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Identification of the viscoelastic properties of an asphalt mixture using a scanning laser Doppler vibrometer

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Abstract Estimating the master curve of the complex modulus of asphalt mixtures is essential for high quality and sustainable mixture and pavement design. There are multiple standard methods to estimate this master curve using hydraulic-pneumatic testing machines. These methods are complex to perform, need expensive equipment, and have constraints over the geometry of the testing samples. Therefore, the investigation of alternative methodologies to overcome these issues is of importance. In this research, an experimental setup coupled with a back-calculation technique is developed to identify the viscoelastic properties of an asphalt mixture using an optical measurement system. Using this system instead of traditional transducers eliminates the inaccuracies caused due to the attachment of a transducer to the specimen and allows feasible measurements on multiple points of the specimen. The developed method is compared with a standard method and an alternative method based on analytical formulas, and the results exhibit a good level of accuracy at a wide range of frequency and temperature. It is also demonstrated that even though this method can provide the master curve of a specimen with arbitrary geometry, the first natural frequency of the specimen at the highest temperature provides the first data point of the master curve at low frequencies. Therefore, the first natural frequency of the specimen should be considered while selecting the geometry of the test object.

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1 Introduction

Understanding the mechanical properties of asphalt mixtures is the first step to a high quality, cost-effective, and sustainable asphalt pavement design. One of the important mechanical properties of asphalt mixtures is its complex modulus of elasticity or $E^*(\omega, T)$. This complex value is presented in Eq. 1, with E' the storage modulus, E'' the loss modulus, and δ the phase angle [1].

$$E^* = E' + iE'' = |E^*|e^{i\delta} \tag{1}$$

European standard EN 12697-26:2018 presents multiple methods to characterize the E^*. These methods consist of bending, direct or indirect tensile tests on asphalt mixtures with specific shapes (prismatic, trapezoidal, or cylindrical) under cyclic sinusoidal or haversine loading [2]. Using the complex modulus of elasticity of the material at different temperatures and frequencies acquired by these experiments and applying the Time-Temperature Superposition (TTS) principle, it is possible to form the master curve of the complex modulus of the asphalt mixtures. However, these methods are relatively complex and require expensive instruments. Furthermore, depending on the experimental setup and the equipment, they are only applicable to specimens with certain geometries.

An alternative technique to compute the complex modulus of elasticity of a material is through experimental modal analysis [3]. Modal analysis is the process of determining the inherent dynamic characteristics of a specimen in terms of natural frequencies, damping ratios, and mode shapes and using these to formulate a mathematical model for its dynamic behavior [4]. This information can then be used to identify the material properties of the specimen. This method has been used for material characterization in different fields of engineering [5], including the complex modulus estimation of asphalt specimens [6].

The results of the modal analysis experiments are typically used in two ways to characterize the complex modulus of elasticity of asphalt mixtures. The first approach, called the forward-calculation method, relies on some analytical formulas to compute the complex modulus of elasticity from modal parameters. Based on the analytical formulas used, the complex modulus of elasticity can be determined for the fundamental natural frequency [7] or multiple natural frequencies [8]. The second approach is called the back-calculation or inverse method. The principle of back-calculation for material identification is to update the unknown material parameters iteratively in a Finite Element Model (FEM) of the test specimens to minimize the distance between the modal experiment output and FEM simulations [9]. This method has been applied to asphalt mixtures, and the main advantage of using the back-calculation methodology over the forward-calculation methods is that the specimens do not have the same strict geometry limitations of the standard methods [10].

The main objectives of this research are to develop an automatic testing setup, based on an optical measurement technique, and apply a back-calculation methodology to explore the complex behavior of asphalt mixtures and quantify their mechanical properties through non-destructive experiments in the lab. Therefore, a novel experimental setup combined with a new back-calculation methodology is proposed to estimate the master curve of asphalt mixtures. In this method, the specimens are excited using a modal shaker instead of a hammer [11] to have better control over the excitation force and improve the signal-to-noise ratio of the measurements. Furthermore, the measurements are conducted using an optical measurement system to have a full field vibration profile on the surface of the specimens. Having the full field vibration allows for the comparison of the mode shapes acquired from the experiments and FEM, and therefore the computation of the Modal Assurance Criteria (MAC). Since this method is based on FEM simulations and not any geometry dependent theory (e.g., Timoshenko's beam theory [8]), it can be applied to specimens with different or even random geometries. Furthermore, this method uses all measured modal parameters of the specimens as opposed to some previous methods that only use the fundamental resonance frequency [12, 13] or only the flexural mode shapes [8] of the specimens.

Section 2 of this paper presents the material production followed by a description of the developed back-calculation procedure and its different steps. Section 3 presents the results of the developed method and the comparison of the estimated master curve with the master curves computed with two other techniques. Finally, the conclusions of this research are given in Section 4.

2 Materials and Methods

A general overview of the materials and methods used in this research is provided in this section. First, in Subsect. 2.1, the overall research methodology is explained. Information about the materials and specimens used in the experiments are presented in Subsect. 2.2. Subsect. 2.3 includes the experimental setup followed by details of the developed back-calculation procedure in Subsect. 2.4. Subsect. 2.5 describes the FEM simulations and the models used to anticipate the viscoelastic properties of asphalt mixtures. Finally the defined cost function and the optimization technique used in this study are explained in Subsect. 2.6.

