DEPARTMENT OF ENGINEERING MANAGEMENT

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Abstract

Design models for multiproduct batch plants define the optimal size and number of batch equipment needed to produce predetermined product quantities over a strategic planning horizon, and such for every stage of the production process. However, although these design models incorporate realistic equipment costs and process information, a realistic spread of product deliveries over the (long) planning horizon, is mostly not considered. In single period design models, product deliveries are not modelled: it is assumed that finished products can be stored (for free) until delivery, or are delivered directly after production. Multiperiod plant design models assume in general that product quantities are delivered at the end of every repetitive (short) production period, but the influence of a variable delivery scheme has barely been investigated.

The aim of this paper is to study the design of a batch plant for a periodic delivery scheme with variable product quantities, and compare the results for two modes of operation: single product and mixed product campaigns, both in a multiperiod context. Additionally, we will evaluate the impact of (limited) storage capacity for finished products and variable product mixes over the periods. All appropriate batch plant design models are solved with mixed integer techniques. The objective is to minimise capital and startup costs, while inventory holding costs are calculated ex post.

From the exploratory study performed, it appears that the cost of periodic deliveries with variable product quantities, seems not that high, especially when end-of-period inventory is allowed. Concerning the mode of operation, the mixed product campaign mode turns out to be very complex to calculate, especially in a multiperiod context, without distinct cost advantages for the plant. Therefore we conclude that, if feasible for the production process, producing in multiperiod single product campaigns is often the cheapest way to operate a batch plant that allows a variable delivery scheme.

1 Introduction

Specialty chemicals, such as food additives, agrichemicals and lubricants (Sharratt, 1997), represent more than 25 % of the EU chemical sales in 2017 (Cefic, 2018) and are preferably produced in batch plants. The construction of such batch plants gives rise to the strategic Batch Plant Design Problem (BPDP) which entails the determination of the optimal number and size of equipment units for every production stage, as well as the associated operational planning guidelines, such as the dedication of products to specific equipment and the optimal batch sizes. The aim of most BPDP is to minimise total costs, while satisfying both demand and design related constraints that state that the designed plant should be large enough to produce the given product quantities within a given (long) production horizon.

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Important to realise is that, to solve the BPDP, the operational use of the plant must always be incorporated, usually denoted as the mode of operation. In academic literature, three modes of operation are introduced: Single Product Campaign (SPC), Mixed-Product Campaigns (MPC) and Network planning (modelled as Resource-task (RTN) or State-task network (STN)). A visualisation of all three modes is included in Appendix A. In case of SPC (Verbiest et al. (2017, 2019a)), the total quantity of each product is produced in one single production run over the whole horizon, before switching to another product. This SPC mode of operation is based on an approximation of the process cycle time for every product, and needs no explicit sequencing nor specific start and end times of the production batches (Applequist et al., 1997). Though, producing every single product only once over a strategic planning horizon might be inappropriate from an inventory or delivery point of view. Also the MPC mode relies upon approximated process cycle times, but a mixed product campaign consists of a combination of batches of different products, which is repeated cyclically over the planning horizon. Because of these mixed campaigns, and in case of minor changeover times, machine idle times can be reduced, meaning smaller and cheaper plant designs can sometimes be achieved (Birewar and Grossmann, 1989; Biegler et al., 1997; Fumero et al., 2011). Furthermore, since products are no longer made in one single run, a more steady supply of products over the planning horizon is assured. Note that only in few BPDP mentioned above, changeover costs or times are taken into account, since explicit scheduling of batches is avoided. Finally, the network planning mode of operation is the most detailed one (Barbosa-Póvoa and Macchietto, 1994; Pinto-Varela, 2015). It includes (detailed) modelling of the exact process times and is closely related to scheduling. As a consequence, when applying a network mode for the design (and scheduling) of multiproduct batch plants (Verbiest et al. (2019b); Van Den Heever and Grossmann (1999)), accurate solutions and efficient resource usage are obtained, but the models become rapidly very large and intractable to solve.

In all aforementioned papers, it is assumed that the total amount required for every product needs to be fulfilled by the end of a given strategic (long) production horizon. In fact, the production horizon is assumed to be one period, which is why these design problems are denoted as single period models for which a delivery scheme over the horizon is simply not considered. As this assumption is inappropriate from a logistic and customer service point of view, we prefer multiple periods models (illustrated in Figure 1). In multiperiod BPDPs, the entire production horizon is divided into multiple periods with, at the end of every period, a delivery point at which specific amounts of products are guaranteed to be ready. In addition, specific production planning characteristics that arise in a multiperiod context, such as allowing inventory from one period to the next, can be taken into account.

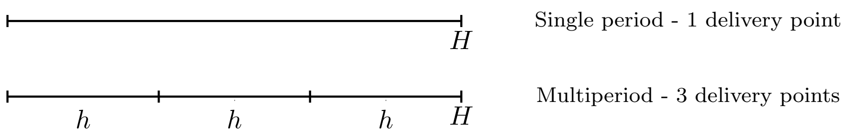


Figure 1: Schematic overview single and multiperiod problems

Multiperiod optimisation models for different modes of operation have been introduced in the batch plant design literature since the 1990s. However, most papers focussed on the modelling and solution methods, rather than on the impact of the multi-periodicity on the design. Varvarezos et al. (1992) used an MINLP model for the multiperiod BPDP in SPC mode and proposed a decomposition technique to tackle computational complexity. They also compared the impact of optimising all periods simultaneously versus solving only the period with lowest or highest demand. Van Den Heever and Grossmann (1999) looked into multiperiod planning of general chemical process systems and investigated the use of disjunctive programming to improve the computational efficiency. More recently, Moreno et al. (2007) formulated a novel MILP model for the design and planning of multiperiod batch plants with semi-continuous equipment and intermediate storage. García-Ayala et al. (2012) investigated capacity expansions and retrofitting of existing plants under a multiperiod scenario. In order to keep the problem tractable, they introduced a disjunctive programming model and solution strategies such as priority branching. Regarding the multiperiod BPDP operating in MPC mode, Fumero et al. (2016) proposed an MILP model but, also in this case, the focus was not on the effect of applying an MPC mode, nor on the operational characteristics. Last but not least, multiperiod BPDPs operating in the network planning mode have only been studied to a limited extent, i.e., by assuming the same demand and repeated operations for every period, thus simplifying the operation of such batch plants (Pinto-Varela, 2015). To conclude, even though design models have been addressed, the above overview shows that the impact on the plant design of the mode of operation (SPC, MPC, network) in a multiperiod context has barely been studied, neither as variable delivery schemes.

In this paper, we investigate the plant design for a multiperiod SPC- or MPC mode of operation, denoted M-SPC and M-MPC respectively, as opposed to S-SPC and S-MPC for single period BPDPs. On top of this, the impact of the following multiperiod characteristics are studied: (1) variability of the product delivery quantities at the end of every period; (2) availability of end-of-period inventory, allowing to keep stock of products at the end of every period; and (3) existence of product mix restrictions that indicate if every product has to be produced in every period. The network planning mode is not further investigated, as the calculations in a multiperiod setting become (too) arduous.

The remainder of this paper is organised as follows: in Section 2, a general description of the multiperiod BPDP and multiperiod characteristics is provided. In Section 3 the mathematical models for both multiperiod SPC and MPC mode of operation are defined. In Section 4, an exploratory study investigates the influence on the plant design of the M-SPC and M-MPC mode and the impact of the different multiperiod characteristics is evaluated. Finally, we conclude with a discussion and conclusion in Sections 4.5 and 5.

2 Problem description and assumptions

In this section, first a general description of the multiperiod BPDP is given. Next, the different multiperiod characteristics are explained.

2.1 Multiperiod BPDP

Irrespective of the number of periods and the mode of operation, all models assume that the batch plant to be designed consists of J stages j, necessary to produce P products i. The total quantity for every product (Q_i) to be fulfilled by the end of the total planning horizon (H) is known upfront. When we consider a multiperiod context, this horizon H is divided into Np number of periods h with equal period lengths Lp. Also the amounts to be delivered of every product at the end of every period (Q_{ih}) are known but can be different, together adding up to the given total quantity (Q_i). The size factors (S_{ij}) and batch processing times (τ_{ij}) are known for every product i at every stage j and identical in all periods. Size factors correspond to the equipment volume needed at stage j to produce a unit mass of product i. The batch processing times are assumed to be independent of the product batch size. For the detailed nomenclature, we refer to Appendix B.

Further general assumptions, as essentially already specified by Voudouris and Grossmann (1992), are:

1. Production process/recipes are known upfront;
2. Unlimited access to raw materials;
3. Only batch equipment is explicitly considered since this is most expensive;
4. Zero-wait policy between the batch stages, meaning a batch is immediately transferred from one stage to the next;

5. Overlapping mode of operation to eliminate idle times (no need to wait until one batch has completely passed through all stages);

6. At most N identical parallel equipment per stage, operating out-of-phase, i.e. batches are supplied from the previous stage shifted in time (see App. A for an illustration);

7. Discrete set of S equipment sizes v_s for all stages j to choose from.

Within this setting, our multiperiod BPDP determines for every stage the optimal number and size of equipment units to be installed (and kept over all periods). Simultaneously the operational planning guidelines per period, such as number of batches and batch sizes per product, are determined. The aim is still to minimise the total cost, while satisfying design and demand constraints in every period. In this paper, the total cost is the sum of capital and startup costs. The capital costs represent the one-time acquisition and installation cost of the batch equipment units, whereas the startup costs represent the costs for preparing equipment units for a new production run (a sequence of batches of the same product), summed over the periods. Finally, in this paper, the inventory holding costs of finished products, during and at the end of every period, are calculated ex post, since optimising them is arduous. A detailed discussion of the cost function is presented in Section 3.

Last but not least, different additional assumptions are needed for SPC- and MPC mode. As stated in the introduction, an SPC mode of operation implies that, in every period, all batches of one product are produced before switching to another product. In addition, product cycle times are used to calculate the fulfilment of the constraints in every period. These product cycle times correspond to the longest stage cycle time over all stages for every product:[1] $\max_{j=1,\ldots,J} \tau_{ij}/N_j$ (Voudouris and Grossmann, 1992). In a MPC mode of operation, on the other hand, batches of various products are produced in a mixed campaign according to a certain sequence, which may vary over the periods. These campaigns are cyclically repeated over the length of every period. The additional assumptions, as specified by Fumero et al. (2013), are: (1) instead of product cycle times, campaign cycle times are defined. These correspond to the longest stage cycle time over all stages for an entire campaign; (2) for every product i a maximum number of batches in a campaign (NBC_i^{UB}) is specified upfront; and (3) a slot-based continuous time formulation is used to account for starting- and finishing times of batches in every campaign in every period. These slots correspond to time intervals of variable unknown length, to which batches are assigned. Note that as the computational performance strongly depends on the number of slots allowed for each equipment unit, an upper bound $E = \sum_i NBC_i^{UB}$ is specified. E is an upper bound, since in the extreme case of a single equipment unit per stage and the number of batches per campaign for every product corresponding to their maximum, $\sum_i NBC_i^{UB}$ time slots are needed.

2.2 Multiperiod characteristics

As announced earlier, the multiperiod characteristics considered in this paper are: Variability of delivery quantities, End-of-period inventory and Product mix restriction.

The first characteristic, Variability of delivery quantities, defines if the amounts to deliver of every product at the end of the periods are equal or not, with the latter accounting for (deterministic) demand variations. To be able to compare the different scenarios per example instance, the sum of the delivery quantities per period will be equal for all scenarios for each product.

[1] The time it takes a product i to pass through a stage j is denoted as the stage cycle time. This time is given by τ_{ij}/N_j, since each unit is charged in turn by batches from the preceding stage (out-of-phase operation - assumption 6). Additionally, since an overlapping mode of operation is assumed (assumption 5), the time interval between two successive batches of a product is determined by the stage having the longest stage cycle time. This is referred to as the cycle time of a product and, hence, corresponds to the time necessary to produce one batch (omitting the begin- and end effects of production).

Next, we assume that all products are delivered at the end of a period, but the characteristic End-of-period inventory defines if it is allowed to keep stock for the next periods or not. However, to avoid an unrealistic accumulation of products over the periods, an upper bound on the amount stored per product during the periods is imposed (see Sec. 3). Storage tanks are not explicitly modelled.

Finally, the characteristic Product mix restriction indicates whether or not there are restrictions on the product mix in a period. A ”fixed product mix” means that every product should be made in every period, whereas for a ”variable product mix” the number of products made in every period can be optimised.

3 Mathematical models

In this section, the mathematical descriptions of the multiperiod BPDP for the different modes of operation, incorporating the multiperiod characteristics, are given.

The multi-period model for SPC mode (M-SPC) is mainly based on the MILP model proposed by Moreno et al. (2007), while for an MPC mode (M-MPC), the MILP model is adopted from Fumero et al. (2016). However, in both cases we adapted their models: the aim of our BPDPs is to minimise total costs while deterministic delivery quantities for every product need to be ready at the end of every period. In Moreno et al. (2007) and Fumero et al. (2016), on the contrary, the amounts to be delivered may vary between lower and upper bounds, while the aim is to maximise profit by considering variations in prices, costs and availability of raw materials over the periods. Also, we are only concerned with the batch equipment, whereas Moreno et al. (2007) model semi-continuous equipment (i.e. pipelines) and short-term intermediate storage tanks as well.

Hereafter, in order to avoid extended mathematical descriptions, only the constraints representing the multiperiod characteristics, the objective function and the expression to calculate the ex post inventory holding costs are described. The description of the complete mathematical MILP models for both modes of operation are given in Appendices C and D. Note that the characteristic Variability of delivery quantities does not need a specific constraint formulation, but is tested by using different data sets.

3.1 M-SPC

3.1.1 Constraints taking into account the multiperiod characteristics

The design constraints, horizon constraints and boundaries for the multiperiod BPDP that do not depend on multiperiod characteristics can be found in Appendix C, as well as the linearisation constraints. The remaining constraints are described hereafter.

End-of-period inventory

Batch equipment constraints
If end-of-period inventory is allowed, the amount produced of product i in period h must no longer be equal to the amount to deliver (Q_{ih}) but is a variable that needs to be determined. Hence, for every stage j, the size of the equipment units v_s should be large enough to hold the batches of every product i in every period h, multiplied by the size factor S_{ij}. Eq. (1) is obtained by using the expression q_{ih}/n_{ih} to represent the batch size of product i in period h, with n_{ih} the number of batches and q_{ih} the amount produced.

$$n_{ih} \geqslant \sum_{s=1}^{S} \frac{q_{ih} S_{ij}}{v_s} u_{js} \qquad \forall i, j, h \qquad (1)$$

If end-of-period inventory is not allowed, $q_{ih} = Q_{ih}$ and Eq. (1) can be replaced by Eq. (2).

$$n_{ih} \geqslant \sum_{s=1}^{S} \frac{Q_{ih} S_{ij}}{v_s} u_{js} \qquad \forall j, i, h \tag{2}$$

Inventory constraints
For every product i the delivery quantity required at the end of every period h can be fulfilled through the production in that period and the inventory kept at the end of the previous period (Eq. (3)). Consequently, the inventory at the end of every period h equals the sum of the inventory from the previous period and the amount produced in this period minus the amount delivered in this period (Eq. (4)). Eq. (5) limits the total amount to be stored at the end of every period, just before delivery, to the maximum amount required over all periods, with $Q_i^{UB} = \max_h(Q_{ih})$. The starting inventory for every product i (I_{i0}) is set to zero (Eq. (6)).

$$I_{i(h-1)} + q_{ih} \geqslant Q_{ih} \qquad \forall i, h \tag{3}$$

$$I_{ih} = I_{i(h-1)} + q_{ih} - Q_{ih} \qquad \forall i, h \tag{4}$$

$$I_{i(h-1)} + q_{ih} \leqslant Q_i^{UB} \qquad \forall i, h \tag{5}$$

$$I_{i0} = 0 \qquad \forall i \tag{6}$$

As stated in Section 2, storage tanks are not explicitly modelled.

