cdot (\mathbf{c}_s - \mathbf{c}_t)$. This is equivalent to $\tilde{u}_s - \tilde{u}_t \leq \tilde{\mathbf{p}}_t \cdot (\mathbf{c}_s - \mathbf{c}_t)$ where $(\beta^R)^{T-t+1} \tilde{\mathbf{p}}_t = \boldsymbol{\rho}_t$.

Finally, this corresponds to the conditions of *ICRES* in which $u_s^R = \tilde{u}_s$ and $u_t^R = \tilde{u}_t$; and $\tilde{\mathbf{p}}_t^R = \tilde{\mathbf{p}}_t$. (The other unknowns can be set to zero: $u_s^E = u_t^E = \tilde{p}_{n,t}^E = 0$.)

A.1.6. *Proof of Corollary 3.* We again prove this result by providing two data sets, one which satisfies the RP restrictions of intertemporal consumption with anticipating and experiencing selves (*ICAES)* but violates the restrictions of *ICRES*, and another which satisfies the restrictions of *ICRES* but violates the restrictions of *ICAES*. We can resort back to our simulation exercise, in particular the consumption time series provided in Table 1. The consumption time series generated by *ICAES* violates the RP conditions of the *ICRES* model, whereas the consumption data generated from *ICRES* violates the RP characterization of *ICAES*.

A.2. *Revealed Preference Tests of Scenarios II and III.*

A.2.1. *Calculations scenario II.* First, conditions (12)–(14) imply $\tilde{p}^i_{1,2} \leq \tilde{p}^i_{1,3}$ and $\tilde{p}^i_{1,2} \leq \tilde{p}^i_{1,1}$. Next, conditions (15) imply that

$$\begin{aligned}
(\beta^A)\tilde{p}^A_{1,1} + (\beta^R)^4\tilde{p}^R_{1,1} + \tilde{p}^E_{1,1} &= 1.025\\
(\beta^A)^2\tilde{p}^A_{1,2} + (\beta^R)^3\tilde{p}^R_{1,2} + \tilde{p}^E_{1,2} &= \sqrt{1.025} + 0.001\\
(\beta^A)^3\tilde{p}^A_{1,3} + (\beta^R)^2\tilde{p}^R_{1,3} + \tilde{p}^E_{1,3} &= 1.
\end{aligned}$$

Replacing all shadow prices with their counterparts from observation 2 gives

$$\begin{aligned}
(\beta^A)\tilde{p}^A_{1,2} + (\beta^R)^4\tilde{p}^R_{1,2} + \tilde{p}^E_{1,2} &\leq 1.025\\
(\beta^A)^2\tilde{p}^A_{1,2} + (\beta^R)^3\tilde{p}^R_{1,2} + \tilde{p}^E_{1,2} &= \sqrt{1.025} + 0.001\\
(\beta^A)^3\tilde{p}^A_{1,2} + (\beta^R)^2\tilde{p}^R_{1,2} + \tilde{p}^E_{1,2} &\leq 1.
\end{aligned}$$

Then construct two new conditions, by subtracting the equality from the first inequality and the last inequality from the equality, to demonstrate the following differences:

$$\begin{aligned}
((\beta^A) - (\beta^A)^2)\tilde{p}^A_{1,2} + ((\beta^R)^4 - (\beta^R)^3)\tilde{p}^R_{1,2} &\leq 1.024 - \sqrt{1.025}\\
((\beta^A)^2 - (\beta^A)^3)\tilde{p}^A_{1,2} + ((\beta^R)^3 - (\beta^R)^2)\tilde{p}^R_{1,2} &\geq \sqrt{1.025} - 0.999.
\end{aligned}$$

The RHS of the first condition is smaller than the RHS of the second condition. Furthermore, $(\beta^A)^2 - (\beta^A)^3 \leq (\beta^A) - (\beta^A)^2 \leq 0$ whereas $(\beta^R)^4 - (\beta^R)^3 \geq (\beta^R)^3 - (\beta^R)^2 \geq 0$. It is not possible to find values for $\tilde{p}^A_{1,2}, \tilde{p}^R_{1,2} \geq 0$ that simultaneously satisfy the above pair of inequalities.