2.1 Methodology

In this research, a back-calculation procedure is proposed that can provide the master curve of the complex modulus and phase angle of asphalt mixtures from modal analysis experiments performed at different temperatures. Then, these E^* and phase angle master curves are compared with the master curves of the same asphalt mixture estimated using a forward-calculation method [8], and the standard Four-Point Bending test on Prismatic specimens (4PB-PR) [2]. For more information on these two methods, refer to [8].

After the results are validated using these two methods, the Coefficient of Variance (CV) for the three E^* master curves are plotted and the repeatability of each method at different frequencies is investigated. CV is the standard deviation over mean, and for material testing on asphalt mixtures, it is normally between 0.15 and 0.30. A CV equal to 0.15, 0.3, and 0.45 means low variation, high variation, and extremely high variation between the measurements, respectively [14]. The results show that the proposed method has a CV of less than 0.15 at frequencies higher than 0.5 Hz hence it can be used as an alternative for traditional testing methods.

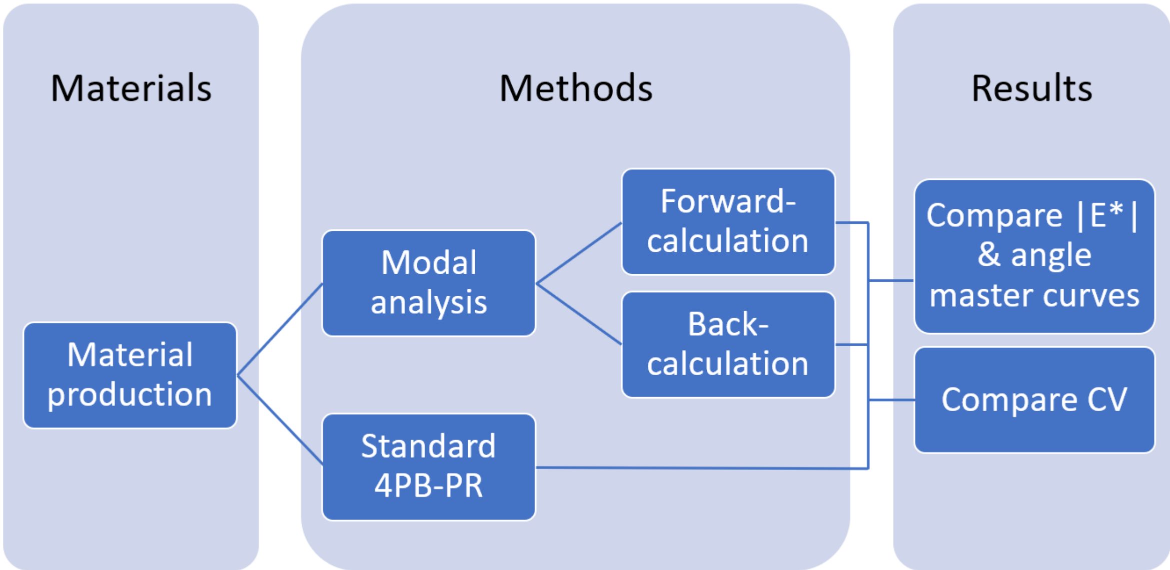


Fig. 1: Research methodology.

Table 1: Basic properties of the 35/50 penetration grade bitumen.

Penetration (1/10 mm)	37
Softening point (°C)	54.3
Fraass breaking point (°C)	-10

Table 2: Mixture composition (by aggregate mass).

Limestone 6.3/14	39.8 %
Limestone 2/6.3	14.0 %
Limestone 0/2	30.0 %
River sand 0/1	7.5 %
Filler (filler 15)	8.7 %
Total	100 %
Bitumen 35/50	4.3 %

The general methodology of this research is illustrated in Figure 1.

2.2 Material production

The asphalt mixture used in this research previously has been used in [8] to develop a forward-calculation methodology to compute the master curve of E^*. This mixture is a typical base layer developed in the EMIB research group of University of Antwerp as a reference for common tests. It is an asphalt concrete with a maximum aggregate size of 14 mm (AC14) and contains 4.3% of bitumen by mass of aggregates. The bitumen is a common 35/50 penetration grade bitumen and its basic properties are available in Table 1. These properties are based on the data sheets of the supplier and confirmed by lab tests according to the standards NBN EN 1426:2015 [15], NBN EN 1427:2015 [16], and NBN EN 12593:2015 [17]. Table 2 shows the composition of the mixtures and Figure 2 illustrates the final grading curve of the mixture.

Six prismatic beams were prepared in this study. Three ($B1 - B3$) for the modal analysis experiments and three ($R1 - R3$) for the reference four-point bending tests. The beams were cut from plates which were produced with the same procedure aiming at similar volumetric properties. The mixture was manufactured according to NBN EN 12697-35:2016 [18] and compacted with a large scale roller compactor using the 2-wheel heavy compaction method, according to NBN EN 12697-33+A1:2007 [19]. The mixing temperature and the compaction temperature were 175 °C and 165 °C, respectively. The dimensions of the beams are presented in Table 3.