Characteristic Product mix restriction

When the product mix is fixed, every product needs to be produced in every period. This generates the following additional constraints: Eq. (7) forces the production of at least one batch of every product i in every period h, whereas Eq. (8) sets a lower bound on the amount produced per period. The lower bounds are given by $Q_i^{LB} = (Q_i T_i^{min})/H$ and $T_i^{min} = \max_j \tau_{ij}/N$[2].

$$n_{ih} \geqslant 1 \qquad \forall i, h \tag{7}$$

$$q_{ih} \geqslant Q_i^{LB} \qquad \forall i, h \tag{8}$$

3.1.2 Objective function

Capital costs
The capital costs, associated with the acquisition or installation of equipment, depend on the number of equipment units installed (indicated by the binary variable z_{jn}) and the size of these units (indicated by the binary variable u_{js} and discrete size option v_s). The value of the capital cost increases non-linearly with increasing size of the equipment unit, where α_j and β_j are stage dependent cost parameters and β_j is typically smaller than one (Sparrow et al., 1975). This one time capital expenditure is assumed to be converted to a uniform cost over the horizon, so that it can be correctly added up with the other cost components. (Jelen et al., 1983)

$$\sum_{j=1}^{J} \sum_{s=1}^{S} \sum_{n=1}^{N} \alpha_j v_s^{\beta_j} z_{jn} u_{js} \tag{9}$$

Startup costs
These costs account for the preparation of the equipment units at the start of every series of batches of product i in every period h. It is modelled as follows:

$$\sum_{i=1}^{P} \sum_{j=1}^{J} \sum_{n=1}^{N} \sum_{h=1}^{Np} Cstart\, z_{jn}\, t_{ih}$$

[2]This lower bound is derived from $\sum_i (Q_i T_i)/B_i \leqslant H$, so for every product i: $(Q_i T_i^{min})/B_i \leqslant H$ applies. Reordering this equation gives $(Q_i T_i^{min})/H \leqslant B_i$ and the minimum is the used lower bound.

where $Cstart$ is a stage and product independent startup cost and t_{ih} indicates whether or not product i is produced in period h. Hence, these costs are affected by the number of products made in a period, as well as the number of equipment units installed in parallel per stage (given by z_{jn}). Note that it is assumed that this cost does not depend on the size of the units.

Finally, the binary variable t_{ih} is linked via the following constraints to the number of batches made of product i in period h, where NB_i^{UB} is a specified upper bound on the number of batches:

$$t_{ih} \leqslant n_{ih} \qquad \forall i, h \tag{10}$$

$$t_{ih} NB_i^{UB} \geqslant n_{ih} \qquad \forall i, h \tag{11}$$

Inventory holding costs
In a multiperiod context the delivery dates are end-of-period, meaning that all products are stored until the end-of-period. Hence, the cost of this product accumulation should be taken into account. Since this calculation generates a non-convex objective function that makes the optimisation arduous, the calculation of this cost is done ex post.

However, a small correction term is included in the objective function to penalise and avoid excess end-of-period inventory that is not asked for:

$$\sum_{h=1}^{Np} \sum_{i=1}^{P} InvCorr I_{ih} \tag{12}$$

with $InvCorr \in]0, 0.001]$ a given correction coefficient.

The ex post inventory is calculated according to the inventory patterns visualised in Figure 2, which shows two periods in which two products are produced:

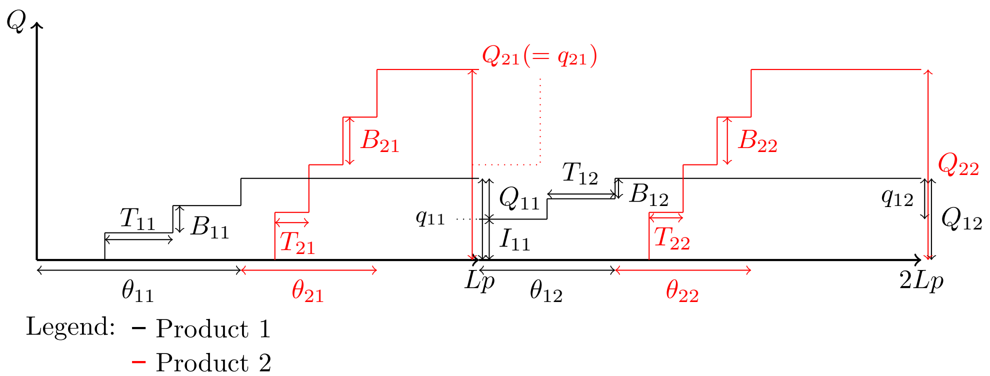


Figure 2: Schematic representation of the ex post inventory calculations for M-SPC

Hence, this cost becomes:

$$\sum_{h=1}^{Np} \sum_{i=1}^{P} \left[\underbrace{((n_{ih} - 1)\theta_{ih} - T_{ih}(\frac{n_{ih}(n_{ih} - 1)}{2}))B_{ih}}_{\text{I}} \right. \\ \left. + \underbrace{(Lp - \sum_{k}^{i} \theta_{kh}) q_{ih}}_{\text{II}} + \underbrace{Lp\, I_{i(h-1)}}_{\text{III}} \right] InvCost \tag{13}$$

with $InvCost$ the inventory holding cost per unit of product per time. The first term (I) corresponds to the production part for every product in every period, in which the batches of every product i are produced. The second term (II) corresponds to that part of the period length in which the total amount produced of every product is waiting to be delivered. Since the SPC mode of operation does not give a sequence in which products are produced, it is assumed that products are produced in increasing order of production quantities. As such, the product with the largest amount produced is kept in inventory the shortest time (as $InvCost$ is considered the same for all products). Lastly, the third term III represents the end-of-period inventory kept when going from one period to the other.

Remark that the first and second term (I and II) are zero in case a product is not produced in a certain period. Indeed, if the inventory stored at the end of the previous period is large enough to meet the delivery quantity in the next period, no production is needed and no batches of this product are made ($q_{ih}, n_{ih} = 0$).

3.2 M-MPC

3.2.1 Constraints of multiperiod characteristics

Also for the models in M-MPC mode, the main batch equipment design, horizon, scheduling, timing and necessary linearisation constraints are included in Appendix D. Only the constraints reflecting specific multiperiod characteristics are defined here.

End-of-period inventory

Batch equipment design constraint:
For every stage j, the size of the equipment units v_s should again be large enough to hold a batch of every product i, multiplied by its size factor S_{ij}. A batch of product i in period h can be written as the amount produced of product i in period h (q_{ih}) divided by the number of batches produced of that product in that period. Moreover, it is assumed that the number of batches of product i produced in one campaign in period h, i.e. n_{ih}, can be written as the selection of one option m via the binary variable c_{imh}. Consequently, the total number of batches of product i produced in a period h corresponds to the number of batches produced in one campaign ($\sum_m mc_{imh}$) times the number of campaign repetitions in that period (NN_h). Rearranging all the previous information, gives Eq. (14). Note that the amount produced of product i in period h (q_{ih}) is a decision variable, since end-of-period inventory is allowed in this model. Hence the constraint to determine the size of the equipment units becomes:

$$NN_h \geqslant \sum_{s=1}^{S} \sum_{m=1}^{NBC_i^{UB}} \frac{S_{ij} q_{ih}}{v_s m} u_{js} c_{imh} \qquad \forall i, j, h \tag{14}$$

Demand and inventory constraints:
Similar as in the M-SPC context, deliveries of specified amounts of every product are promised at the end of every period. Additionally, end-of-period inventory is allowed so that the amount produced of product i in period h does no longer need to be equal to the amount required. To incorporate these features, Eqs. (3)-(6) from the M-SPC model are included.

Finally, Eqs. (15)-(16) enforce that if no amount of product i is produced in period h, no batches are made and vice versa.

$$1 - c_{i0h} \leqslant q_{ih} \qquad \forall i, h \tag{15}$$

$$Q_i^{UB}(1 - c_{i0h}) \geqslant q_{ih} \qquad \forall i, h \tag{16}$$

No end-of-period inventory

In case no end-of-period inventory is allowed, $I_{ih} = 0, q_{ih} = Q_{ih} \quad \forall i, h$. Hence, Eqs. (3)-(6) are again substantially simplified. Moreover, Eq. (14) becomes Eq. (17), as q_{ih} is replaced with Q_{ih}.

$$NN_h \geqslant \sum_{s=1}^{S} \sum_{m=1}^{NBC_i^{UB}} \frac{S_{ij} Q_{ih}}{v_s m} u_{js} c_{imh} \qquad \forall i, j \tag{17}$$

Fixed product mixes

When the product mix is fixed, every product needs to be produced in every period. This generates the following additional constraints: Eq. (18) forbids to produce zero batches of a product, whereas Eq. (19) forces to produce at least one batch of every product i in a campaign in every period h. Finally, Eq. (20) sets a lower bound on the amount produced per period. The lower bound Q_i^{LB} is formulated similarly as for the M-SPC case. Hence, we set $Q_i^{LB} = (Q_i T_i^{min})/H$ and $T_i^{min} = \max_j \tau_{ij}/N$.

$$c_{i0h} = 0 \qquad \forall i, h \tag{18}$$

$$\sum_{m=1}^{NBC_i^{UB}} c_{imh} = 1 \qquad \forall i, h \tag{19}$$

$$q_{ih} \geqslant Q_i^{LB} \qquad \forall i, h \tag{20}$$

3.2.2 Objective function

Similar as for the M-SPC model, the aim of the multiperiod M-MPC BPDP is to minimise the total cost, which consists here of capital plus startup costs. The inventory holding costs are calculated ex post.

Capital costs:
The capital costs are the same as for the M-SPC models and correspond to Eq. (9).

Startup costs:
Comparable as for the M-SPC models, in which a fixed startup cost is incurred every time a series of batches of one product (i.e. a run) starts, the startup costs are formulated as a fixed cost per campaign per equipment unit installed. Hence, it is assumed that it is not necessary to setup within the campaign, but always at the beginning of a new campaign.

The formulation of this cost is:

$$\sum_{j=1}^{J} \sum_{n=1}^{N} \sum_{b=1}^{B} \sum_{h=1}^{Np} Cstart\, BB_b\, NNR_{bh}\, z_{jn} \tag{21}$$

with $\sum_{b=1}^{B} BB_b\, NNR_{bh}$ the choice for a specific number of campaign repetitions (BB_b) in period h, out of a discrete set of repetitions.

Ex post inventory holding costs:
To illustrate the calculation of this ex post inventory holding cost, the inventory pattern of two products is first shown in Figure 3 for a single period context (S-MPC), assuming that all products are kept in inventory until the end of the production horizon.

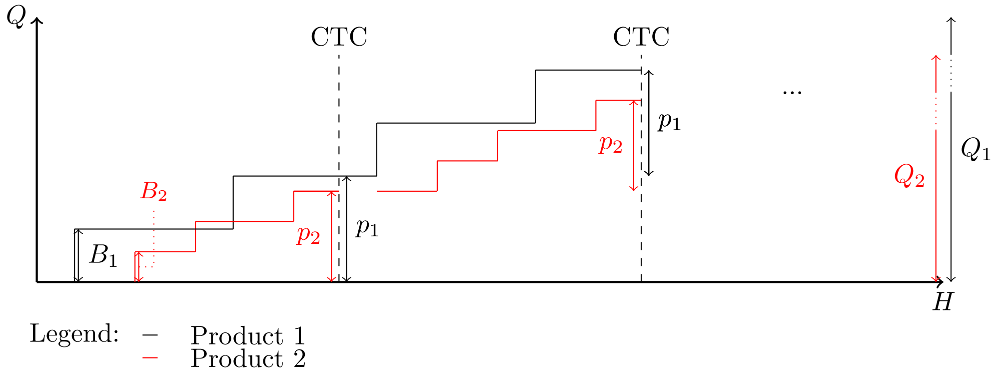


Figure 3: Schematic representation of the ex post inventory calculations for S-MPC

Given this pattern, the inventory holding cost calculations are the following:

$$\sum_{i=1}^{P}\Bigg[\underbrace{(n_iCTC-\sum_{n=1}^{N}\sum_{e=1}^{E}(TF_{Jen}-TI_{Je_{first}n})y_{iJen})B_iNN}_{\text{I}} + \underbrace{p_i((NN-1)H-CTC(\frac{NN(NN-1)}{2}))}_{\text{II}}\Bigg]InvCost \tag{22}$$

with $InvCost$ a known product independent inventory holding cost. The first term (I) corresponds to the production part: as can be seen on Figure 3, different products may be produced alternately and, consequently, batches of one product can be made throughout the entire campaign length. Hence, for every batch, the finishing time on the final stage J is used to mark the point at which a step of size B_i is taken. Note that these finishing times are shifted over the starting time of the first batch of a campaign on this stage, as these begin effects are not taken into account. The second term (II) corresponds to that part of the horizon in which the amount produced is waiting to be delivered, with p_i the amount produced of product i per campaign and NN the number of campaign repetitions.

The calculation of this cost for a multiperiod context is fairly similar to that for the single period one and an illustration of the inventory pattern of two products over two periods is shown in Figure 4.