A.2.2. *Calculations scenario III.* Conditions (13) and (14) imply $\tilde{p}^E_{1,2} \leq \tilde{p}^E_{1,1} \leq \tilde{p}^E_{1,3}$ and $\tilde{p}^R_{1,2} \leq \tilde{p}^R_{1,1} \leq \tilde{p}^R_{1,3}$. Next, conditions (15) imply that

$$\begin{aligned}
(\beta^R)^4\tilde{p}^R_{1,1} + \tilde{p}^E_{1,1} &= 1.175\\
(\beta^R)^3\tilde{p}^R_{1,2} + \tilde{p}^E_{1,2} &= 1.08\\
(\beta^R)^2\tilde{p}^R_{1,3} + \tilde{p}^E_{1,3} &= 1,
\end{aligned}$$

and thus,

$$\begin{aligned}
(\beta^R)^4\tilde{p}^R_{1,2} + \tilde{p}^E_{1,2} &\leq 1.175\\
(\beta^R)^3\tilde{p}^R_{1,2} + \tilde{p}^E_{1,2} &= 1.08\\
(\beta^R)^2\tilde{p}^R_{1,2} + \tilde{p}^E_{1,2} &\leq 1.
\end{aligned}$$

We subtract the equality from the first inequality and the last inequality from the equality to show,

$$\begin{aligned}
((\beta^R)^4 - (\beta^R)^3)\tilde{p}^R_{1,2} &\leq 0.095\\
((\beta^R)^3 - (\beta^R)^2)\tilde{p}^R_{1,2} &\geq 0.08.
\end{aligned}$$

We can finally rewrite this as $\tilde{p}^R_{1,2} \leq \frac{0.095}{(\beta^R)^4-(\beta^R)^3}$ and $\tilde{p}^R_{1,2} \geq \frac{0.08}{(\beta^R)^3-(\beta^R)^2}$. These two conditions are mutually exclusive for high levels of β^R, more specifically when $\beta^R \geq 19/16$.

A.3. *Extension to Infinite Horizon.* Throughout the main article, we have restricted attention to the case where the horizon $\mathcal{T}$ is finite. However, we can easily generalize our theoretical framework to a case where $\mathcal{T} = \mathbb{N}_0$. To that end, we will readjust the consumer's overall utility, as seen from decision period $\tau \geq \tau_0$, as follows:

$$\text{(A.13)} \qquad \sum_{t\geq\tau} \delta^{t-\tau}\big[\omega^A(\tau)D^A(t-\tau)u^A(\mathbf{c}_t) + \omega^R(\tau)D^R(t)u^R(\mathbf{c}_t) + u^E(\mathbf{c}_t)\big].$$

A few remarks are in order at this point. First note that in this case, the time function D^R is simply a function of t and not of the length of the planning period (as T is infinite). In accordance with the finite-horizon setting, we merely assume that D^R is decreasing in t. Next, the main difference between (A.13) and the objective function in the main article is the presence of the exponential discount factor δ, contained in the (open) unit interval (0,1). Although this factor is assumed to be uniform, time functions D^A and D^R remain heterogeneous across selves. We can then show that, under the same sufficient condition as in Proposition 2, the dynamic consistency result remains robust:

PROPOSITION A.1. *Let $(\mathbf{c}^*_t)_{t\geq\tau}$ be the solution to maximizing (A.13) subject to $\sum_{t=\tau}^{+\infty} \boldsymbol{\rho}_t \cdot \mathbf{c}_t \leq y_\tau$ and let $(\hat{\mathbf{c}}_t)_{t\geq\tau'}$ be the solution to the same problem with the decision period shifted to $\tau' \neq \tau$. Then, $\hat{\mathbf{c}}_t = \mathbf{c}^*_t$ for all $t \in [\tau', +\infty)$ if*

$$\text{(A.14)} \qquad \log\omega^A(\tau) = -\log D^A(t-\tau) + \alpha(t), \ \textit{for all } t \geq \tau, \ \textit{and } \tau \in \mathcal{T}$$

for some mapping $\alpha : \mathbb{R}_+ \to \mathbb{R}$, and $\frac{\partial\omega^R(\tau)}{\partial\tau} = 0$.