These dimensions were selected considering the European standards and the restrictions of the experimental setup. According to standard NBN EN 12697-26:2018 [2], the width h_y and height h_z of the samples must be at least three times the maximum grain size of the mixture. Therefore, the width h_y and height h_z of all the beams used in this study were more than 4.2 cm. The dimensions of the reference specimens $R1$ to $R3$ were chosen in accordance with NBN EN 12697-26:2018 [2]. Furthermore, considering the size of the climate chamber available at this study and the frame

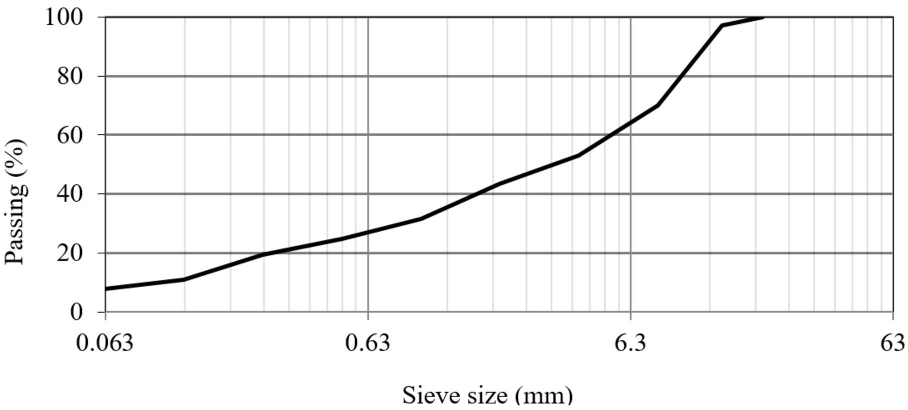


Fig. 2: Grading curve of the designed mixture.

Table 3: Dimensions of the beams used for modal analysis ($B1-B3$), and the beams for the reference standard experiments ($R1-R3$).

Specimen	**Height (H) [cm]**	**Width (W) [cm]**	**Length (L) [cm]**
B1	5.23	5.40	40.0
B2	5.27	5.37	39.9
B3	5.26	5.51	40.0
R1	5.02	5.07	44.8
R2	4.95	5.07	44.8
R3	4.34	5.00	44.7

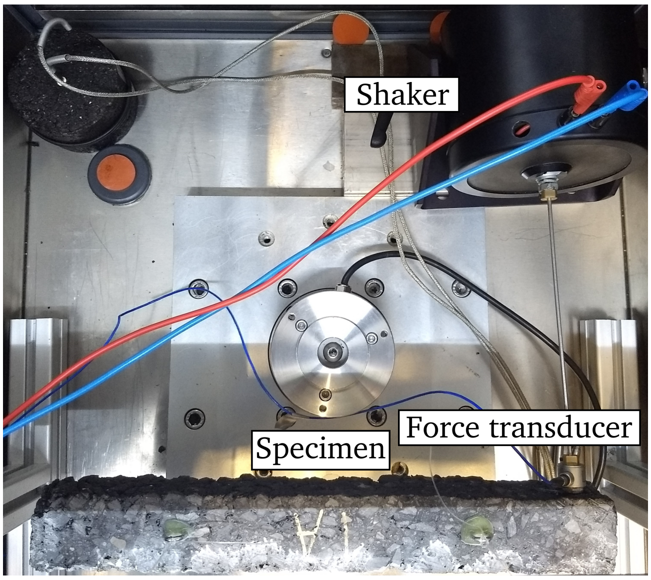
installed inside of the climate chamber for specimen suspension (see Figure 3b), the length L of specimens $B1$ to $B3$ had to be less than 45 cm.

After the production of beams $B1$ to $B3$, their front sides were painted by a white spray paint (Ardrox® 9D1B aerosol) to improve the LDV measurement quality as suggested in the literature [20].

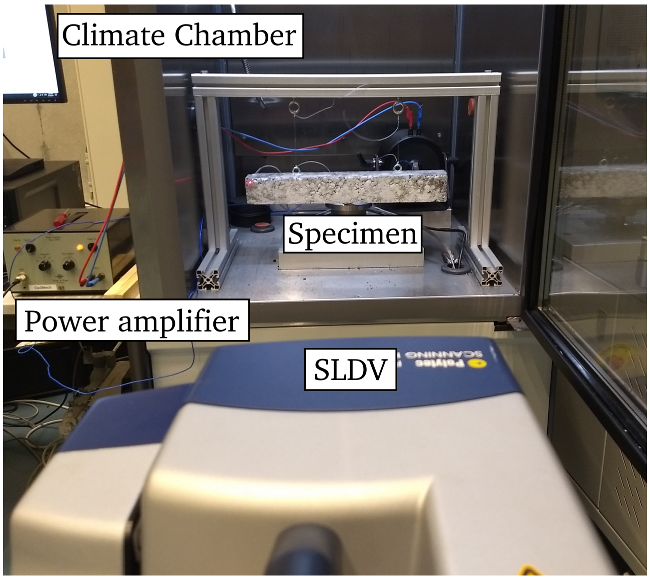
2.3 Experimental setup

First, a small frame was built to fit inside the climate chamber of the Universal Testing Machine (UTM). Then, the specimens were suspended from this frame using two screw eyes and fishing lines to simulate free-free conditions (see Figure 3b). The reason for choosing this boundary condition was the ease in setting up a free-free modal experiments and making a FEM that represents the boundary conditions appropriately. The experiments were conducted at five temperatures (5, 10, 15, 20, and 30 °C). These temperatures were chosen to cover a wide range of the master curve while taking into account the operational temperature limitations of the shaker, which cannot go lower than 0 °C or exceed 40 °C. If measurements at a more extensive temperature range are required, there exist shakers that have a larger temperature range or it is possible to place the shaker outside of the climate chamber and connect it to the specimen with a stinger that gets through a small hole.