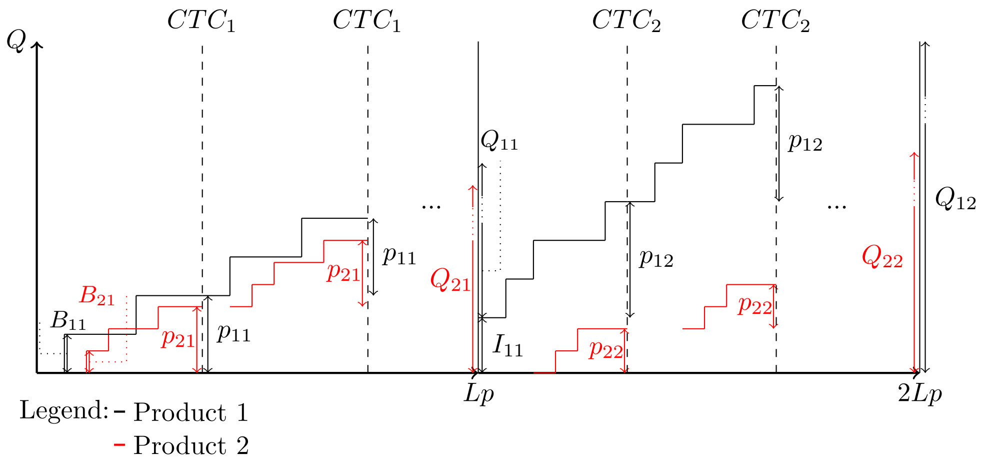


Figure 4: Schematic representation of the ex post inventory calculations for M-MPC

Consequently, the cost expression is given by:

$$\sum_{h=1}^{Np}\sum_{i=1}^{P}\Bigg[\underbrace{(n_{ih}CTC_h - \sum_{n=1}^{N}\sum_{e=1}^{E}((TF_{Jenh} - TI_{Je_{first}nh})y_{iJenh}))B_{ih}NN_h}_{\text{I}} + \underbrace{p_{ih}((NN_h - 1)Lp - CTC_h(\frac{NN_h(NN_h-1)}{2}))}_{\text{II}} + \underbrace{Lp\, I_{i(h-1)}}_{\text{III}}\Bigg] InvCost \tag{23}$$

Terms I and II are the same as in the S-MPC expression, except for the inclusion of the extra dimension of periods h and the substitution of the horizon H by the period length Lp. Moreover, these terms can be discarded if product i is not produced in period h ($B_{ih}, p_{ih} = 0$). Finally, term III is included to account for the possibility of having inventory over the periods with I_{ih} the amount of end-of-period inventory.

Correction terms

Finally, to deal with some degrees of freedom present in the problem, two correction terms are included in the optimisation: an inventory correction term and an initialisation correction term.

As in the M-SPC model, the inventory correction term avoids the accumulation of inventory without demand. This is done via:

$$\sum_{h=1}^{Np}\sum_{i=1}^{P} InvCorr I_{ih} \tag{24}$$

with $InvCorr \in]0, 0.001]$ a given coefficient.

To enforce the M-MPC campaign timings to start at zero, an initialisation correction term is given by:

$$\sum_{j=1}^{J}\sum_{n=1}^{N}\sum_{e=1}^{E}\sum_{h=1}^{Np} InitCorr\, TF_{jenh} \tag{25}$$

with $InitCorr \in]0, 0.001]$ a small, known parameter.

4 Examples

4.1 Data

The impact on the batch plant design of the mode of operation and different combinations of multiperiod characteristics, is analysed for 5 examples. The dimensions of these examples are shown in Table 1. For all examples, the planning horizon is set to 4 periods of 1 month each (20 days per month, 24 h per day). Delivery quantities of every product are specified for every period. In case of variable delivery quantities, for every product the lowest delivery quantity required over the periods is assumed to be less than half of the highest delivery quantity, but it is also possible that products are not required in a period. This highest delivery quantity is also the upper bound for the amounts stored per period. Finally, regarding the objective function, on top of the capital costs, two magnitudes of startup costs are tested for every example: low and high, with high assuming the same order of magnitude for the total capital and startup costs. More detailed information on the input data is given in Tables E.1-E.3 in Appendix E.

Table 1: Dimensions of the examples

	Products (P)	Stages (ST)	Sizes (S)	Max par. equip. (N)
Example 1	3	3	8	3
Example 2	3	4	5	3
Example 3	3	4	6	4
Example 4	4	3	5	3
Example 5	6	4	7	4

4.2 Scenarios

First all 5 examples are solved in a single period context for the two modes of opeartion (S-SPC and S-MPC), using the models from Voudouris and Grossmann (1992) and Fumero et al. (2013) respectively. Next, all examples are solved in a multiperiod context (M-SPC and M-MPC) for different combinations of Variability of delivery quantities (equal or variable), End-of-period inventory (inv or no inv) and Product mix restriction (fix or var). If end-of-period inventory is not allowed, the product mix restriction is not imposed, as indicated in Table 2, since for equal deliveries the product mix is automatically fixed, and in case of variable delivery quantities the product mix is fixed, except if a product is not required in a period.

Each example is optimised for 3 different objective functions: (1) capital costs, (2) capital and low startup costs and (3) capital and high startup costs. For all solutions, the inventory holding costs are calculated ex post. This means that for every example, 6 single period + 36 multiperiod = 42 models are solved (see Tab. 2). Combined with/without ex post calculation, this leads to 84 scenarios per example or $84 \times 5 = 420$ instances in total.

All MILP optimisations use the Gurobi Optimiser 7.0 (library for C++) with a time limit set to 21 h, executed on an Intel(R) Core(TM) i7-4790 CPU, 3.60 GHz and 16 GB of RAM under a windows operating system. Within this time limit, 87 % of the instances were solved to optimality and for 64 % of the remaining instances the optimality gap is lower than 5 %, and for 82 % lower than 10 %. The latter happens merely for the models in MPC mode.

Table 2: Scenarios tested for every example in a multiperiod context

Modelling options	#	Possibilities
Mode of operation	2	SPC or MPC
Characteristics of the multiperiod context:		
- Variability of delivery qty	2	equal or variable
- End-of-period inventory & product mix	3	no inv, inv-fix, inv-var
Objective function	3	(1a) cap. (2a) cap.+ startup(low) (3a) cap.+ startup(high)
Objective function + Ex post calculations	3	(1b) cap.+ expost inv. (no startup cost included) (2b) cap.+ startup(low)+ expost inv. (3b) cap.+ startup(high)+ expost inv.
Total scenarios per example	36	
Total scenarios incl. ex post calculations per example	36	

4.3 General results

In the next subsections, we review in depth the impact of multiperiodicy and mode of operations on respectively the capital costs, startup costs and ex post inventory holding costs. Note that caution must be applied when generalising the results: they are based on 5 examples and not statistically tested (yet). Notwithstanding these limitations, an attempt to analyse trends found over all examples is made.

As the examples have different dimensions and the resulting costs can have a different order of magnitude, in the following sections normalised costs are used to assess the implications of the mode of operation and multiperiod characteristics. For every example, the solutions for all scenarios are first related to the solution of the single period scenario in SPC mode (S-SPC) with minimised capital costs. Then, an average of the relative solutions over all examples per scenario is taken to give an idea of the trends.

Detailed results for 3 examples are given in the next section.

4.3.1 Impact of mode of operation and multiperiod characteristics on the total cost of the plant design, for different objectives

For each mode of operation, the relative impact of the multiperiod characteristics on the total cost of the plant design is depicted in Tables 3 and 4 respectively. For both the multiperiod models in SPC mode and MPC mode, we observe that producing according to equal delivery quantities is most efficient for all objectives, "capital cost", "capital + low startup cost", as well as "capital + high startup cost". When including variability in the delivery quantities, the total cost increases, but can partially be avoided by allowing end-of-period inventory. Of course, when adding the ex post inventory holding cost ("a"-cases versus "b"-cases), the total cost is always higher, and increases more spectacular for the single than for the multiperiod models.

Table 3: Average relative *total costs* over 5 examples in SPC mode, with S-SPC as reference point

Avg. rel. total cost in SPC mode:	S-SPC	M-SPC					
		Equal delivery qty			Variable delivery qty		
a) Minimised objective b) Incl. ex post inv. holding costs	single period	no inv	inv fix mix	inv var mix	no inv	inv fix mix	inv var mix
(1a) cap.	1	1.0181	1.0181	1.0181	1.1912	1.0338	1.0338
(1b) cap.+expost inv.	2.4011	1.3557	1.3557	1.3557	1.5030	1.4114	1.4104
(2a) cap.+startup(low)	1.0326	1.1485	1.1485	1.1485	1.3108	1.1553	1.1509
(2b) cap.+startup(low)+expost inv.	2.4298	1.4842	1.4842	1.4842	1.6575	1.5313	1.5240
(3a) cap.+startup(high)	1.1366	1.5475	1.5475	1.5475	1.7002	1.5470	1.5271
(3b) cap.+startup(high)+expost inv.	2.5441	1.8824	1.8824	1.8824	2.0503	1.9243	1.9086

Table 4: Average relative *total costs* over 5 examples in MPC mode, with S-SPC as reference point

Avg. rel. total cost in MPC mode:	S-MPC	M-MPC					
		Equal delivery qty			Variable delivery qty		
a) Minimised objective b) Incl. ex post inv. holding costs	single period	no inv	inv fix mix	inv var mix	no inv	inv fix mix	inv var mix
(1a) cap.	1.0078	1.0078	1.0078	1.0078	1.1864	1.0248	1.0248
(1b) cap.+expost inv.	2.3609	1.3456	1.3456	1.3456	1.5391	1.4176	1.4074
(2a) cap.+startup(low)	1.1327	1.1816	1.1645	1.1645	1.3215	1.1914	1.1914
(2b) cap.+startup(low)+expost inv.	2.4845	1.5217	1.5198	1.5198	1.6454	1.5771	1.5701
(3a) cap.+startup(high)	1.5146	1.6952	1.6178	1.6178	1.7521	1.6788	1.6788
(3b) cap.+startup(high)+expost inv.	2.8935	2.0108	1.9619	1.9619	2.0780	2.0695	2.0670

In case of (1a) and (1b) startup costs are not taken into account, although they could be calculated ex post.

4.3.2 Impact of the mode of operation and multiperiod characteristics on the capital cost of the plant design, for different objectives

For all objectives, the relative impact of the mode of operation and multiperiod characteristics on the capital cost is depicted in Tables 5 and 6. Since ex post inventory costs do not influence the capital cost, the "b" scenarios are omitted.

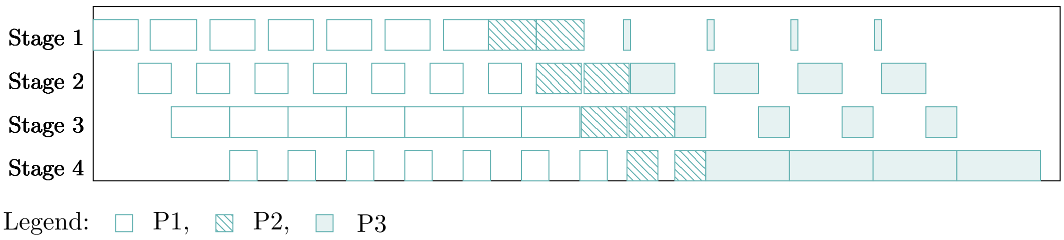
Table 5: Average relative *capital costs* over 5 examples for SPC scenarios with 4 periods, with the single period SPC scenario (S-SPC) as the point of reference

Avg. rel. capital cost in SPC mode:	S-SPC	M-SPC					
		Equal delivery qty			Variable delivery qty		
Minimised objective:	single period	no inv	inv fix mix	inv var mix	no inv	inv fix mix	inv var mix
(1a) cap.	1	1.0181	1.0181	1.0181	1.1912	1.0338	1.0338
(2a) cap.+startup(low)	1	1.0181	1.0181	1.0181	1.1912	1.0338	1.0338
(3a) cap.+startup(high)	1	1.0343	1.0343	1.0343	1.1912	1.0338	1.0338

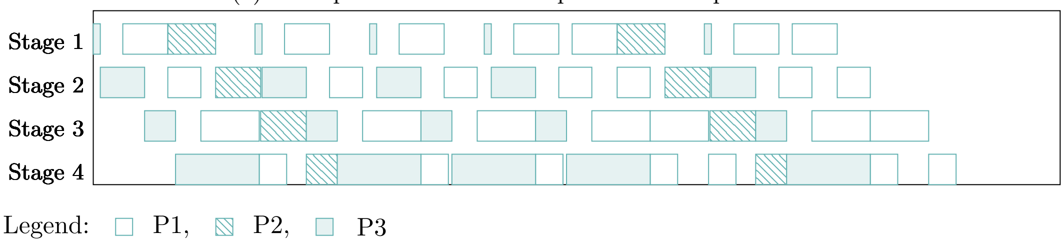
Table 6: Average relative *capital costs* over 5 examples for MPC scenarios with 4 periods, with the single period SPC scenario (S-SPC) as the point of reference

Avg. rel. capital cost in MPC mode:	S-MPC	M-MPC					
		Equal delivery qty			Variable delivery qty		
Minimised objective:	single period	no inv	inv fix mix	inv var mix	no inv	inv fix mix	inv var mix
(1a) cap.	1.0078	1.0078	1.0078	1.0078	1.1864	1.0248	1.0248
(2a) cap.+startup(low)	1.0146	1.0146	1.0167	1.0167	1.1890	1.0248	1.0248
(3a) cap.+startup(high)	1.0677	1.0890	1.1056	1.1056	1.2024	1.1148	1.1148

Single period models for both modes of operation (S-SPC versus S-MPC): For single period models, the S-MPC mode results on average in slightly more expensive designs than S-SPC, but the difference is small. Indeed, for 2 examples lower capital costs are found in S-MPC, since the complementary processing times allow to create a better "puzzle" of batches in a mixed campaign. An illustration of such a "puzzle" is given in Figure 5. In SPC mode (Fig. 5a), idle time occurs mainly in stage 4 for product 1, whereas for product 3 this is the most occupied stage. Stage 1, on the other hand, has a long processing time for product 1 and the shortest for product 3. Through alternating production of these products (Fig. 5b), less time is needed to produce the same number of batches. As a result, the total time can be used to produce more but smaller batches, that fit in smaller equipment units.



(a) Example S-SPC mode of operation for 3 products



(b) Example S-MPC mode of operation for 3 products (visualisation of 1 campaign)

Figure 5: Comparison SPC- and MPC mode of operation for 3 products in a single period

However, for 1 example the same design (and capital cost) is found for S-SPC and S-MPC, and the 2 remaining examples show a more expensive design for S-MPC. This might be explained as follows: firstly, without complementary processing times, the advantages of an interwoven MPC "puzzle" can not be obtained. Secondly, the fixed proportion of the number of batches of every product in every MPC-campaign, which means that the required production quantity is sometimes exceeded. Indeed, for MPC mode the decision maker has to define upfront the upper bounds of the number of batches per product per campaign. As stated in Section 2, this upper bound influences the maximum number of time slots considered by the model. When this upper bound is too high, it makes the problem intractable to solve, whereas a low value might miss the best campaign compositions. This upper bound also affects the possible number of campaign repetitions considered. To our best knowledge, the importance of this setting has not been stressed enough in the literature on MPC mode.

When high startup costs are included in the objective, the difference in capital costs between models in S-SPC and S-MPC mode becomes larger (rows 2 and 3 in Tabs. 5-6). This effect is discussed in the next section.

Single versus multiperiod SPC (S-SPC versus M-SPC): When turning S-SPC into M-SPC, higher capital costs are found on average over these 5 examples, as indicated by column 1 versus the other columns in Table 5. This can partly be attributed to the more constrained problem in case of multiple periods. Indeed, multiperiod models assume that at the end of every period, an integer number of batches needs to be produced whereas in a single period model, only at the end of the production horizon, an integer number of batches is required. Furthermore, the multiple delivery points disrupt the optimal S-SPC production runs of a product. The influence of the different multiperiod characteristics is discussed later.