The proof of this proposition is very similar to the proof of Proposition 2. For completeness, we will replicate it here. Assume, by way of contradiction, that the solution $(\mathbf{c}^*_t)_{t\geq\tau}$ to optimizing (A.13) subject to $\sum_{t\geq\tau} \boldsymbol{\rho}_t \cdot \mathbf{c}_t \leq y_\tau$ is no longer optimal from decision period τ' onward. This implies there must exist another sequence $(\hat{\mathbf{c}}_t)_{t\geq\tau'}$ such that,

$$\begin{aligned} &\textstyle\sum_{t\geq\tau'} \delta^{t-\tau'}\big[\omega^A(\tau')D^A(t-\tau')u^A(\hat{\mathbf{c}}_t) + \omega^R(\tau')D^R(t)u^R(\hat{\mathbf{c}}_t) + u^E(\hat{\mathbf{c}}_t)\big] \\ \text{(A.15)} \qquad &> \textstyle\sum_{t\geq\tau'} \delta^{t-\tau'}\big[\omega^A(\tau')D^A(t-\tau')u^A(\mathbf{c}^*_t) + \omega^R(\tau')D^R(t)u^R(\mathbf{c}^*_t) + u^E(\mathbf{c}^*_t)\big], \end{aligned}$$

and given assumption (A.14), we can write $a(t) = \omega^A(\tau')D^A(t-\tau')$ and $b(t) = \omega^R(\tau')D^R(t)$. Multiplying both sides of (A.15) by $\delta^{\tau'-\tau}$, we obtain the following:

$$\begin{aligned} &\textstyle\sum_{t\geq\tau'} \delta^{t-\tau}\big[\omega^A(\tau')D^A(t-\tau')u^A(\hat{\mathbf{c}}_t) + \omega^R(\tau')D^R(t)u^R(\hat{\mathbf{c}}_t) + u^E(\hat{\mathbf{c}}_t)\big] \\ \text{(A.16)} \qquad &> \textstyle\sum_{t\geq\tau'} \delta^{t-\tau}\big[\omega^A(\tau')D^A(t-\tau')u^A(\mathbf{c}^*_t) + \omega^R(\tau')D^R(t)u^R(\mathbf{c}^*_t) + u^E(\mathbf{c}^*_t)\big]. \end{aligned}$$

Adding $\sum_{t=\tau}^{\tau'-1} \delta^{t-\tau}\left[\omega^A(\tau')D^A(t-\tau')u^A(\mathbf{c}^*_t) + \omega^R(\tau')D^R(t)u^R(\mathbf{c}^*_t) + u^E(\mathbf{c}^*_t)\right]$ to both sides of the inequality in (A.16), we obtain:

$$\begin{aligned} &\textstyle\sum_{t\geq\tau} \delta^{t-\tau}\big[a(t)u^A(\hat{\mathbf{c}}_t) + b(t)u^R(\hat{\mathbf{c}}_t) + u^E(\hat{\mathbf{c}}_t)\big] \\ \text{(A.17)} \qquad &?\ \textstyle\sum_{t\geq\tau} \delta^{t-\tau}\big[a(t)u^A(\mathbf{c}^*_t) + b(t)u^R(\mathbf{c}^*_t) + u^E(\mathbf{c}^*_t)\big]. \end{aligned}$$

But (A.17) is in contradiction with the optimality of $(\mathbf{c}^*_t)_{t\geq\tau}$. We therefore conclude that $(\mathbf{c}^*_t)_{t\geq\tau'}$ is also optimal for τ'. Given that τ, τ' were chosen randomly, we have the desired result.