To model the asphalt mixture as a linear viscoelastic material, the strain of the specimen must remain under a certain level. Therefore, using a shaker or an automatic impact hammer to have complete control over the load is advantageous. This is because the modal analysis tests with SLDV have to be repeated multiple times according to the number of scanning points and the number of desired averages, and it is necessary to have the same loading profile for every measurement. Furthermore, using a hammer to conduct modal analysis measurements on a suspended asphalt specimen with an SLDV led to poor results due to significant movements of the specimen combined with the speckle error caused by the rough surface of the asphalt specimen. Therefore, in this research, a modal shaker was used to excite the specimens. The shaker was a Brüel & Kjær Vibration Exciter type 4809, exciting the specimens with a periodic chirp signal between the frequency range of 800 to 19200 Hz (see Figure 3a). Signals were generated using a Polytec on-board signal generator and amplified by a Brüel & Kjær power amplifier type 2706. A Brüel & Kjær force transducer type 8230-001 was placed between the tip of the stinger of the shaker and the specimen to measure the force, and a Polytec PSV-400 He-Ne Scanning Laser Doppler



(a)



(b)

Fig. 3: Experimental setup: (a) Top view inside the climate chamber. (b) Overall view from the back of the SLDV.

Vibrometer (SLDV) was used to measure the vibration velocity of 95 scanning points on the surface of the specimens (see Figure 3b). The SLDV is a popular optical measurement system that operates by measuring the velocity of one or multiple point on the surface of an object. The principle of its operation is the detection of the Doppler shift in the frequency of the light scattered from the target point. This instrument has been used in many different engineering fields [21, 22] and in recent years, its ability to conduct measurements on the pavement has been demonstrated in the literature [8, 20, 23].

Using the velocity of the scanning points, measured by the SLDV, and the measured excitation force, the FRFs of the samples $B1 - B3$ at each scanning point were calculated. To increase the quality of the method, each measurement was conducted five times on each of the scanning points and the average of the FRFs were computed. These average FRFs at each point of the of the specimens were the input for the developed back-calculation methodology. Measurements at each temperature took about 10 minutes and there was a waiting time of at least four hours (similar to the standards 4PB-PR test [2]) for the specimen to reach the target temperature between each experiment.

2.4 Back-calculation procedure

Back-calculation is a powerful technique that can be used to estimate the mechanical properties of a material by comparing a set of measurements with simulations. During the past years, the back-calculation methodology has been used by many researchers, especially to develop methods to predict pavement properties with FWD [24, 25]. In more recent research, back-calculation has also been used to estimate the viscoelastic properties of asphalt mixtures using laboratory modal tests [6, 26]. Back-calculation method to estimate the mechanical properties of asphalt mixtures consists of three main steps [1]:

- Measurements: In this step, the specimen is excited, and the vibrations are measured. Typically, the excitation is done using a weight, hammer, or shaker, and the measurements are conducted with accelerometers or geophones.
- Simulation: In the second step, a certain set of material properties is assumed to simulate the response of the material with a numerical method. This model should be able to represent the behavior of the material accurately.
- Optimization: In the last step, by varying the material properties in the simulations, the difference between the measurements and simulations (cost function) is minimized.

The measurements to back-calculate the properties of asphalt mixtures are either conducted in-situ using an FWD for excitation and geophones to measure the deflection on the surface of the road [27] or in the laboratory with a hammer and accelerometers to conduct modal analysis experiments [28]. In this research, a modal analysis approach using a shaker for excitation

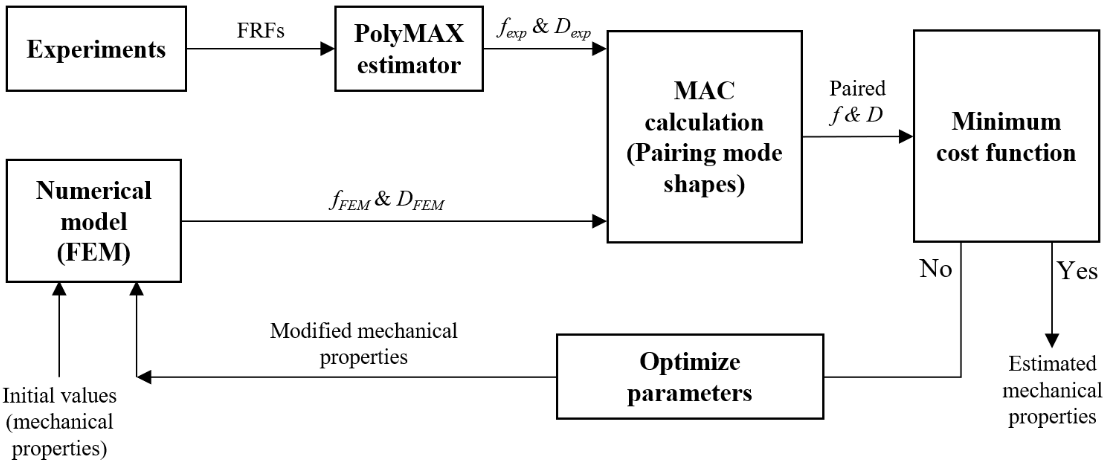


Fig. 4: Overview of the back-calculation methodology used in this research to find the desired properties of the pavement mixtures (f is the vector of natural frequencies, computed from ω_k , and D is the vector of percentage of damping ratios, calculated from ξ_k).

and an SLDV for vibration measurements on multiple points on the surface of the object was used.