Single versus multiperiod MPC (S-MPC versus M-MPC): Conversely, for all 5 examples, the M-MPC models with no and low startup costs and equal delivery quantities, result in the same design as the S-MPC models. A possible explanation lies in the fact that the S-MPC model already tends towards a multiperiod model, since batches of every product are made in each MPC-campaign, which is repeated over the horizon. Hence, single and multiperiod MPC models with equal deliveries are more alike, except that in S-MPC models no intermediate deliveries are specified, and no changes in the MPC-campaign composition may occur over the horizon. For high startup costs, however, this similarity in design is no longer observed. However, it should be noted that, for all 5 examples, the campaign lengths in the S-MPC models appeared to be shorter than the M-MPC period length. If S-MPC campaign lengths are longer than the period length, it is possible to obtain more expensive designs for M-MPC models, irrespective of the height of the startup costs.

Multiperiod models if only capital cost is optimised and startup costs is neglected: Overall, for these 5 examples, when minimising only capital costs, lower averages are obtained for M-MPC than M-SPC models. However, when startup costs interact, this conclusion no longer holds, as explained in further on and explicitly discussed in the next section.

Multiperiod models with equal delivery quantities (M-SPC and M-MPC): In case of equal delivery quantities, no difference in design is observed between all modes and options, if only capital cost is optimised (row (1a) of columns 2-4 in Tabs. 5-6). Indeed, if startup costs are neglected and the same amount of every product needs to be produced in every period, there is no benefit in keeping inventory. Also, since every product is requested in every period anyway, the restriction of fixed product mix is not limiting. When (high) startup costs are included in the objective, this is no longer true.

Multiperiod models with variable delivery quantities (M-SPC and M-MPC): When variable delivery quantities are required over the periods, more expensive designs are needed for both modes of operation. However, if inventory is allowed, the increased design cost can be partially compensated for. Indeed, from Tables 5 and 6, the increase of 19 and 18 % in capital cost seen in case of variable delivery quantities in SPC and MPC mode (column 5 vs 2-4) respectively, is reduced to +1.6 and +1.7 % when end-of-period inventory is allowed (columns 6-7 vs 2-4).

4.3.3 Impact on the capital cost of including startup costs in the objective function, for different modes of operation and multiperiod characteristics

Including startup costs in single period models (S-SPC versus S-MPC): Not surprisingly, Tables 3 and 4, show that startup costs are on average higher in MPC mode than in SPC mode, and even more apparent for high startup costs (shown by rows 2 and 3). Consequently, the impact of startup costs on the design (capital cost) is higher for all MPC models as well, as depicted in Tables 5 and 6. Note that in this paper the startup costs are included per MPC-campaign. If additional startups were also required within an

MPC-campaign, the startup costs for MPC mode would be even larger.

Including startup costs in multiperiod models (M-SPC and M-MPC): When evolving from a single to a multiperiod SPC model, the impact of (high) startup costs is large, since there are more frequent production run startups. Indeed, for equal delivery quantities (without end-of-period inventory and for all product mix options), the startup costs are at least Np times higher (depending on the corresponding design decisions). In MPC mode, on the contrary, the increase in startup costs is modest and even not observed for all multiperiod models. After all, also in S-MPC mode, MPC-campaigns are already repeated, and the difference in costs depends on the number of campaigns over a long horizon versus the number of repeated campaigns over all periods.

Including startup costs in case of equal delivery quantities with/without end-of-period inventory: For M-MPC models with equal delivery quantities (Tab. 4, columns 2-4), the total costs with and without end-of-period inventory, are no longer the same. Indeed, by shifting part of the production to earlier periods and keeping it in inventory, the total number of batches, and consequently the number of campaign repetitions, can be lowered. However, despite fewer startups, the accompanied design (capital cost) is more expensive (Table 6, columns 2-4). These differences are not observed in the M-SPC models, as shifts of the entire production run to another period are retained due to the restrictive upper bound on the inventory.

Including startup costs in case of variable delivery quantities with/without product mix restrictions: When delivery quantities vary over the periods, the influence of the fixed product mix restriction becomes apparent for the M-SPC models when high startup costs are included (shown in the bottom rows of the last two columns of Table 3). After all, it might be that not every product is required in every period. For M-MPC models, however, no difference is observed between the fixed and variable product mix cases: the total number of campaign repetitions was not altered and the product mix restriction was not limiting regarding startup costs. If, as aforementioned, additional startups were required within a MPC campaign, this would probably no longer be true.

4.3.4 Impact of mode of operation and multiperiod characteristics on the inventory holding cost of a plant design, for different objectives

As explained earlier, due to the non-linearity of the cost function, the inventory holding cost component discussed hereafter is calculated ex post.

Inventory holding costs for single period versus multiperiod models: From Tables 3 and 4, it can be seen that for both SPC- and MPC modes, the same conclusion holds: inventory holding costs are much higher in a single period context, as we assume that products are accumulated in stock until the end of the horizon. It appeared that S-MPC models have, on average, lower inventories than S-SPC models, but this is not observed for all examples. Additionally, in the multiperiod context, when comparing M-MPC to M-SPC, there is no clear sense of the impact of the mode of operation on the inventory holding costs.

Inventory holding costs for different multiperiod characteristics: When analysing M-SPC and M-MPC models for the multiperiod characteristics, no conclusive distinctions can be observed. For example, in case of equal delivery quantities without end-of-period inventory, the same inventory pattern occurs in every period. In case of variable delivery quantities without end-of-period inventory, periods with lower production are alternated with periods with higher production. Overall, there is no indication when total inventory holding costs will be higher. Also, depending on the delivery quantities and end-of-period inventory allowed or not, not only the number of batches may change but also the product/campaign cycle times. Finally, in case of varying delivery quantities, product mix restrictions result, on average over these 5 examples, in small inventory holding cost increases.

Inventory holding costs and startup costs: For M-MPC models with equal delivery quantities and startup costs included, it was found that inventory holding costs were higher when end-of-period inventory was allowed than without this inventory. As discussed in the previous section, it appeared that larger designs were chosen to partially shift production in order to reduce startup costs. This shift is, however, accompanied with an increase of the inventory holding costs that partly offset the advantage.

4.4 Detailed examples

In this section, we discuss three examples to illustrate some findings from the previous section in detail. The first example concerns the impact of including startup costs in the objective. For the remaining two examples, we only concentrate on some particularities. The input data of all examples are shown in Appendix E (Tables E.1 and E.2). The results are depicted in Tables 7 - 14.

Example 1: 3 products over 3 stages with complementary processing times

Minimising only capital costs
From Table 7 (SPC mode), it can be seen that, for equal delivery quantities, single and multiple period SPC models result in the same design, independently of inventory allowed or product mix restrictions. Also for MPC mode (Table 8), single and multiperiod models with equal delivery quantities, obtain the same design regardless of inventory and product mix. In fact, both modes of operation require more expensive designs when delivery quantities vary over the periods, but these increases are (partly) avoided when end-of-period inventory is allowed. Note that the product mix restriction is not limiting for this example, neither for equal or variable delivery quantities, since every product is required in every period.

When comparing the capital costs for this example, producing in MPC mode is cheaper for all combinations of characteristics (first row of Tables 7 and 8). In fact, for this case, the processing times of the different products turn out to be complementary, hence a compact "puzzle" can be constructed so that more but smaller batches are produced within the given production time, and smaller equipment units can be chosen. Indeed, for S-MPC, the 3 products are produced in 50, 150 and 50 batches respectively, while for S-SPC, 25, 116 and 33 batches are produced. Inventory holding costs, however, are higher for all MPC models, outweighing the decrease in capital costs, and making MPC mode more expensive for this example. Remember that startup costs are not taken into account in Table 7.

A special note must be made on the problem sizes and corresponding computation times: MPC calculations require much more computation time than SPC calculations, even more significantly when turning an S-MPC model into an M-MPC one. Indeed, Van Den Heever and Grossmann (1999) showed already that for multiperiod problems, implying that binary variables increase with every additional time period, the computation times quickly become intractable.

Table 7: Example 1: 3 products - *capital costs*, ex post inventory holding costs and optimal design decisions, for a plant operating in SPC mode when optimising only capital cost

Capital cost for Ex.1 in SPC mode:	S-SPC	M-SPC					
		Equal delivery qty			Variable delivery qty		
Objective: Minimise capital cost	single[a] period	no inv	inv fix mix	inv var mix	no inv	inv fix mix	inv[b] var mix
Capital costs	763 251	763 251	763 251	763 251	899 292	763 251	763 251
Expost inventory costs	1 066 084	244 318	244 318	244 318	249 339	275 209	275 209
Total costs[c]	*1 829 335*	*1 007 569*	*1 007 569*	*1 007 569*	*1 148 631*	*1 038 460*	*1 038 460*
Design decisions (written as size (number)):							
stage 1	6200(1)		6200(1)		6200(2)	6200(1)	
stage 2	7000(1)		7000(1)		7000(1)	7000(1)	
stage 3	4800(2)		4800(2)		5100(2)	4800(2)	
Total end-of-period inventory	0	0	0	0	0	78 766	78 766

[a]Problem size S-SPC: 149 variables (105 binary); 194 constraints, solved in 0.047 s
[b]Problem size M-SPC: 588 variables (105 binary); 1422 constraints, solved in 0.608 s
[c]In monetary units

Table 8: Example 1: 3 products - *capital costs*, ex post inventory holding costs and optimal design decisions, for a plant operating in MPC mode when optimising only capital cost

Capital cost for Ex.1 in MPC mode	S-MPC	M-MPC					
		Equal delivery qty			Variable delivery qty		
Objective: Minimise capital cost	single[a] period	no inv	inv fix mix	inv var mix	no inv	inv fix mix	inv[b] var mix
Capital costs	742 044	742 044	742 044	742 044	876 383	754 799	754 799
Expost Inventory costs	1 101 448	281 292	281 292	281 292	293 198	297 034	297 034
Total costs[c]	*1 843 492*	*1 023 336*	*1 023 336*	*1 023 336*	*1 169 581*	*1 051 833*	*1 051 833*
Design decisions (written as size (number)):							
stage 1	4800(2)		4800(2)		6200(2)	5100(2)	
stage 2	5700(1)		5700(1)		7000(1)	6200(1)	
stage 3	3700(2)		3700(2)		4800(2)	3700(2)	
Total end-of-period inventory	0	0	0	0	0	55 556	55 556

[a]Problem size S-MPC: 1341 variables (1054 binary); 6749 constraints, solved in 61.85 s
[b]Problem size M-MPC: 6459 variables (3877 binary); 31 229 constraints, after 21 h - gap of 1.7 %
[c]In monetary units

Minimising capital and high startup costs
In the second part of this example, startup costs are included in the optimisation. The results are presented in Table 9 for SPC mode and in Table 10 for MPC mode.

When comparing Tables 7 and 8 with Tables 9 and 10, it is obvious that the impact of startup costs on the optimal design is higher for MPC than for SPC mode. In MPC all but one model obtained a different design, whereas in SPC mode no changes in the design decisions are made. Due to these more expensive designs, the MPC mode is often not beneficial for this example with respect to capital costs. Due to the campaign structure, the startup costs itself are much higher in MPC than in SPC mode. Especially in S-MPC mode, these costs are 12 times higher than for the S-SPC case.

Also, from Table 10, it can be seen that the M-MPC cases with equal delivery quantities do not obtain the same design for all options. It is beneficial to increase capital costs and use end-of-period inventory in order to reduce startup costs. Inventory holding costs are higher due to these production shifts.

Table 9: Example 1: 3 products - capital, startup, ex post inventory holding costs and optimal design decisions, for a plant operating in SPC mode when optimising capital and (high) startup costs

Capital cost for Ex.1 in SPC mode:	S-SPC	M-SPC					
		Equal delivery qty			Variable delivery qty		
Objective: Minimise capital+startup cost	single[a] period	no inv	inv fix mix	inv var mix	no inv	inv fix mix	inv[b] var mix
Capital costs	763 251	763 251	763 251	763 251	899 292	763 251	763 251
Startup costs	60 000	240 000	240 000	240 000	300 000	240 000	240 000
Expost Inventory costs	1 066 084	244 318	244 318	244 318	249 339	275 209	275 209
Total costs[c]	*1 889 335*	*1 247 569*	*1 247 569*	*1 247 569*	*1 448 631*	*1 278 460*	*1 278 460*
Design decisions (written as size (number)):							
stage 1	6200(1)		6200(1)		6200(2)	6200(1)	
stage 2	7000(1)		7000(1)		7000(1)	7000(1)	
stage 3	4800(2)		4800(2)		5100(2)	4800(2)	
Total end-of-period inventory	0	0	0	0	0	78 766	78 766

[a]Problem size S-SPC: 149 variables (105 binary); 195 constraints, solved in 0.081 s
[b]Problem size M-SPC: 708 variables (225 binary); 1551 constraints, solved in 0.515 s
[c]In monetary units

Table 10: Example 1: 3 products - capital, startup, ex post inventory holding costs and optimal design decisions, for a plant operating in MPC mode when optimising capital and startup cost

Capital cost for Ex.1 in MPC mode:	S-MPC	M-MPC					
		Equal delivery qty			Variable delivery qty		
Objective: Minimise capital+startup cost	single[a] period	no inv	inv fix mix	inv var mix	no inv	inv fix mix	inv[b] var mix
Capital costs	839 347	791 145	876 383	876 383	876 383	817 543	817 543
Startup costs	600 000	1 000 000	750 000	750 000	875 000	875 000	875 000
Inventory costs	1 117 995	272 830	342 679	342 679	286 986	297 212	297 212
Total costs[d]	*2 557 342*	*2 063 975*	*1 969 062*	*1 969 062*	*2 038 369*	*1 989 755*	*1 989 755*
Design decisions (written as size (number)):							
stage 1	7000(1)	5100(2)	6200(2)		6200(2)	5700(2)	
stage 2	7000(1)	5100(1)	7000(1)		7000(1)	6200(1)	
stage 3	5700(2)	4300(2)	4800(2)		4800(2)	4300(2)	
Total end-of-period inventory	0	0	176 000	176 000	0	74 286	74 286

[a]Problem size S-MPC: 1593 variables (1306 binary); 7002 constraints, solved in 19.10 s
[b]Problem size M-MPC: 7251 variables (4669 binary); 32 022 constraints, solved in 4019 s
[c]ex post inventory holding costs
[d]in monetary units

Example 2: 3 products over 4 stages with products that are not required every period

Minimising capital and startup costs
For this example we only consider the case of optimised capital plus startup costs. The results are shown in Table 11 for the SPC mode and Table 12 for MPC mode.

When looking at the capital costs, the same conclusions as in the first example hold for most of the characteristics. But in this example, not all products are demanded in every period. Hence in M-SPC mode, startup and inventory holding costs are higher when the product mix is fixed (Table 11). In M-MPC mode, on the contrary, higher inventory holding costs were encountered, but no reduction of the number of campaign repetitions.