A.4. *Additional Results.*

A.4.1. *Predictability of prices.* In this appendix, we discuss the time series of spot prices (Figure A.1), interest rates (Figure A.2), and log discounted prices (Figure A.3) for our sample, along the lines of Blow et al. (2021). We normalize all prices by using the corresponding mean prices in 1992 as the base. Figure A.1 shows that all spot prices are increasing over the observation period. One exception is the price of petrol, which declines between 1985 and 1988. Figure A.2 summarizes the evolution of the yearly interest rate on consumer loans. This interest rate varies between 10% and 20%. Yet it is worth noting that, due to compounding of interest across time, the discount rate is systematically increasing over time. Because of this, the discounted prices in Figure A.3 are declining. The log discounted price curves are moreover fairly linear. This reflects a relatively uniform (negative) growth rate over time.

Like Blow et al. (2021), we subsequently study predictability by regressing the (log) discounted prices of each commodity on a linear time trend. We do this for every commodity and for every consumer separately, because the sequence of observations differs by consumer. This leads to 16,416 linear regressions. The R^2 of each regression reflects the degree of (temporal) variation in prices that is captured by the time trend. Higher R^2 values imply better predictability. The histogram in Figure A.4 shows the distribution of the R^2 among 16,416 consumer–commodity pairs. The median R^2 among all consumer–commodity regressions is as high as 93%.

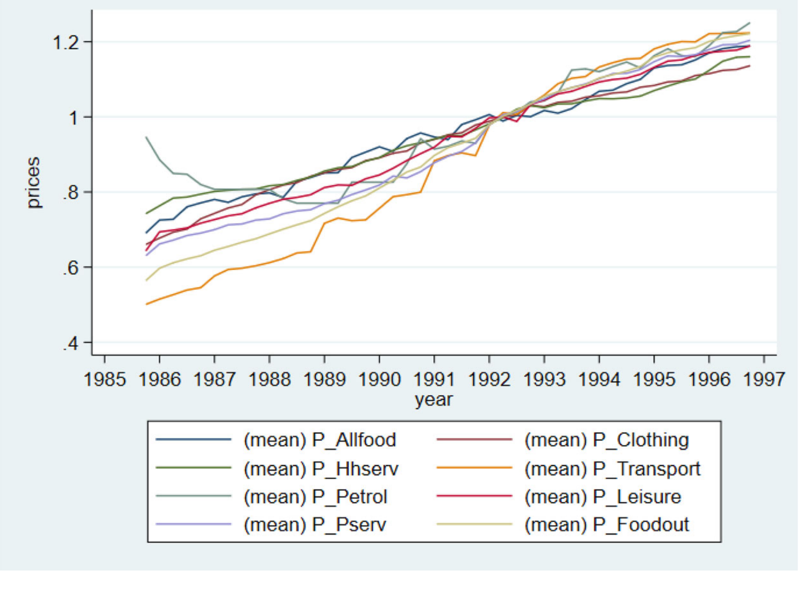

FIGURE A.1

TIME SERIES OF SPOT PRICES.