The output of the experiments are the Frequency Response Function (FRF) of the specimens (see Figure 4). The FRFs are the input to the PolyMAX, also called the polyreference least-squares complex frequency-domain method [29], that uses a least-squares approach to fit a rational fraction polynomial model to the FRFs. The PolyMAX estimator is able to estimate the modal parameters, including the natural frequencies, damping ratios, and the mode shapes of the system [30]. The poles of the system s_k can be found from this algorithm and are in relation with eigenfrequencies ω_k and damping ratios ξ_k, as shown in Eq. 2:

$$s_k = -\xi_k \pm i\sqrt{1 - \xi_k^2}\omega_k \tag{2}$$

Using MAC, the acquired experimental modal parameters are paired with the modal parameters computed by a FEM developed in COMSOL Multiphysics 5.1. The FEM is initially developed using initial guesses for the unknown mechanical properties of the material and will be updated later. The details of the FEM and the formulas used to simulate the specimens used in this research are available in Subsect. 2.5. Afterward, a cost function is defined based on the paired modal parameters (see Subsect. 2.6). By minimizing this cost function and optimizing the initial inputs of the FEM iteratively, the mechanical properties of the material are estimated.

2.5 FEM Simulations

The simplest method to model the behavior of asphalt mixtures is to assume them as homogeneous, isotropic, and linear elastic materials. This assumption leads to the calculation of a simple module of elasticity for the material and is the base of the static back-calculation procedure [24]. Another procedure to estimate properties of asphalt mixtures is the dynamic back-calculation. In this method, the material is assumed to be homogeneous, isotropic and linear viscoelastic and the dynamic response of the pavement is used to find one or multiple unknown properties, such as elastic moduli, thickness, Poisson's ratios, mass densities, and damping ratios of different layers of pavement. In newer pavement design procedure, usually, the dynamic master curve of asphalt mixtures is used. Back-calculating this property for asphalt mixtures is possible with FWD [27] or modal analysis experiments [31]. In these studies, the master curves of asphalt mixtures are modeled using different functions: Sigmoidal function [32], 2S2P1D model [33, 34], and Havriliak-Negami (HN) model [35]. A comparison between the 2S2P1D and HN models is available in [11]. In this research, the HN model is used to simulate the viscoelastic properties of asphalt mixtures. This method has been successfully used to model the master curve of complex modulus and complex Poisson's ratio of different viscoelastic materials such as polymers [36] or asphalt mixtures [10]. Eq. 3 presents the HN model formula used to model the complex modulus of elasticity in the FEM simulations (E^*) [35].

$$E^*(\omega,T) = E_\infty + \frac{(E_0 - E_\infty)}{[1+(i\omega\alpha_T(T)\tau)^\alpha]^\beta} \tag{3}$$

where E_0 is the static low-frequency modulus when $\omega \to 0$, E_∞ is the high-frequency modulus when $\omega \to \infty$, α describes the frequency dependency, β governs the asymmetry of the loss factor peak, τ is the relaxation time, and α_T is the shifting factor (see Eq. 4) according to the Williams-Landel-Ferry (WLF) [37] approach.

$$\log \alpha_T = -\frac{C_1(T - T_{ref})}{C_2 + T - T_{ref}} \tag{4}$$

where C_1 and C_2 are the empirical values determined from experiments.

Eq. 3 shows that to model the complex modulus of elasticity of a viscoelastic material in the frequency domain, seven unknown parameters (E_0, E_∞, C_1, C_2, τ, α, and β) have to be identified. In this research, the back-calculation is performed assuming $E_0 = 100$ MPa. The Poisson's ratio is also assumed to follow the HN model with $\nu_0 = 0.5$, and $\nu_\infty = 0.3$. These assumptions were essential to make the time-consuming optimization process possible, and are based on the literature [26] and the fact that the cost function is less sensitive to these parameters.

2.6 Optimization

In this subsection, a cost function or objective function is defined based on the measurement results and the simulations. The objective is to change the unknown parameters of the simulation in an efficient way to minimize this cost function. The cost function could be based on the error between the simulated and measured FRFs [28] or expressed in terms of the residuals between the simulated and measured modal parameters [38].

In this research, the optimization is based on a single cost function with equal weight for each of the modes but different weights between the natural frequencies and damping ratios. The cost function (CF) used in this research is:

$$CF = \sum_{z=1}^{m} K(z)\,[\frac{W\frac{CF_f}{\frac{\sum_{i=1}^{n(z)} f_{FEM}(z,i)}{n(z)}} + \frac{CF_D}{\frac{\sum_{i=1}^{n(z)} D_{FEM}(z,i)}{n(z)}}}{W+1}] \tag{5}$$

with

$$\begin{cases} CF_f = \sqrt{\frac{\sum_{i=1}^{n(z)}(f_{exp}(z,i)-f_{FEM}(z,i))^2}{n(z)}} \\ CF_D = \sqrt{\frac{\sum_{i=1}^{n(z)}(D_{exp}(z,i)-D_{FEM}(z,i))^2}{n(z)}} \end{cases}$$