When comparing Table 11 (SPC) with Table 12 (MPC), the same designs are obtained for S-SPC and S-MPC and similar or better designs for M-MPC in comparison to M-SPC. Also, as expected, for the M-MPC models with equal delivery quantities (Table 12), reductions in startup costs are achieved when end-of-period inventory was allowed. This is at the expense of slightly higher inventory holding costs, but without an increase in design. Finally, the S-MPC model generates startup costs that are 4 times higher than for the S-SPC model. Comparing the multiperiod models, startup costs are still higher in MPC mode, but they differ less. We can conclude that for this example total costs are better in MPC mode for most of the models, except for variable delivery quantities with end-of-period inventory.

Table 11: Example 2: 3 products - capital, startup, ex post inventory holding costs and optimal design decisions, for a plant operating in SPC mode optimising capital and startup costs

	S-SPC	M-SPC					
		Equal delivery qty			Variable delivery qty		
Objective: Minimised capital+startup cost	single[a] period	no inv	inv fix mix	inv var mix	no inv	inv fix mix	inv[b] var mix
Capital costs	210 341	223 071	223 071	223 071	255 544	210 341	210 341
Startup costs	5400	21 600	21 600	21 600	19 800	21 600	19 800
Expost Inventory costs	287 910	70 918	70 918	70 918	69 924	84 409	83 393
Total costs[c]	*503 651*	*315 589*	*315 589*	*315 589*	*345 268*	*316 350*	*313 534*
Design decisions (written as size (number)):							
stage 1	9000(1)		9000(1)		13500(1)	9000(1)	
stage 2	6000(1)		9000(1)		6000(1)	6000(1)	
stage 3	6000(1)		6000(1)		9000(1)	6000(1)	
stage 4	9000(1)		9000(1)		135000(1)	9000(1)	
Total end-of-period inventory	0	0	0	0	0	68 032	66 452

[a]Problem size S-SPC: 148 variables (92 binary); 223 constraints, solved in 0.158 s
[b]Problem size M-SPC: 731 variables (248 binary); 1583 constraints, solved in 0.593 s
[c]In monetary units

Table 12: Example 2: 3 products - capital, startup, ex post inventory holding costs and optimal design decisions, for a plant operating in MPC mode when optimising capital and startup costs

	S-MPC	M-MPC					
		Equal delivery qty			Variable delivery qty		
Objective: Minimised capital+startup cost	single[a] period	no inv	inv fix mix	inv var mix	no inv	inv fix mix	inv[b] var mix
Capital costs	210 341	210 341	210 341	210 341	252 037	210 341	210 341
Startup costs	21 600	28 800	25 200	25 200	19 800	30 600	30 600
Ex post Inventory costs	245 167	63 280	64 117	64 117	57 109	83 569	75 879
Total costs[d]	*477 108*	*302 421*	*299 658*	*299 658*	*328 946*	*324 510*	*316 820*
Design decisions (written as size (number)):							
stage 1	9000(1)		9000(1)		13500(1)	9000(1)	
stage 2	6000(1)		6000(1)		9000(1)	6000(1)	
stage 3	6000(1)		6000(1)		9000(1)	6000(1)	
stage 4	9000(1)		9000(1)		9000(1)	9000(1)	
Total end-of-period inventory	0	0	2810	2810	0	55 328	41 242

[a]Problem size S-MPC: 3206 variables (2565 binary); 16004 constraints, solved in 832 s
[b]Problem size M-MPC: 12422 variables (7996 binary); 65882 constraints,
after 21 h - gap of 9.4 %, after 72 h - gap of 3.8 %
[d]in monetary units

Example 3: 3 products over 4 stages with non complementary processing times
Minimising only capital costs
The results of minimising only capital costs are presented in Tables 13 and 14, for SPC and MPC mode respectively. Overall, the same conclusions as for the first example can be drawn for the different characteristics. Note again that fixed product mixes are never restrictive as every product is required in each period.

However, conversely to the Example 1 at minimal capital cost, producing in MPC mode results in more expensive designs for the single period and most of the multiperiod models. When examining the input data, we observe indeed no complementary processing times over the products, but, on the contrary, for all products the first stage appeared to be the bottleneck. Therefore, we investigated if the upper bounds on the number of batches per product per MPC-campaign are limiting, and increased them to allow longer campaigns, but the same design as in SPC mode could not be reached. Lastly, when investigating the optimal number of batches obtained in the SPC models, there is no fixed proportion detected between the products. Hence, this might explain the higher capital costs.

Table 13: Example 3: 3 products - total *capital costs* and ex post inventory holding costs and optimal design decisions, for a plant operating in SPC mode

	S-SPC	M-SPC					
		Equal delivery qty			Variable delivery qty		
Objective: Minimised capital cost	single[a] period	no inv	inv fix mix	inv var mix	no inv	inv fix mix	inv[b] var mix
Capital costs	54 108	54 369	54 369	54 369	65 965	58 750	58 750
Expost inventory costs	83 575	21 636	21 636	21 636	20 422	20 962	20 962
Total costs[c]	*137 683*	*76 005*	*76 005*	*76 005*	*86 687*	*79 712*	*79 712*
Design decisions (written as size (number)):							
stage 1	1000(2)	1000(2)			2500(1)	2000(1)	
stage 2	1000(1)	2000(1)			2000(1)	2000(1)	
stage 3	2000(1)	1000(1)			2500(1)	2000(1)	
stage 4	2000(1)	2000(1)			4000(1)	3000(1)	
Total end-of-period inventory	0	0			0	9343	9343

[a]Problem size S-SPC: 164 variables (108 binary); 234 constraints, solved in 0.125 s
[b]Problem size M-SPC: 639 variables (108 binary); 1570 constraints, solved in 0.530 s
[c]In monetary units

Table 14: Example 3: 3 products - total *capital costs* and ex post inventory holding costs and optimal design decisions, for a plant operating in MPC mode

	S-MPC	M-MPC					
		Equal delivery qty			Variable delivery qty		
Objective: Minimised capital cost	single[a] period	no inv	inv fix mix	inv var mix	no inv	inv fix mix	inv[b] var mix
Capital costs	58 750	58 750	58 750	58 750	67 945	58 750	58 750
Expost inventory costs	81 631	20 305	20 305	20 305	22 601	23 900	23 900
Total costs[c]	*140 381*	*79 055*	*79 055*	*79 055*	*90 546*	*82 650*	*82 650*
Design decisions (written as size (number)):							
stage 1	2000(1)	2000(1)			2500(1)	2000(1)	
stage 2	2000(1)	2000(1)			3000(1)	2000(1)	
stage 3	2000(1)	2000(1)			2000(1)	2000(1)	
stage 4	3000(1)	3000(1)			4000(1)	3000(1)	
Total end-of-period inventory	0	0			0	11 010	11 010

[a]Problem size S-MPC: 2210 variables (1688 binary); 12 317 constraints, solved in 54.09 s
[b]Problem size M-MPC: 8762 variables (5312 binary); 45 121 constraints, solved in 2822 s
[c]In monetary units

4.5 Discussion

For our limited set of examples, we can derive the following conclusions on the impact on the plant design by the mode of operation and the multiperiod characteristics.

Regarding capital cost and startup costs, single period batch plant design models yield, on average, the cheapest solutions for both SPC and MPC. However, the assumption of a single delivery point at the end of a long horizon makes these models inappropriate from a industrial point of view.

Next, turning an S-SPC into an M-SPC model with equal delivery quantities per period, results on average for our 5 examples, in slightly more expensive designs. For each mode of operation, in case of pure capital cost optimisation and equal delivery quantities, equal design solutions are obtained regardless of end-of-period inventory or product mix restrictions. But for variable delivery quantities, the capital costs increase with 17 and 18 % respectively for SPC and MPC mode, which can be avoided to a large extent by allowing end-of-period inventory (+1.6% for SPC and +1.7 % for MPC mode).

Once startup costs are included in the optimisation, on average for these examples, the capital costs increase only limited in SPC mode, whereas in MPC mode they increase substantially for all combinations of characteristics. Furthermore, in MPC mode, shifting part of the production by keeping it in inventory appeared beneficial, even when delivery quantities are equal over the periods.

Lastly, the inventory holding costs calculated ex post are incontestably very high for the single period model. For the multiperiod cases, these costs are reasonable, but conclusions for the different multiperiod characteristics are less clear. For example, the aforementioned production shift in M-MPC to reduce startup costs is accompanied with an increase in inventory holding costs.

Comparing the two modes of operation, the following conclusions can drawn for our example set: producing in MPC mode can be beneficial with respect to capital costs when complementary processing times are present. However, once startup costs are included, the aforementioned advantage is negated, and the SPC mode is more advantageous. Moreover, when additional startups would be included within an MPC-campaign, the impact of the startup cost would be even larger.

Although producing in MPC mode does not have distinct advantages with respect to costs, in a single period context it is a more appropriate mode from a commercial point of view than SPC, where a product is made only once over a long horizon. In a multiperiod context, however, this argument looses its strength, as also M-SPC is a realistic way of operating. Additionally, solving models in MPC mode is very sensitive to the input parameters given by the decision maker, which affects their practical usability. Finally M-MPC models become rapidly intractable from the moment the instances become larger.

5 Conclusion

In this paper, the multiperiod BPDP with a periodic delivery scheme is investigated. First, multiperiodicity is introduced in the mathematical models to account for more frequent product delivery points as opposed to single period models, and such for both the single product as the mixed product campaign mode of operation. Next, the following delivery scheme characteristics are introduced into the mathematical multiperiod BPDP models: variability in delivery quantities, end-of-period inventory and product mix restrictions.

For 5 examples (datasets) we solved 36 multiperiod scenarios each, giving us a first insight into the impact on the plant design of the operational mode and these multiperiod characteristics. It is found that, for these examples, delivering equal quantities at the end of every period leads to the cheapest design in terms of total costs. However, the extra cost to be more flexible towards customers' needs, translated in variable delivery quantities, is not that high. Moreover, allowing end-of-period inventory and variable product mixes to accommodate for these variations, limits the extra costs.

Concerning the mode of operation, in a multiperiod context the advantages of the SPC mode are more pronounced. Firstly, from the examples studied, it appeared that producing in MPC mode is more expensive regarding startup, although it can be (slightly) beneficial for the capital cost if the different products have complementary processes. Secondly, from a delivery perspective, a multiperiod SPC campaign structure is as realistic as an MPC mode. Thirdly, MPC computation times increase very rapidly, and the solutions strongly depend on MPC input parameters that need tuning themselves, such as the maximum number of batches for every product in an MPC campaign.

Finally, we are aware of the limited sample size used to come up with these conclusions, mainly due to the arduousness of the MPC models. Therefore, before involving a more extended sample set, a detailed analysis of the setting of the MPC parameters is needed. Besides, future research is needed to introduce additional features to the design models, such as the cost of inventory equipment and, most difficult, a way to account for the impact of sequence dependent changeover costs and times.

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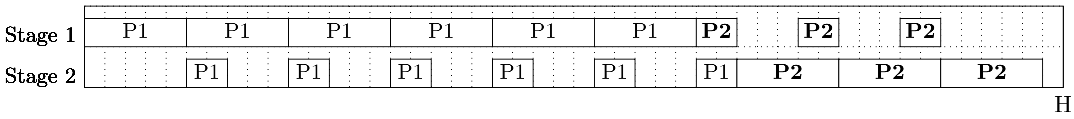
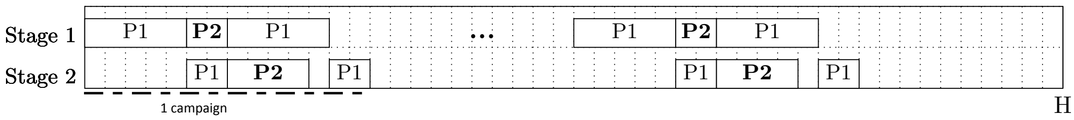
A Terminology

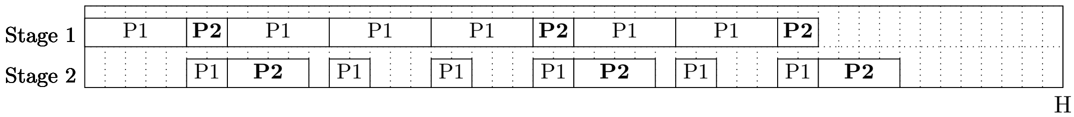
A.1 Mode of operation



(a) SPC mode of operation



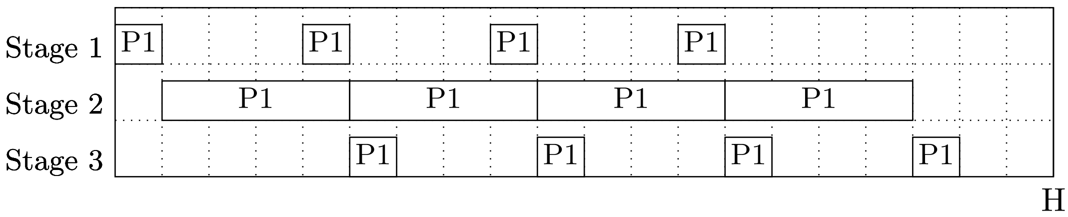
(b) MPC mode of operation



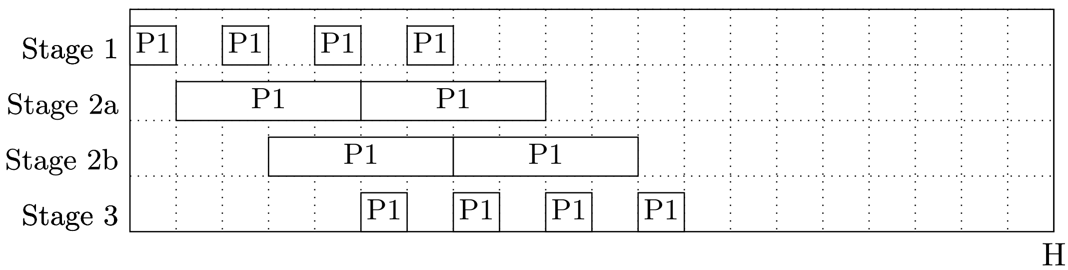
(c) Network planning mode of operation

Figure A.1: Overview modes of operation

A.2 Parallel equipment



(a) No parallel equipment



(b) Parallel equipment installed in stage 2, operating out-of-phase

Figure A.2: Overview parallel equipment

B Nomenclature

B.1 M-SPC

Table B.1: Nomenclature multiperiod BPDP - M-SPC

Indices	
i	products (P)
j	stages (J)
s	discrete sizes (S)
n	number of equipment in parallel (N)
h	periods (Np)
Parameters	
S_{ij}	size factor of product i in stage j
τ_{ij}	processing time of product i in stage j
Q_{ih}	amount of product i to be delivered at the end of period h
Q_i^{UB}	upper bound on the amount of product i produced in period h, with $Q_i^{UB} = 5 \max_h(Q_{ih})$
NB_i^{UB}	upper bound on the number of batches of product i in period h
Q_i^{LB}	lower bound on the amount of product i produced in period h
H	total production horizon
Lp	length of every fixed, equal period h, with $Lp = H/Np$
v_s	equipment size s
I_{i0}	starting inventory of product i
α_j, β_j	cost parameters of stage j
$Cstart$	startup cost per product
$InvCost$	a fixed cost per unit product held in inventory per time
Variables	
Integer	
n_{ih}	number of batches produced of product i in period h
Binary	
t_{ih}	equals 1 if product i is produced in period h
ZT_{ijnh}	product of z_{jn} and t_{ih} (linearisation)
Continuous	
q_{ih}	amount of product i produced in period h
I_{ih}	amount of product i kept in inventory at the end of period h (i.e. end-of-period inventory)
T_{jih}	total time spent on product i on stage j in period h
θ_{ih}	total time spent on product i in period h
B_{ih}	batch size of product i in period h
W_{ijsh}	product of u_{js} and q_{ih} (linearisation)
X_{jinh}	product of z_{jn} and T_{jih} (linearisation)