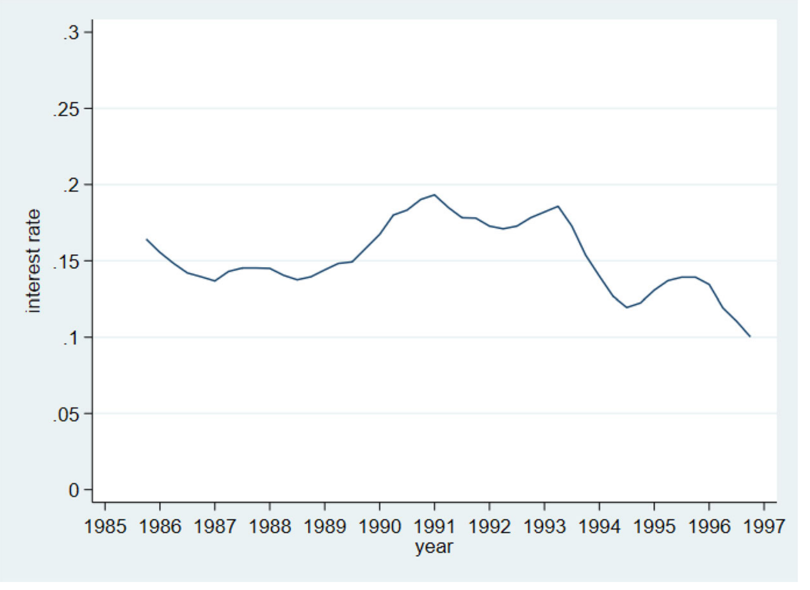

Figure A.2

time series of interest rate on consumer loans.

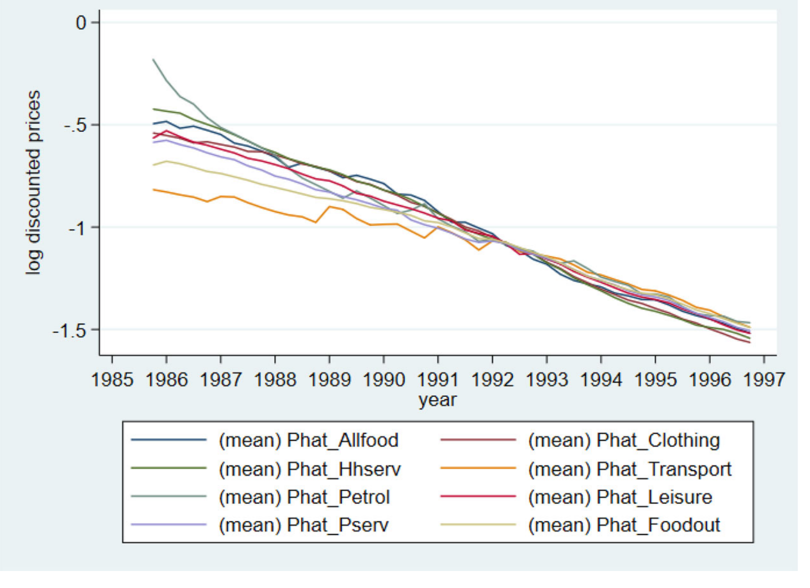

Figure A.3

time series of log discounted prices.

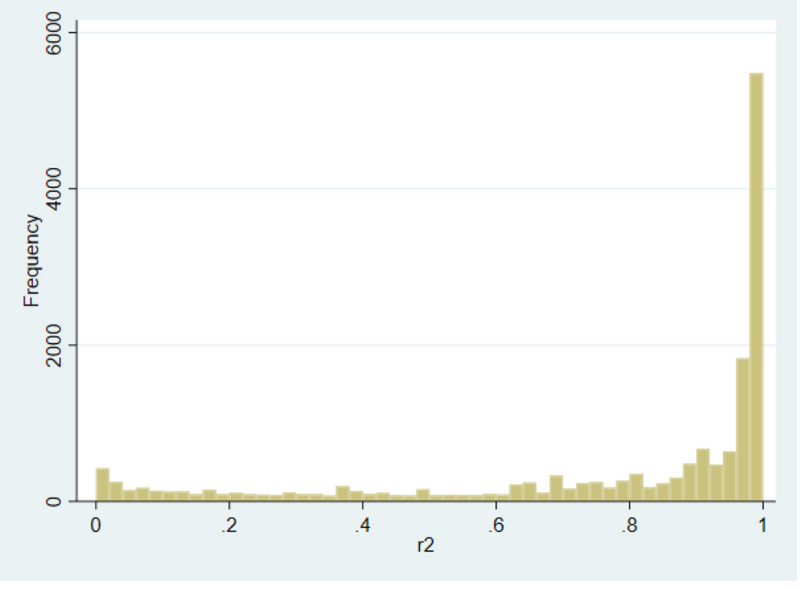

FIGURE A.4

DISTRIBUTION OF THE PROPORTION OF VARIATION IN LOG DISCOUNTED PRICES THAT IS PREDICTABLE FROM A LINEAR TIME TREND.