where

- f_{exp} and D_{exp} are natural frequency and percentage of the damping ratio estimated from the experiments.
- f_{FEM} and D_{FEM} are natural frequency and percentage of the damping ratio computed by the FEM.
- z is the experiment number based on the temperature of the climate chamber.
- m is the number of temperatures the experiments are conducted at (equal to five in this research).
- $n(z)$ is the number of the matched mode shapes with an acceptable MAC at a certain temperature.
- W is a weighting function to increase the importance of matching the natural frequencies with respect to matching the percentage of the damping ratios. In this research, after trying different values, a weighting function of 10 was selected.
- $K(z)$ is a weighting function to increase the possibility of the optimization to use more matched mode shapes. $K(z)$ increases when less mode shapes are used to optimize the parameters. For instance, $K(z)$ is equal to 10 when the algorithm can find nine matching mode shapes, and it increases to 2000 when there are only two matching mode shapes. Therefore, the algorithm tries to find more matching mode shapes instead of minimizing the cost function by eliminating some modes.

To minimize this cost function, different optimization algorithms can be used. These algorithms are usually divided into two groups: global optimization methods and local optimization methods. Global optimization methods are always capable of finding the global optimum, but they need a lot of calculation power. The local optimization methods can converge to the solution more quickly, but it is possible that the solution is a local optimum [39]. In this research, due to the high calculation time of the FEM, a local optimization technique (the multi-variable optimization algorithm *fmincon* from MATLAB) is adopted to compute the optimum values for the unknown parameters. Although to make sure a global optimum is reached, the optimization is done based on multiple initial guesses derived from a simple forward-calculation method and the literature.

3 Results and Discussion

In this section, first, the unknown parameters of the master curves are determined for three specimens $B1$, $B2$, and $B3$, through the back-calculation methodology. Then, the results are compared with two other approaches, and the advantages and disadvantages of each method are described.

3.1 Results of the proposed back-calculation method

The optimized values for the three $B1$ to $B3$ samples are presented in Table 4. Using the averaged values provided in this table, the master curve of the complex modulus of this asphalt concrete is plotted in Figure 5a. Moreover, the shift factor calculated for the mixture is presented in Figure 5b.

3.2 Validation experiments

The trend of the curve is as expected, and the STD is very low. However, to have a better understanding of the accuracy of the method, in the next section, this master curve is compared with the master curves of the same mixture acquired from two other methods.

Figure 6 presents the master curve of an asphalt concrete estimated by three different methods. These curves include the master curve derived from the proposed back-calculation method for $B1 - B3$ specimens, the master curve of the forward-calculation methods for $B1 - B3$ specimens, and the master curve of $R1 - R3$ samples derived from 4PB-PR experiments. More details on using the forward-calculation method and 4PB-PR can be found in [2, 8]. This figure shows that all three master curves follow a similar trend. Especially at frequencies above 25 Hz, they match with each other, with the master curve of the back-calculation method being closer to the master curve of the 4PB-PR. However, $|E^*|$ estimated by the back-calculation method does not correspond very well with the other two master curves at frequencies lower than 3 Hz. This is due to the fact that all the natural frequencies of the beams are between 0.80 kHz and 15.8 kHz. Therefore, even after shifting the data of the experiments at high temperatures to the left, the proposed back-calculation method is not able to predict the behavior of the material at low frequencies with high accuracy. This problem can be solved by conducting experiments at higher temperatures or on a longer beam, as explained in Section 3.3.

Figure 7 shows that the phase angles of the specimens acquired with all three different methods at high frequencies are also similar to each other. The standard 4PB-PR test provides information for a more extensive frequency range than the other two methods. The back-calculation method can extrapolate the curve for low frequencies as well, but due to the lack of data points at low frequencies, it is not a good fit.

Finally, the CV of the proposed method was computed and compared with the CV of the other two methods. Figure 8 presents the comparison of the CV of the three methods. The CV of the back-calculation method is less than 0.45 for all frequencies. However, it is higher than 0.3 in the frequency range below 0.07 Hz. Moreover, the CV of the back-calculation method is lower than the CV of the forward-calculation method above 0.1 Hz and lower than the CV of the 4PB-PR method above 6.6 Hz. It proves that the proposed method has a high repeatably at high frequencies but a slightly lower repeatably at low frequencies, which in many applications is very important. Solutions to overcome this issue are discussed in Section 3.3.