B.2 M-MPC

Table B.2: Nomenclature multiperiod BPDP - M-MPC

Indices	
i	products (P)
j	stages (J)
s	discrete sizes (S)
n	number of equipment in parallel (N)
e	timeslots ($E{=}\sum_{i=1}^{P} NBC_i^{UB}$)
m	number of batches per campaign (NBC_i^{UB})
b	discrete options for the number of mixed campaign repetitions (B)
h	periods (Np)
Parameters	
S_{ij}	size factor of product i in stage j
τ_{ij}	processing time of product i in stage j
Q_i	total amount to produce of product i
H	horizon, total available production time
v_s	tank size s
BB_b	number of repetitions b

Q_{ih}	amount of product i to be delivered at the end of period h
Q_i^{UB}	upper bound on the amount of product i produced in period h, with $Q_i^{UB} = \max_h(Q_{ih})$
NBC_i^{UB}	upper bound on the number of batches per campaign of product i in period h
Q_i^{LB}	lower bound on the amount of product i produced in period h
H	total production horizon
Lp	length of every fixed, equal period h, with $Lp = H/Np$
I_{i0}	starting inventory of product i
$InvCost$	a fixed cost per unit product held in inventory per time
$InitCorr$	a small, positive coefficient to tackle the degrees of freedom with respect to the timings
$InvCorr$	a small, positive coefficient to tackle the degrees of freedom with respect to end-of-period inventory
Variables	
Continuous:	
q_{ih}	amount of product i produced in period h
I_{ih}	amount of product i kept in inventory at the end of period h (i.e. end-of-period inventory)
TF_{jenh}	finishing time of slot e in equipment unit n of stage j in period h
TI_{jenh}	starting time of slot e in equipment unit n of stage j in period h
CTC_h	cycle time of a campaign in period h
H_h	optimised period length of period h
B_{ih}	batch size of product i in period h
p_{ih}	amount of product i produced per campaign in period h
WQ_{ijsmh}	product of q_{ih} and W_{ijsmh} (linearisation)
WW_{bh}	product of NNR_{bh} and CTC_h (linearisation)
Binary:	
c_{imh}	equals 1 if product i is produced in m batches in a campaign of period h
NNR_{bh}	equals 1 if a campaign is repeated BB_b times in period h
y_{ijenh}	equals 1 if product i is assigned to slot e and processed in equipment unit n of stage j in period h
x_{jenh}	equals 1 if slot e of equipment unit n of stage j is used in period h
r_{ieh}	equals 1 if product i is produced in slot e in period h
W_{ijsmh}	product of u_{js} and c_{imh} (linearisation)
WX_{jnbh}	product of z_{jn} and NNR_{bh} (linearisation)
Integer:	
n_{ih}	number of batches of product i per campaign of period h
NN_h	number of times a campaign is repeated in period h

C Mathematical model - M-SPC

C.1 Constraints

C.1.1 End-of-period inventory

The MILP model for the multiperiod case with end-of-period inventory is presented below.

Batch equipment design constraints:
Eqs. (26)-(28) are related to the number and size of equipment units installed in every stage: Eq. (26) states that at least one unit in every stage j should be installed. Eq. (27) indicates that equipment unit $n+1$ of stage j can only be installed if equipment unit n already exists. Eq. (28) defines that for every stage j one size s is to be chosen. Finally, for every stage j, the capacity of the equipment units v_s should be large enough to hold a batch of every product i in every period h, multiplied by its size factor S_{ij}. Eq. (29) is obtained through the incorporation of the discrete set of sizes and the expression q_{ih}/n_{ih} to represent a batch of product i in period h, with q_{ih} the amount of product i produced in period h and

n_{ih} the number of batches.

$$\sum_{n=1}^{N} z_{jn} \geqslant 1 \qquad \forall j \qquad (26)$$

$$z_{jn} \geqslant z_{j(n+1)} \qquad \forall j, n \text{ with } n < N \qquad (27)$$

$$\sum_{s=1}^{S} u_{js} = 1 \qquad \forall j \qquad (28)$$

$$n_{ih} \geqslant \sum_{s=1}^{S} \frac{q_{ih} S_{ij}}{v_s} u_{js} \qquad \forall i, j, h \qquad (29)$$

The incurred nonlinearity $q_{ih}u_{js}$ of Eq. (29) is addressed through the introduction of a continuous variable $W_{ijsh} = q_{ih}u_{js}$ and the following linearisation constraints:

$$W_{ijsh} \leqslant q_{ih} \qquad \forall i, j, s, h \qquad (30)$$

$$W_{ijsh} \leqslant Q_i^{UB} u_{js} \qquad \forall i, j, s, h \qquad (31)$$

$$W_{ijsh} \geqslant q_{ih} - Q_i^{UB}(1 - u_{js}) \qquad \forall i, j, s, h \qquad (32)$$

$$W_{ijsh} \geqslant 0 \qquad \forall i, j, s, h \qquad (33)$$

with $Q_i^{UB} = \max_h(Q_{ih})$. Given this, Eq. (29) can be rewritten as Eq. (34).

$$n_{ih} \geqslant \sum_{s=1}^{S} \frac{W_{ijsh} S_{ij}}{v_s} \qquad \forall i, j, h \qquad (34)$$

Horizon constraints:
The total time spent on every product i on stage j in period h corresponds to the time spent per batch (stage cycle time) multiplied by the number of batches in that period (Eq. (35)). Furthermore, the total time spent on every product i in period h corresponds to the longest time spent on a stage, as given by Eq. (36). Finally, total production time per period should not exceed the fixed period length Lp (Eq. (37)).

$$\sum_{n=1}^{N} z_{jn} T_{jih} = n_{ih} \tau_{ij} \qquad \forall j, i, h \qquad (35)$$

$$\theta_{ih} \geqslant T_{jih} \qquad \forall j, i, h \qquad (36)$$

$$\sum_{i=1}^{P} \theta_{ih} \leqslant Lp \qquad \forall h \qquad (37)$$

The nonlinear Eq. (35), due to the product $z_{jn}T_{jih}$, is replaced by its linear equivalent Eq. (38) via the continuous variable $X_{jinh} = z_{jn}T_{jih}$:

$$\sum_{n=1}^{N} X_{jinh} = n_{ih} \tau_{ij} \qquad \forall i, j, h \qquad (38)$$

and via the following linearisation constraints:

$$X_{jinh} \leqslant Lp\, z_{jn} \qquad \forall i, j, n, h \qquad (39)$$

$$X_{jinh} \leqslant T_{jih} \qquad \forall i, j, n, h \qquad (40)$$

$$X_{jinh} \geqslant T_{jih} - Lp(1 - z_{jn}) \qquad \forall i, j, n, h \qquad (41)$$

$$X_{jinh} \geqslant 0 \qquad \forall i, j, n, h \qquad (42)$$

Demand and inventory constraints:
Since deliveries of specified amounts of every product are guaranteed at the end of every period, additional constraints are formulated in comparison to the single period context. Moreover, as end-of-period inventory is allowed in this model, this needs to be accounted for as well.

The total delivery quantity required of every product i at the end of every period h can be fulfilled through a combination of production in that period and inventory that was kept at the end of the previous period (Eq. (43)). Consequently, the amount kept in inventory of every product i at the end of every period h equals the sum of the amount in inventory from the previous period and the amount produced in this period minus the amount required in this period (Eq. (44)). Eq. (45) limits the total amount to be stored at the end of every period, just before delivery, to the maximum amount required over all periods, with $Q_i^{UB} = \max_h(Q_{ih})$. Note that it is assumed that the amount held in inventory per product, given its upper bound, will fit in a storage tank. As stated in Section 2, these tanks are not explicitly modelled, since they represent a smaller cost in comparison to the batch equipment. Finally, the starting inventory for every product i (I_{i0}) is set to zero (Eq. (46)).

$$I_{i(h-1)} + q_{ih} \geqslant Q_{ih} \qquad \forall i, h \qquad (43)$$
$$I_{ih} = I_{i(h-1)} + q_{ih} - Q_{ih} \qquad \forall i, h \qquad (44)$$
$$I_{i(h-1)} + q_{ih} \leqslant Q_i^{UB} \qquad \forall i, h \qquad (45)$$
$$I_{i0} = 0 \qquad \forall i \qquad (46)$$

Boundaries:
Eqs. (47)-(50) pose an upper bound on the variables, with NB_i^{UB} the upper bound on the number of batches. This is formulated as $(Q_i^{UB} S_i^{max})/v_1$.

$$n_{ih} \leqslant NB_i^{UB} \qquad \forall i, h \qquad (47)$$
$$q_{ih} \leqslant Q_i^{UB} \qquad \forall i, h \qquad (48)$$
$$T_{jih} \leqslant Lp \qquad \forall j, i, h \qquad (49)$$
$$\theta_{ih} \leqslant Lp \qquad \forall i, h \qquad (50)$$

C.1.2 Adaptations for different multiperiod characteristics

As already mentioned before, modifications are made to the mathematical model depending on the combination of characteristics.

No end-of-period inventory In this case, no end-of-period inventory is allowed, resulting in $I_{ih} = 0$ and $q_{ih} = Q_{ih} \quad \forall i, h$. Hence, Eqs. (43)-(45) are considerably simplified. Moreover, Eq. (29) can be replaced by the linear constraint Eq. (51) and the linearisation constraints (Eqs. (30)-(33)) can be omitted.

$$n_{ih} \geqslant \sum_{s=1}^{S} \frac{Q_{ih} S_{ij}}{v_s} u_{js} \qquad \forall j, i, h \qquad (51)$$

Fixed product mixes When the product mix is fixed, every product needs to be produced in every period. This generates the following additional constraints: Eq. (52) forces the production of at least one batch of every product i in every period h, whereas Eq. (53) sets a lower bound on the amount produced per period. The lower bounds are given by $Q_i^{LB} = (Q_i T_i^{min})/H$ and $T_i^{min} = \max_j \tau_{ij}/N$[3].

$$n_{ih} \geqslant 1 \qquad \forall i, h \qquad (52)$$
$$q_{ih} \geqslant Q_i^{LB} \qquad \forall i, h \qquad (53)$$

[3]This lower bound is derived from $\sum_i (Q_i T_i)/B_i \leqslant H$, so for every product i: $(Q_i T_i^{min})/B_i \leqslant H$ applies. Reordering this equation gives $(Q_i T_i^{min})/H \leqslant B_i$ and the minimum is the used lower bound.

C.2 Objective function

Capital costs
The aim of this optimisation problem is to minimise capital costs. These costs are associated with the acquisition or installation of equipment. As can be seen, the capital costs increase in a nonlinear manner with increasing size of the equipment units, where α_j and β_j are stage dependent cost coefficients and β_j is typically smaller than one for every stage j (Sparrow et al., 1975). This one time capital expenditure is assumed to be already converted to a uniform cost per horizon, so that it can be correctly added up with the other cost components. (Jelen et al., 1983)

$$\sum_{j=1}^{J}\sum_{s=1}^{S}\sum_{n=1}^{N}\alpha_j v_s^{\beta_j} z_{jn} u_{js} \tag{54}$$

The incurred nonlinearity $z_{jn}u_{js}$ is tackled via the binary variable: $Y_{jsn} = z_{jn}u_{js}$ and the following constraint:

$$Y_{jsn} \geqslant z_{jn} + u_{js} - 1 \qquad \forall j, s, n \tag{55}$$

$$Y_{jsn} \in \{0, 1\} \qquad \forall j, s, n \tag{56}$$

Startup costs:
This cost accounts for the preparation of the equipment units at the start of every series of batches of product i in every period h. It is modelled as follows:

$$\sum_{i=1}^{P}\sum_{j=1}^{J}\sum_{n=1}^{N}\sum_{h=1}^{Np} Cstart\, z_{jn}\, t_{ih}$$

where $Cstart$ is a stage and product independent startup cost and t_{ih} indicates whether or not product i is produced in period h. This variable is accompanied with the following additional constraints to make the link with the number of batches made of product i in period h:

$$t_{ih} \leqslant n_{ih} \qquad \forall i, h \tag{57}$$

$$t_{ih} NB_i^{UB} \geqslant n_{ih} \qquad \forall i, h \tag{58}$$

In order to linearise this startup cost, the binary variable $ZT_{ijnh} = z_{jn}t_{ih}$ is introduced together with the constraints below:

$$ZT_{ijnh} \geqslant z_{jn} + t_{ih} - 1 \qquad \forall i, j, n, h \tag{59}$$

$$ZT_{ijnh} \in \{0, 1\} \qquad \forall i, j, n, h \tag{60}$$

Hence, the startup cost becomes:

$$\sum_{i=1}^{P}\sum_{j=1}^{J}\sum_{n=1}^{N}\sum_{h=1}^{Np} Cstart_i\, ZT_{ijnh} \tag{61}$$

Besides influencing the number of equipment units installed in parallel per stage (given by z_{jn}), this cost component may result in production shifts in a multiperiod context. After all, it may be beneficial to increase production of one or more products in a certain period, and to keep it as end-of-period inventory, so that in another period these products do not need to be made ($t_{ih} = 0$) and startup costs can be avoided.

Inventory holding costs
In the multiperiod context with specified delivery points, all products are kept in inventory until the end-of-period delivery dates, hence, this product accumulation should be taken into account. Since

this calculation generates a non-convex objective function that makes the optimisation arduous, the calculation of this cost is done ex post and corresponds to:

$$\sum_{h=1}^{Np}\sum_{i=1}^{P}\Bigg[\underbrace{((n_{ih}-1)\theta_{ih}-T_{ih}(\frac{n_{ih}(n_{ih}-1)}{2}))B_{ih}}_{\text{I}} + \underbrace{(Lp-\sum_{k}^{i}\theta_{kh})q_{ih}}_{\text{II}} + \underbrace{Lp\, I_{i(h-1)}}_{\text{III}}\Bigg] InvCost \tag{62}$$

with $InvCost$ the inventory holding cost per unit of product per time. The first term (I) corresponds to the production part for every product in every period, in which the batches of every product i are produced. The second term (II) corresponds to that part of the period length in which the total amount produced of every product is waiting to be delivered. Since production in an SPC mode of operation does not give a sequence in which products are produced, it is assumed that products are produced in increasing order of production quantities. Hence, the product with the largest amount produced is kept in inventory the shortest time. Lastly, the third term III represents the end-of-period inventory kept when going from one period to the other.