A.4.2. *Explaining consistency with* ICAES.

TABLE A.1
REGRESSION OF CONSISTENCY WITH *ICAES* ON TOTAL INTERTEMPORAL BUDGET, AGE, HOUSEHOLD COMPOSITION, AND YEAR DUMMIES.

	ICAES
ICAES	
allexp	0.000
	(0.000)
Between 26 and 35	0.142
	(0.164)
Between 36 and 45	0.094
	(0.169)
Between 46 and 55	−0.173
	(0.189)
Older than 55	−0.240
	(0.190)
Couple without children (under 14 y.o)	−0.123
	(0.120)
Couple with a child	−0.177
	(0.119)
Couple with two children	−0.311***
	(0.116)
Couple with three or more children	−0.443***
	(0.146)
Observations	2052

NOTE: Age data are available in intervals 26–35 (dummy 2), 36–45 (dummy 3), 46–55 (dummy 4), and older than 55 (dummy 5). The sample consists of all consumers. Year dummies, all insignificant, are suppressed for compactness.

A.4.3. *Extensions (Pass rates).*

TABLE A.2
PASS RATES FOR DIFFERENT SPECIFICATIONS OF *ICARES*, WITH LIQUIDITY CONSTRAINTS, UNCERTAINTY, MEASUREMENT ERROR, AND FOR SUBSAMPLES THAT START IN THE FIRST QUARTER.

Pass Rates	Singles				Couples			
	ICES	*ICAES*	*ICRES*	*ICARES*	*ICES*	*ICAES*	*ICRES*	*ICARES*
Baseline	0.093	0.494	0.820	0.948	0.035	0.455	0.795	0.964
Liq Constr	0.174	0.512	0.988	0.994	0.088	0.468	0.988	0.995
Uncertainty								
$\alpha = 0.99$	0.116	0.552	0.895	0.965	0.055	0.546	0.878	0.978
$\alpha = 0.975$	0.238	0.738	0.965	0.977	0.196	0.747	0.927	0.986
$\alpha = 0.95$	0.541	0.913	0.971	0.983	0.444	0.918	0.957	0.991
Price error								
$\sigma_p = 0.005$	0.087	0.477	0.791	0.942	0.038	0.455	0.784	0.965
$\sigma_p = 0.01$	0.081	0.477	0.785	0.942	0.041	0.447	0.757	0.960
$\sigma_p = 0.05$	0.047	0.407	0.570	0.837	0.044	0.355	0.513	0.833
Quantity error								
$\sigma_c = 0.01$	0.093	0.494	0.820	0.953	0.035	0.451	0.795	0.964
$\sigma_c = 0.05$	0.093	0.477	0.837	0.959	0.039	0.451	0.791	0.965
$\sigma_c = 0.1$	0.087	0.459	0.831	0.959	0.044	0.453	0.796	0.965
First quarter	0.118	0.490	0.843	0.922	0.043	0.480	0.799	0.977

SUPPORTING INFORMATION

Additional supporting information may be found online in the Supporting Information section at the end of the article.

Data S1

REFERENCES

ADAMS, A., L. CHERCHYE, B. DE ROCK, and E. VERRIEST, "Consume Now or Later? Time Inconsistency, Collective Choice, and Revealed Preference," *American Economic Review* 104 (2014), 4147–83.

AFRIAT, S. N., "The Construction of Utility Functions from Expenditure Data," *International Economic Review* 8 (1967), 67–77.

AGUIAR, V. H., and N. KASHAEV, "Stochastic Revealed Preferences with Measurement Error," *Review of Economic Studies* 88 (2021), 2042–93.

AMBRUS, A., and K. ROZEN, "Rationalising Choice with Multi-Self Models," *Economic Journal* 125 (2014), 1136–56.