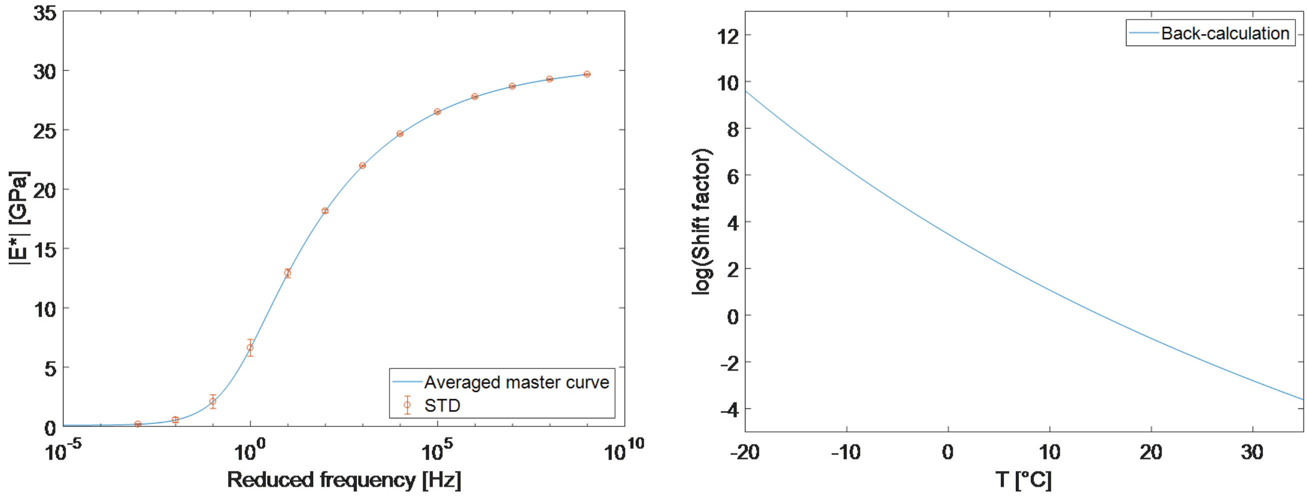
3.3 Advantages and limitations of the proposed method

The main advantage of the proposed method over the traditional methods is the less strict limitations on the dimensions of the testing samples. It is possible to conduct modal analysis on an asphalt mixture with any geometry and model its behavior in a FEM. However, most traditional methods use analytical formulas developed for specimens with certain geometries. The price of the equipment for the proposed back-calculation technique is competitive with a basic stiffness testing system to conduct the standard experiments. However, the post-processing of the data in the proposed method is more time-consuming than the standard methods and good initial values are required to reach the global minimum of the cost function in a reasonable time. Moreover, the master curves of the proposed method are based on all the mode shapes of the specimens that were estimated with good quality. Although, in the forward-calculation methods, usually, only the fundamental natural frequency [7] or the flexural mode shapes of the specimens [8] are used. Lastly, while conducting standard tests in high temperatures (around 40 °C), it is possible to have unwanted creep deformation in the specimens. But due to the lower level of vibration created in the back-calculation tests, this issue does not happen.

As explained in previous sections, the master curves derived from the proposed back-calculation method were based on the data of the modal analysis experiments, which lead to data points at high frequencies for these samples. This limitation does not exist in the

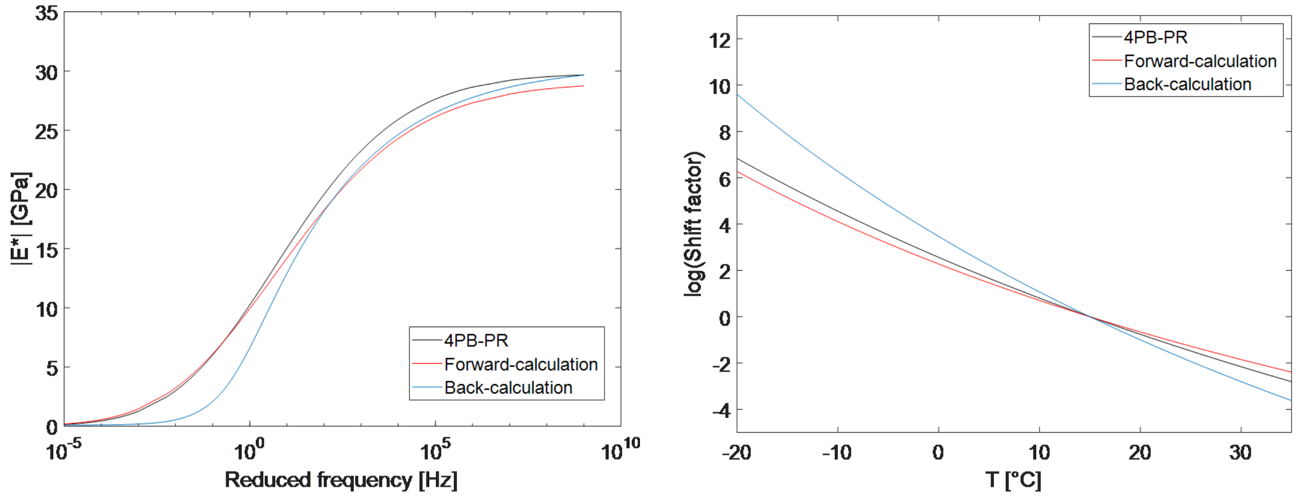
Table 4: The optimized values of the parameters of the HN model and shift factors of the WLF equation (see Eq. 3 and 4) for three $B1 - B3$ specimens.

Scan	E_∞ **[GPa]**	α	β	τ [s]	C_1	C_2
B1	30.63	0.69	0.23	0.26	25.0	146.9
B2	30.52	0.72	0.24	0.21	31.5	150.1
B3	30.53	0.61	0.26	0.33	31.3	128.4
Average	30.56	0.67	0.24	0.26	29.3	141.8
STD	0.05	0.05	0.01	0.05	3.0	9.6



(a) (b)

Fig. 5: (a) Averaged master curve of the specimens $B1 - B3$, at the reference temperature of 15 °C computed by the back-calculation technique, (b) the shift factor according to the WLF approach.



(a) (b)

Fig. 6: (a) Comparison of the averaged master curves using three different methods at the reference temperature of 15 °C. (b) Comparison between the shifting factors derived using the three methods.