Note that, apart from this ex post cost, a small correction term is included in the objective function to avoid non-zero end-of-period inventory that is not asked for:

$$\sum_{h=1}^{Np}\sum_{i=1}^{P} InvCorr I_{ih} \tag{63}$$

with $InvCorr \in]0, 0.001]$ a given correction coefficient.

D Mathematical model - M-MPC

D.1 Constraints

D.1.1 End-of-period inventory

The MILP model for the multiperiod context and a mixed-product campaign mode of operation with end-of-period inventory is presented.

Batch equipment design constraints:
Eqs. (64)-(66) are related with the number and size of equipment units installed in every stage. These constraints are, as stated, independent of the periods and similar to Eqs. (26)-(28) from the M-SPC model. The number of batches of product i produced in a campaign in period h, i.e. n_{ih}, can be written as the selection of one option m via the binary variable c_{imh} (Eqs. (67)-(68)). Note that, in comparison to a single period model, there is the possibility to choose zero batches. Finally, for every stage j, the size of the equipment units v_s should be large enough to hold a batch of every product i, multiplied by its size factor S_{ij} (Eq. (69)). As the amount produced of product i in period h is a decision variable now,

the capacity constraint is nonlinear in three factors (u_{js}, c_{imh} and q_{ih}).

$$\sum_{n=1}^{N} z_{jn} \geqslant 1 \qquad \forall j \qquad (64)$$

$$z_{jn} \geqslant z_{j(n+1)} \qquad \forall j, n \text{ with } n < N \qquad (65)$$

$$\sum_{s=1}^{S} u_{js} = 1 \qquad \forall j \qquad (66)$$

$$\sum_{m=0}^{NBC_i^{UB}} c_{imh} = 1 \qquad \forall i, h \qquad (67)$$

$$n_{ih} = \sum_{m=0}^{NBC_i^{UB}} m c_{imh} \qquad \forall i, h \qquad (68)$$

$$NN_h \geqslant \sum_{s=1}^{S} \sum_{m=1}^{NBC_i^{UB}} \frac{S_{ij} q_{ih}}{v_s m} u_{js} c_{imh} \qquad \forall i, j, h \qquad (69)$$

Firstly, to overcome the product of two binary variables u_{js} and c_{imh}, the binary variable $W_{ijsmh} = u_{js} c_{imh}$ is introduced, together with the linearisation constraints:

$$W_{ijsmh} \leqslant c_{imh} \qquad \forall i, j, s, m, h \qquad (70)$$

$$W_{ijsmh} \leqslant u_{js} \qquad \forall i, j, s, m, h \qquad (71)$$

$$W_{ijsmh} \geqslant c_{imh} + u_{js} - 1 \qquad \forall i, j, s, m, h \qquad (72)$$

$$W_{ijsmh} \in \{0, 1\} \qquad \forall i, j, s, m, h \qquad (73)$$

With this, Eq. (69) becomes Eq. (74) which is still nonlinear ($W_{ijsmh}\, q_{ih}$). Hence, secondly, the continuous variable $WQ_{ijsmh} = W_{ijsmh}\, q_{ih}$ is introduced together with constraints (75)-(78) to replace the nonlinear expression Eq. (74) by Eq. (79). Note that Q_i^{UB} is introduced as an upper bound on q_{ih}, with $Q_i^{UB} = \max_h(Q_{ih})$.

$$NN_h \geqslant \sum_{s=1}^{S} \sum_{m=1}^{NBC_i^{UB}} \frac{S_{ij} q_{ih}}{v_s m} W_{ijsmh} \qquad \forall i, j, h \qquad (74)$$

$$WQ_{ijsmh} \leqslant Q_i^{UB} W_{ijsmh} \qquad \forall i, j, s, m, h \qquad (75)$$

$$WQ_{ijsmh} \leqslant q_{ih} \qquad \forall i, j, s, m, h \qquad (76)$$

$$WQ_{ijsmh} \geqslant q_{ih} - Q_i^{UB}(1 - W_{ijsmh}) \qquad \forall i, j, s, m, h \qquad (77)$$

$$WQ_{ijsmh} \geqslant 0 \qquad \forall i, j, s, m, h \qquad (78)$$

$$NN_h \geqslant \sum_{s=1}^{S} \sum_{m=1}^{NBC_i^{UB}} \frac{S_{ij}}{v_s m} WQ_{ijsmh} \qquad \forall i, j, h \qquad (79)$$

Horizon constraints:
The product of the campaign cycle time CTC_h and the number of campaign repetitions NN_h in a period can not exceed the given period length Lp (Eq. (80)).

$$CTC_h\, NN_h \leqslant Lp \qquad (80)$$

To overcome the product of a continuous and an integer variable $CTC_h\, NN_h$, the number of campaign repetitions NN_h is discretised via B options, and the selection of one option in every period is indicated via the binary variable NNR_{bh} and accompanying Eqs. (81)-(82), with:

$$NNR_{bh} = \begin{cases} 1 & \text{if the campaign is repeated } BB_b \text{ times in period } h \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{b=1}^{B} NNR_{bh} = 1 \qquad \forall h \tag{81}$$

$$NN_h = \sum_{b=1}^{B} BB_b\, NNR_{bh} \qquad \forall h \tag{82}$$

By replacing NN_h in Eq. (80) with Eq. (82), a product of a continuous and a binary variable remains ($CTC_h\, NNR_{bh}$). To overcome this, the continuous variable $WW_{bh} = CTC_h\, NNR_{bh}$ is introduced together with linearisation constraints Eqs. (83)-(86).

$$WW_{bh} \leqslant Lp\, NNR_{bh} \qquad \forall b, h \tag{83}$$

$$WW_{bh} \leqslant CTC_h \qquad \forall b, h \tag{84}$$

$$\sum_{b=1}^{B} WW_{bh} = CTC_h \qquad \forall h \tag{85}$$

$$WW_{bh} \geqslant 0 \qquad \forall b, h \tag{86}$$

Eventually, Eq. (80) can be replaced by its linear equivalent Eq. (87).

$$\sum_{b=1}^{B} BB_b\, WW_{bh} \leqslant Lp \qquad \forall h \tag{87}$$

Scheduling constraints:
These constraints are used to obtain the assignment of product (batches) to equipment units. In order to formulate the constraints, the following binary variables are introduced:

$$y_{ijenh} = \begin{cases} 1 & \text{if product } i \text{ is assigned to slot } e \text{ and processed} \\ & \text{in equipment unit } n \text{ of stage } j \text{ in period } h \\ 0 & \text{otherwise} \end{cases}$$

$$x_{jenh} = \begin{cases} 1 & \text{if slot } e \text{ of equipment unit } n \text{ of stage } j \text{ is used in period } h \\ 0 & \text{otherwise} \end{cases}$$

$$r_{ieh} = \begin{cases} 1 & \text{if product } i \text{ is processed in slot } e \text{ in period } h \\ 0 & \text{otherwise} \end{cases}$$

If equipment unit n of stage j does not exist, then none of its slots can be used in any period (Eq. (88)) and no product batches can be assigned to it (Eq. (89)). If, on the other hand, this equipment unit exists, then at least one of its slots must be occupied by a product in a period (Eq. (90)).

$$x_{jenh} \leqslant z_{jn} \qquad \forall j, e, n, h \tag{88}$$

$$y_{ijenh} \leqslant z_{jn} \qquad \forall i, j, e, n, h \tag{89}$$

$$\sum_{i=1}^{P} \sum_{e=1}^{E} y_{ijenh} \geqslant z_{jn} \qquad \forall j, n, h \tag{90}$$

Eq. (91) states that if slot e on equipment unit n of stage j is used in period h, this slot can not be occupied by the other equipment units of that stage in that period. Analogously, Eq. (92) indicates that if that slot is occupied by product i in period h, that slot can not be occupied by the other equipment units of that stage in that period to produce another product i'.

$$x_{jen'h} \leqslant 1 - x_{jenh} \qquad \forall j, e, h, n, n' \text{ with } n \neq n' \tag{91}$$

$$y_{i'jen'h} \leqslant 1 - y_{ijenh} \qquad \forall i, i', j, e, h, n, n' \text{ with } n \neq n' \tag{92}$$

Eqs. (93)-(94), (95)-(96) and (97) define the relations among variables y_{ijenh} and respectively r_{ieh}, x_{jenh} and both. If slot e is not occupied by product i in period h, then no equipment unit n can use this slot in period h to produce product i (Eq. (93)), whereas if slot e is used to produce product i in period h, it must be produced on one of the equipment units n (Eq. (94)). If slot e of equipment unit n is not used in period h, then no product can be assigned to it in that period (Eq. (95)). Conversely, if that slot is used, then exactly one product is assigned to it (Eq. (96)). Finally, Eq. (97) states that product i is only assigned to slot e of equipment unit n of stage j in period h if this slot is used both on equipment unit n and by product i in period h.

$$y_{ijenh} \leqslant r_{ieh} \qquad \forall i, j, e, n, h \qquad (93)$$

$$\sum_{n=1}^{N} y_{ijenh} = r_{ieh} \qquad \forall i, j, e, h \qquad (94)$$

$$y_{ijenh} \leqslant x_{jenh} \qquad \forall i, j, e, n, h \qquad (95)$$

$$\sum_{i=1}^{P} y_{ijenh} = x_{jenh} \qquad \forall j, e, n, h \qquad (96)$$

$$y_{ijenh} \geqslant x_{jenh} + r_{ieh} - 1 \qquad \forall i, j, e, n, h \qquad (97)$$

The total number of slots e used by product i in period h equals the number of batches per campaign of this product in this period.

$$\sum_{e=1}^{E} r_{ieh} = n_{ih} \qquad \forall i, h \qquad (98)$$

In order to reduce computational complexity, it is assumed that the time slots in every equipment unit in every period are occupied in ascending order. Hence, Eq. (99) states that slot e is occupied before slot $e+1$ can be occupied. Furthermore, Eq. (100) is included to avoid alternative solutions (i.e. a symmetry breaking constraint), whereas Eq. (101) defines a preordering constraint: the products assigned to slots follow the same order in all the stages per period.

$$\sum_{i=1}^{P} r_{ieh} \geqslant \sum_{i=1}^{P} r_{i(e+1)h} \qquad \forall h, e \text{ with } e < E \qquad (99)$$

$$\sum_{e=1}^{E} 2^e x_{jenh} \geqslant \sum_{e=1}^{E} 2^e x_{je(n+1)h} \qquad \forall j, h, n \text{ with } n < N \qquad (100)$$

$$\sum_{i=1}^{P} \sum_{n=1}^{N} i\, y_{ijenh} = \sum_{i=1}^{P} \sum_{n=1}^{N} i\, y_{ij'enh} \qquad \forall j, j', e, h \text{ with } j < j' \qquad (101)$$

Timing constraints:
The timing constraints are given by Eqs. (102)-(107). Eq. (102) defines the finishing time of every slot e on equipment unit n of stage j in period h (TF_{jenh}), being equal to the starting time of that slot (TI_{jenh}) plus the processing time of the product assigned to that slot in that period. Eq. (103) forces the finishing time of slot e of equipment unit n to be equal or earlier than the starting time of the next slot $e+1$. Furthermore, Eq. (103), together with Eq. (104), state that if slot $e+1$ of equipment unit n is not used in period h, then the finishing time of slot e and the starting time of slot $e+1$ coincide in that period. Since a zero-wait policy is assumed, Eqs. (105) and (106) enforce the finishing time of a slot e on a stage in a period to be equal to the starting time of that slot on the next stage, taken into account the usage of the slots. Finally, to obtain the cycle time CTC_h of a campaign in period h, the differences between the finishing time of the last slot and the starting time of the first slot of every equipment unit n are calculated. The longest stage cycle time gives the eventual value for the campaign cycle time per

period. This is given by Eq. (107).

$$
\begin{aligned}
&TF_{jenh} = TI_{jenh} + \sum_{i=1}^{P} \tau_{ij} y_{ijenh} && \forall j,e,n,h && (102)\\
&TF_{jenh} \leqslant TI_{j(e+1)nh} && \forall j,e,n,h \text{ with } e < E && (103)\\
&TF_{jenh} - TI_{j(e+1)nh} \geqslant -Lp\, x_{j(e+1)nh} && \forall j,e,n,h \text{ with } e < E && (104)\\
&TF_{jenh} - TI_{(j+1)en'h} \geqslant Lp\,(x_{jenh} + x_{(j+1)en'h} - 2) && \forall j,e,n,n',h \text{ with } j < J && (105)\\
&-TF_{jenh} + TI_{(j+1)en'h} \geqslant Lp\,(x_{jenh} + x_{(j+1)en'h} - 2) && \forall j,e,n,n',h \text{ with } j < J && (106)\\
&CTC_h - TF_{j(E)nh} + TI_{jenh} \geqslant Lp((x_{jenh} - 1) - \sum_{e'=1}^{e-1} x_{je'nh}) && \forall j,e,n,h && (107)
\end{aligned}
$$

Demand and inventory constraints:
In the multiperiod context, deliveries of specified amounts of every product are promised at the end of every period. Additionally, if end-of-period inventory is allowed, the amount produced of product i in period h does no longer need to be equal to the amount required. To incorporate these features, additional constraints on the fulfilment of the delivery quantities are included.

The total delivery quantity required of every product i in every period h can be fulfilled through a combination of production in that period and inventory that was kept at the end of the previous period (Eq. (108)). Consequently, the amount kept in inventory of every product i at the end of every period h equals the sum of the amount in inventory from the previous period and the amount produced in this period minus the amount required in this period (Eq.(109)). Eq. (110) limits the total amount available at the end of every period to the maximum amount required over all periods, with $Q_i^{UB} = \max_h Q_{ih} \quad \forall i$. Similar to the previous chapter, it is assumed that this upper bound will fit in a storage tank. Eq. (111) sets the starting inventory for every product i (I_{i0}) to zero.
Finally, Eqs. (112)-(113) enforce that if no amount of product i is produced in period h, no batches are made and vice versa.

$$
\begin{aligned}
&I_{i(h-1)} + q_{ih} \geqslant Q_{ih} && \forall i,h && (108)\\
&I_{ih} = I_{i(h-1)} + q_{ih} - Q_{ih} && \forall i,h && (109)\\
&I_{i(h-1)} + q_{ih} \leqslant Q_i^{UB} && \forall i,h && (110)\\
&I_{i0} = 0 && \forall i && (111)
\end{aligned}
$$

$$
\begin{aligned}
&1 - c_{i0h} \leqslant q_{ih} && \forall i,h && (112)\\
&Q_i^{UB}(1 - c_{i0h}) \geqslant q_{ih} && \forall i,h && (113)
\end{aligned}
$$

D.1.2 Adaptations for other multiperiod characteristics

The modifications made to the mathematical model to account for different MPDF characteristics are presented here.