ANDREONI, J., and C. SPRENGER, "Risk Preferences are Not Time Preferences," *American Economic Review* 102 (2012), 3357–76.

BAUCELLS, M., and S. BELLEZZA, "Temporal Profiles of Instant Utility during Anticipation, Event, and Recall," *Management Science* 63 (2017), 729–48.

BEATTY, T., and I. CRAWFORD, "How Demanding is the Revealed Preference Approach to Demand," *American Economic Review* 101 (2011), 2782–95.

BENHABIB, J., and A. BISIN, "Modeling Internal Commitment Mechanisms and Self-Control: A Neuroeconomics Approach to Consumption-Saving Decisions," *Games and Economic Behavior* 52 (2005), 460–92.

BLOW, L., M. BROWNING, and I. CRAWFORD, "Nonparametric Analysis of Time-Inconsistent Preferences," *Review of Economic Studies* 88 (2021), 2687–734.

BLUNDELL, R., L. PISTAFERRI, and I. PRESTON, "Consumption Inequality and Partial Insurance," *American Economic Review* 98 (2008), 1887–921.

———, and J.-M. ROBIN, "Latent Separability: Grouping Goods without Weak Separability," *Econometrica* 68 (2000), 53–84.

BRONARS, S., "The Power of Nonparametric Tests of Preference Maximization," *Econometrica* 55 (1987), 693–98.

BROWNING, M., "A Nonparametric Test of the Life-Cycle Rational Expections Hypothesis," *International Economic Review* 30 (1989), 979–92.

———, and M. COLLADO, "Habits and Heterogeneity in Demands: A Panel Data Analysis," *Journal of Applied Econometrics* 22 (2007), 625–40.

CAPLIN, A., and J. LEAHY, "Psychological Expected Utility Theory and Anticipatory Feelings," *Quarterly Journal of Economics* 116 (2001), 55–79.

CASADO, J. M., "From Income to Consumption: Measuring Households Partial Insurance," *Empirical Economics* 40 (2011), 471–95.

CASTILLO, M. I., S. SUN, M. A. FRANK-CRAWFORD, and J. C. BORRERO, "Save the Best for Last I: Young Adults Demonstrate Negative Time Preference—A Replication and Extension," *Behavior Analysis: Research and Practice* 22 (2022), 143–63.

CHAPMAN, G. B., "Expectations and Preferences for Sequences of Health and Money," *Organizational Behavior and Human Decision Processes* 67 (1996), 59–75.

CHERCHYE, L., B. DE ROCK, R. GRIFFITH, M. O'CONNELL, K. SMITH, and F. VERMEULEN, "A New Year, A New You? Within-Individual Variation in Food Purchases," *European Economic Review* 127 (2020), 1034–78.

CHIAPPORI, P. A., "Rational Household Labor Supply," *Econometrica* 56 (1988), 63–89.

———, "Collective Labor Supply and Welfare," *Journal of Political Economy* 100 (1992), 437–67.

CRAWFORD, I., "Habits Revealed," *Review of Economic Studies* 77 (2010), 1382–402.

———, and ———, "Empirical Revealed Preference," *Annual Review of Economics* 6 (2014), 503–24.

———, and M. POLISSON, "Testing for Intertemporal Nonseparability," *Journal of Mathematical Economics* 52 (2014), 46–49.

DEAN, M., and A. SAUTMANN, "Credit Constraints and the Measurement of Time Preferences," *Review of Economics and Statistics* 103 (2021), 119–35.

DEMUYNCK, T., "Statistical Inference for Measures of Predictive Success," *Theory and Decision* 79 (2015), 689–99.

———, and E. VERRIEST, "I'll Never Forget My First Cigarette: A Revealed Preference Analysis of the Habits as Durables Model," *International Economic Review* 54 (2013), 717–38.

Drouhin, N., "Non-Stationary Additive Utility and Time Consistency," *Journal of Mathematical Economics* 86 (2020), 1–14.