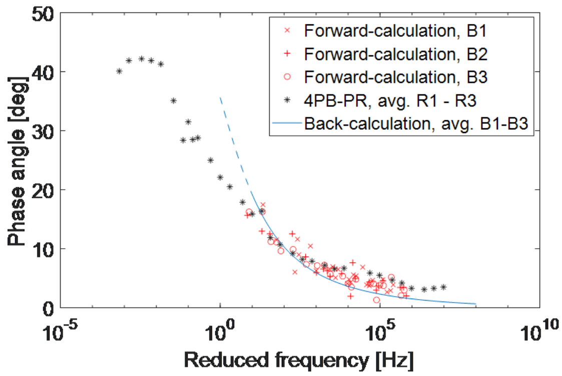


Fig. 7: Comparison of the phase angles derived from three methods at reference temperature of 15 °C. The result of the forward-calculation on three beams are presented separately since the natural frequencies of the beams are not the same, and therefore, it is not possible to take an average of the phase angles at certain frequencies.

4PB-PR method and can be considered as a drawback for the proposed method. However, there are two ways to tackle this issue. The first solution is to select the geometry of the sample in a way that its first natural frequency is lower than the natural frequency of the samples used in this research. The second method is conducting experiments at higher temperatures, which correspond to low frequencies after the frequency shift. Following these two points leads to more information at low frequencies (the left side of the master curve) and, therefore, a better prediction of the $|E^*|$ at low frequencies. For instance, FEM simulations on an asphalt concrete beam of the same mixture composition, with the dimensions of $0.6 \times 0.042 \times 0.042$ m, demonstrated that it is possible to have a data point at 0.5 Hz for the master curve plotted at 15 °C. This increases the accuracy of both forward-calculation and back-calculation methods to estimate $|E^*|$ at low frequencies. However, it also provides a limitation on the geometry of the specimens. Although contrary to the standard methods that conduct experiment up to the maximum frequency of 30 Hz, the proposed method can provide more accurate information of the asphalt mixture at high frequencies and low temperatures since it uses mode shapes with high frequencies (up to 15.8 kHz for the specimens of this research).

4 Conclusions

In this research, a novel method is proposed to calculate the master curves of the complex modulus and phase angle of asphalt mixtures using an optical measurement system and back-calculation methodology. This method is applicable to specimens with arbitrary geometries since it uses a Finite Element Model (FEM) instead of analytical formulas to model the specimens. The FEM is developed using the Havriliak-Negami model and Williams-Landel-Ferry method to simulate the viscoelastic behavior of the asphalt mixture at different temperatures. The master curves acquired by the proposed method are expected to have better accuracy than the master curves of the standard method at high frequencies since instead of shifting the results of low temperatures, the specimens are excited at high frequencies as well. The results show that the master curve of the complex modulus estimated by the proposed method is in between the master curves acquired by the other two methods in frequencies higher than 135 Hz. Furthermore, according to the comparisons between the master curves and looking at the coefficient of variance of the master curves, it is concluded that all the methods predict similar $|E^*|$ at high frequencies, but the back-calculation method has some limitations at low frequencies. Besides, the high repeatability of the back-calculation method was verified by showing that its coefficient of variance was lower than that of the forward-calculation method at most frequencies and that of the standard method at frequencies higher than 5.8 Hz. Similar to the master curve of $|E^*|$, the phase angle of the back-calculation method corresponds well with the phase angle of the other methods at frequencies higher than 7 Hz.

The main limitation of the back-calculation technique was demonstrated to be the less accurate results at low frequencies for a specimen with this geometry (lower than 3 Hz at 15 °C for this material). For instance, $|E^*|$ calculated by the back-calculation method is up to 4 GPa lower than that of the other two methods in some frequencies. This is a geometric limitation, and a simple solution to overcome this issue was proposed by modeling a longer or thinner beam in the FEM. It is also possible to conduct experiments at higher temperatures, as long as it does not cause creep deformations in the specimens.

Further research is needed to improve the optimization technique using more complex weighting factors to take into account the accuracy of the estimated modal parameters using the PolyMAX estimator as well. Implementing a global optimization method to prevent incorrect results due to the presence of local minima could also be interesting. Moreover, in the next step, this method should be applied to specimens with other geometries to demonstrate its applicability on specimens with different shapes.

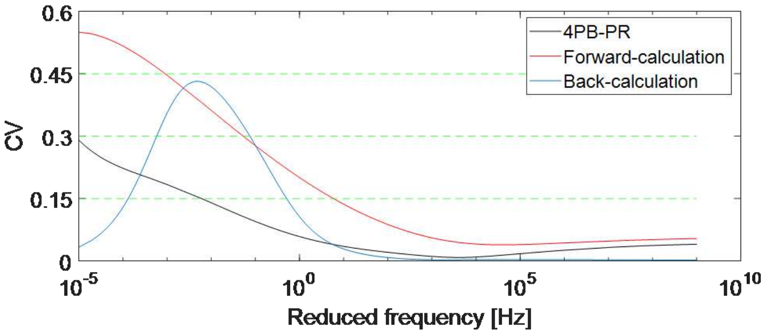


Fig. 8: Coefficient of variance computed for three different methods. All methods have high repeatability at frequencies higher than 10 Hz.

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Conflict of interest

The authors declare that they have no conflict of interest.

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