Dziewulski, P., "Revealed Time Preference," *Games and Economic Behavior* 112 (2018), 67–77.

Echenique, F., T. Imai, and K. Saito, "Testable Implications of Models of Intertemporal Choice: Exponential Discounting and Its Generalizations," *American Economic Journal: Microeconomics* 12 (2020), 114–43.

Elster, J., and G. Loewenstein, "Utility from memory and anticipation," in *Choice Over Time* (New York: Russell Sage Foundation, 1992), 213–34.

Frederick, S., ———, and T. O'Donoghue, "Time Discounting and Time Preference: A Critical Review," *Journal of Economic Literature* 40 (2002), 351–401.

Gilboa, I., A. Postlewaite, and L. Samuelson, "Memorable Consumption," *Journal of Economic Theory* 165 (2016), 414–55.

Guo, L., "Anticipatory Consumptions," *Management Science* 66 (2020), 3717–34.

Hai, R., D. Krueger, and A. Postlewaite, "On the Welfare Cost of Consumption Fluctuations in the Presence of Memorable Goods," *Quantitative Economics* 11 (2020), 1177–214.

Houthakker, H. S., "Revealed Preference and the Utility Function," *Economica* 17 (1950), 159–74.

Jackson, M. O., and L. Yariv, "Present Bias and Collective Dynamic Choice in the Lab," *American Economic Review* 104 (2014), 4184–204.

Kahneman, D., P. P. Wakker, and R. Sarin, "Back to Bentham? Explorations of Experienced Utility," *Quarterly Journal of Economics* 112 (1997), 375–406.

Koopmans, T., "Stationary Ordinal Utility and Impatience," *Econometrica* 28 (1960), 287–309.

Kőszegi, B., "Utility from Anticipation and Personal Equilibrium," *Economic Theory* 44 (2009), 415–44.

Laajaj, R., "Endogenous Time Horizon and Behavioral Poverty Trap: Theory and Evidence from Mozambique," *Journal of Development Economics* 127 (2017), 187–208.

Loewenstein, G., "Anticipation and the Valuation of Delayed Consumption," *Economic Journal* 97 (1987), 666–84.

———, and D. Prelec, "Preferences for Sequences of Outcomes," *Psychological Review* 100 (1993), 91–108.

———, and N. Sicherman, "Do Workers Prefer Increasing Wage Profiles?" *Journal of Labor Economics* 9 (1991), 67–84.

Manzini, P., M. Mariotti, and L. Mittone, "Choosing Monetary Sequences: Theory and Experimental Evidence," *Theory and Decision* 69 (2010), 327–54.

May, K. O., "Intransitivity, Utility, and the Aggregation of Preference Patterns," *Econometrica* 22 (1954), 1–13.

Morewedge, C. K., "Utility," in *The Wiley Blackwell Handbook of Judgment and Decision Making* (Chichester West Sussex: Wiley, 2015), 295–330.

O'Donoghue, T., and M. Rabin, "Doing It Now or Later," *American Economic Review* 89 (1999), 103–24.

Prelec, D., and G. Loewenstein, "Decision Making over Time and under Uncertainty: A Common Approach," *Management Science* 37 (1991), 770–86.

Rockafellar, R. T., *Convex Analysis* (Princeton: Princeton University Press, 1970).

Samuelson, P., "A Note on Measurement of Utility," *Review of Economic Studies* 4 (1937), 155–61.

———, "A Note on the Pure Theory of Consumer's Behavior," *Economica* 5 (1938), 61–71.

Selten, R., "Properties of a Measure of Predictive Success," *Mathematical Social Sciences* 21 (1991), 153–67.

Varian, H. R., "The Nonparametric Approach to Demand Analysis," *Econometrica* 50 (1982), 945–74.

———, "Non-Parametric Tests of Consumer Behaviour," *Review of Economic Studies* 50 (1983), 99–110.

———, "Nonparametric Analysis of Optimizing Behavior with Measurement Error," *Journal of Econometrics* 30 (1985), 445–58.