No end-of-period inventory In case no end-of-period inventory is allowed, $I_{ih} = 0, q_{ih} = Q_{ih} \quad \forall i,h$. Hence, Eqs. (108)-(110) are substantially simplified. Moreover, Eq. (69) becomes nonlinear in two binary variables u_{js} and c_{imh}, instead of in two binary variables and a continuous one (u_{js}, c_{imh} and q_{ih}.

$$
NN_h \geqslant \sum_{s=1}^{S} \sum_{m=1}^{NBC_i^{UB}} \frac{S_{ij} Q_{ih}}{v_s m} u_{js} c_{imh} \qquad \forall i,j \qquad (114)
$$

Fixed product mixes When the product mix is fixed, every product needs to be produced in every period. This generates the following additional constraints: Eq. (115) forbids to produce zero batches of a product, whereas Eq. (116) forces to produce at least one batch of every product i in a campaign in every period h. Finally, Eq. (117) sets a lower bound on the amount produced per period. The lower bound Q_i^{LB} is formulated similarly as for the M-SPC case. Hence, we set $Q_i^{LB} = (Q_i T_i^{min})/H$ and $T_i^{min} = \max_j \tau_{ij}/N$.

$$c_{i0h} = 0 \qquad \forall i, h \tag{115}$$

$$\sum_{m=1}^{NBC_i^{UB}} c_{imh} = 1 \qquad \forall i, h \tag{116}$$

$$q_{ih} \geqslant Q_i^{LB} \qquad \forall i, h \tag{117}$$

D.2 Objective function

The aim of the multiperiod BPDP is to minimise the total cost. This cost consists of capital and startup costs, whereas inventory holding costs are calculated ex post.

Capital costs:
The capital costs to be minimised are formulated similar as for the M-SPC models:

$$\sum_{j=1}^{J} \sum_{s=1}^{S} \sum_{n=1}^{N} \alpha_j v_s^{\beta_j} z_{jn} u_{js} \tag{118}$$

The incurred nonlinearity of z_{jn} and u_{js} is tackled through the introduction of the binary variable $Y_{jsn} = z_{jn} u_{js}$ and the following constraints:

$$Y_{jsn} \geqslant z_{jn} + u_{js} - 1 \qquad \forall j, s, n \tag{119}$$

$$Y_{jsn} \in \{0, 1\} \qquad \forall j, s, n \tag{120}$$

Startup costs:
Comparable as for the M-SPC models, in which a fixed startup cost is incurred every time a series of batches of one product (i.e. a run) starts, the startup costs are formulated as a fixed cost per campaign. Hence, it is assumed that it is not necessary to setup within the campaign, but always at the beginning of a new campaign.

The formulation of this cost is:

$$\sum_{j=1}^{J} \sum_{n=1}^{N} \sum_{b=1}^{B} \sum_{h=1}^{Np} Cstart\, BB_b\, NNR_{bh}\, z_{jn} \tag{121}$$

The product of NNR_{bh} and z_{jn} is addressed through the introduction of a binary variable: $WX_{jnbh} = NNR_{bh} z_{jn}$ and the following constraints:

$$WX_{jnbh} \geqslant NNR_{bh} + z_{jn} - 1 \qquad \forall j, n, b, h \tag{122}$$

$$WX_{jnbh} \in \{0, 1\} \qquad \forall j, n, b, h \tag{123}$$

This results in:

$$\sum_{j=1}^{J} \sum_{n=1}^{N} \sum_{b=1}^{B} \sum_{h=1}^{Np} Cstart\, BB_b\, WX_{jnbh} \tag{124}$$

Ex post inventory holding costs:
The ex post inventory holding cost expression is given by:

$$\sum_{h=1}^{Np}\sum_{i=1}^{P}\Bigg[\underbrace{(n_{ih}CTC_h - \sum_{n=1}^{N}\sum_{e=1}^{E}((TF_{(Jenh)} - TI_{(Je_{first}nh)})y_{iJenh}))B_{ih}NN_h}_{\text{I}} \\ + \underbrace{p_{ih}((NN_h - 1)Lp - CTC_h(\frac{NN_h(NN_h-1)}{2}))}_{\text{II}} \\ + \underbrace{Lp\, I_{i(h-1)}}_{\text{III}}\Bigg] InvCost \tag{125}$$

with $InvCost$ a known product independent inventory holding cost. The first term (I) corresponds to the production part: in an MPC mode of operation, different products may be produced alternately and, consequently, batches of one product can be made throughout an entire campaign length. Hence, for every batch, the finishing time on the final stage J in period h is used to mark the point at which a step of size B_{ih} is taken. Note that these finishing times are shifted over the starting time of the first batch of a campaign on this stage in a period, as these begin effects are not taken into account. The second term (II) corresponds to that part of the period length in which the amount produced is waiting to be delivered, with p_{ih} the amount produced of product i per campaign per period h. Finally, term III is included to account for the possibility of having inventory over the periods with I_{ih} the amount of end-of-period inventory.

Correction terms
Finally, to deal with some degrees of freedom present in the problem, two correction terms are included in the optimisation: an inventory correction term and an initialisation correction term.

The inventory correction term avoids the accumulation of inventory without use. This is done via:

$$\sum_{h=1}^{Np}\sum_{i=1}^{P} InvCorr I_{ih} \tag{126}$$

with $InvCorr \in]0, 0.001]$ a given coefficient.

To enforce the timings to start at zero, an initialisation correction term is given by:

$$\sum_{j=1}^{J}\sum_{n=1}^{N}\sum_{e=1}^{E}\sum_{h=1}^{Np} InitCorr\, TF_{jenh} \tag{127}$$

with $InitCorr \in]0, 0.001]$ a small, known parameter.

E Input data

Table E.1: Process and demand data - Ex.1-2

Process and demand data - Ex.1

Number of products: 3
Number of stages: 3
Max. number of parallel equip. per stage: 3

Total amount to be produced Q (in kg)

	prod 1	prod 2	prod 3
	215040	1152000	281600

Amount to be produced per period Q_{ih} (in kg)

variability

period 1	38400	288000	70600
period 2	69120	192000	103000
period 3	71680	384000	44000
period 4	35840	288000	64000

Processing time τ_{ij} (in h)

stage 1	10.74	9.83	9.83
stage 2	2.0	4.85	13.20
stage 3	5.22	18.69	6.14

Size factors S_{ij} (in l/kg)

stage 1	0.7	0.7	0.7
stage 2	0.6	0.6	0.65
stage 3	0.5	0.45	0.55

Set S of discrete sizes v_s (in l) =
{3700, 4300, 4800, 5100, 5700, 6200, 6500, 7000}
Cost parameters α = {600, 600, 700}
and β={0.6, 0.6, 0.7}
Startup cost coefficients - low $Cstart$: 800
Startup cost coefficients - high $Cstart$: 5000
Inventory holding cost $InvCost$: 0.0007

Process and demand data - Ex.2

Number of products: 3
Number of stages: 4
Max. number of parallel equip. per stage: 3

Total amount to be produced Q (in kg)

	prod 1	prod 2	prod 3
	156000	78000	104000

Amount to be produced per period Q_{ih} (in kg)

variability

period 1	12000	20000	12000
period 2	60000	38000	20000
period 3	40000	0	32000
period 4	44000	20000	40000

Processing time τ_{ij} (in h)

stage 1	6.4	6.8	1.0
stage 2	4.7	6.4	6.3
stage 3	8.3	6.5	4.4
stage 4	3.9	4.4	11.9

Size factors S_{ij} (in l/kg)

stage 1	7.9	0.7	0.7
stage 2	2.0	0.8	2.6
stage 3	5.2	0.9	1.6
stage 4	4.9	3.4	3.6

Set S of discrete sizes v_s (in l) =
{1000, 4000, 6000, 9000, 13500}
Cost parameters α = {250, 250, 250, 250}
and β={0.6, 0.6, 0.6, 0.6}
Startup cost coefficients - low $Cstart$: 450
Startup cost coefficients - high $Cstart$: 2500
Inventory holding cost $InvCost$: 0.00075

Table E.2: Process and demand data - Ex.3-4

Process and demand data - Ex.3

Number of products: 3			
Number of stages: 4			
Max. number of parallel equip. per stage: 4			
Total amount to be produced Q (in kg)			
	prod 1	prod 2	prod 3
	77376	67968	61536
Amount to be produced per period Q_{ih} (in kg)			
		variability	
period 1	24344	7992	13384
period 2	13344	13992	21384
period 3	23344	23992	19384
period 4	16344	21992	7384
Processing time τ_{ij} (in h)			
stage 1	9.3	8.5	9.7
stage 2	5.4	5.8	5.5
stage 3	4.2	4.1	4.3
stage 4	2.0	2.5	2.1
Size factors S_{ij} (in l/kg)			
stage 1	2.0	0.7	1.2
stage 2	1.6	1.3	1.5
stage 3	1.6	1.6	0.4
stage 4	2.6	2.7	1.6
Set S of discrete sizes v_s (in l) = {500, 1000, 2000, 2500, 3000, 4000}			
Cost parameters α = {135, 148, 140, 150} and β={0.6, 0.6, 0.6, 0.6}			
Startup cost coefficients - low $Cstart$: 200			
Startup cost coefficients - high $Cstart$: 750			
Inventory holding cost $InvCost$: 0.0004			

Process and demand data - Ex.4

Number of products: 4				
Number of stages: 3				
Max. number of parallel equip. per stage: 3				
Total mount to be produced Q (in kg)				
	prod 1	prod 2	prod 3	prod 4
	109714	131657	233143	329143
Amount to be produced per period Q_{ih} (in kg)				
		variability		
period 1	0	20914	23285	37285
period 2	39571	37915	71286	100786
period 3	29571	26914	75286	76286
period 4	40572	45914	63286	114786
Processing time τ_{ij} (in h)				
stage 1	14.0	16.0	12.0	10.0
stage 2	25.0	5.0	15.0	5.0
stage 3	7.0	18.0	4.0	20.0
Size factors S_{ij} (in l/kg)				
stage 1	0.7	0.6	0.7	0.65
stage 2	0.6	0.7	0.65	0.7
stage 3	0.5	0.45	0.55	0.5
Set S of discrete sizes v_s (in l) = {3000, 5600, 6800, 8400, 9800}				
Cost parameters α = {600, 600, 700} and β={0.6, 0.6, 0.7}				
Startup cost coefficients - low $Cstart$: 1000				
Startup cost coefficients - high $Cstart$: 5000				
Inventory holding cost $InvCost$: 0.0009				

Table E.3: Process and demand data - Ex.5

Number of products: 6						
Number of stages: 4						
Max. number of parallel equip. per stage: 4						
Total amount to be produced Q (in kg)						
	prod 1	prod 2	prod 3	prod 4	prod 5	prod 6
	24000	48000	32000	38400	36800	32000
Amount to be produced per period Q_{ih} (in kg)						
		variability				
period 1	6000	11000	10000	6000	12500	10000
period 2	8000	0	4000	9600	7000	5500
period 3	4000	22000	8000	12200	10500	11000
period 4	6000	15000	10000	10600	6800	6500
Processing time τ_{ij} (in h)						
stage 1	3.0	6.0	5.0	10.2	6.6	2.7
stage 2	9.2	2.6	8.0	5.5	3.5	4.3
stage 3	4.5	7.0	3.8	4.9	8.0	8.7
stage 4	7.0	3.0	7.3	6.3	5.0	2.9
Size factors S_{ij} (in l/kg)						
stage 1	0.9	1.6	1.4	1.35	0.8	1.14
stage 2	1.1	1.15	1.0	1.05	1.5	1.14
stage 3	1.3	0.7	1.0	1.05	1.32	1.14
stage 4	1.25	1.21	0.95	1.05	1.20	1.14
Set S of discrete sizes v_s (in l) = {1000, 1200, 1350, 1500, 1650, 1800, 2000}						
Cost parameters α = {650, 720, 460, 500} and β={0.65, 0.7, 0.65, 0.60}						
Startup cost coefficients - low $Cstart$: 500						
Startup cost coefficients - high $Cstart$: 1500						
Inventory holding cost $InvCost$: 0.0017						

F Detailed results - Examples 4 & 5

Example 4: 4 products over 3 stages

Table F.1: Example 4: 4 products - total capital, startup and ex post inventory holding costs and optimal design decisions, for a plant operating in SPC mode

	S-SPC	M-SPC					
		Equal delivery qty			Variable delivery qty		
Min. objective/ incl. inv. costs[c]	single[a] period	no inv	inv fix mix	inv var mix	no inv	inv fix mix	inv[b] var mix
Capital costs	520 336	533 486	533 486	533 486	608 661	533 486	533 486
Startup costs	12 000	48 000	48 000	48 000	45 000	48 000	45 000
Inventory costs	687 160	157 450	157 450	157 450	167 208	188 027	183 871
Total costs[d]	*1 219 496*	*738 936*	*738 936*	*738 936*	*820869*	*769513*	*762357*
Design decisions (written as size (number)):							
stage 1	5600(1)		6800(1)		8400(1)	6800(1)	
stage 2	6800(1)		6800(1)		8400(1)	6800(1)	
stage 3	5600(1)		5600(1)		6800(1)	5600(1)	
Total amount end-of-period inventory	0	0	0	0	0	42 771	61 486

[c]ex post inventory holding costs
[d]in monetary units

Table F.2: Example 4: 4 products - total capital, startup and ex post inventory holding costs and optimal design decisions, for a plant operating in MPC mode

	S-SPC	M-SPC					
		Equal delivery qty			Variable delivery qty		
Min. objective/ incl. inv. costs[c]	single[a] period	no inv	inv fix mix	inv var mix	no inv	inv fix mix	inv[b] var mix
Capital costs	520 336	520 336	533 486	533 486	608 661	533 486	533 486
Startup costs	15 000	60 000	36 000	36 000	36 000	48 000	48 000
Inventory costs	715 489	156 917	167 450	167 450	148 441	166 175	171 461
Total costs[d]	*1 250 825*	*737 253*	*736 936*	*736 936*	*793 102*	*747 661*	*752 947*
Design decisions (written as size (number)):							
stage 1	5600(1)	5600(1)	6800(1)		8400(1)	6800(1)	
stage 2	6800(1)	6800(1)	6800(1)		8400(1)	6800(1)	
stage 3	5600(1)	5600(1)	5600(1)		6800(1)	5600(1)	
Total amount end-of-period inventory	0	0	28 572	28 572	0	21 762	21 286

[c]ex post inventory holding costs
[d]in monetary units

Example 5: 6 Products over 4 stages

Table F.3: Example 5: 6 products - total capital, startup and ex post inventory holding costs and optimal design decisions, for a plant operating in SPC mode

	S-SPC	M-SPC					
		Equal delivery qty			Variable delivery qty		
Min. objective/ incl. inv. costs[c]	single[a] period	no inv	inv fix mix	inv var mix	no inv	inv fix mix	inv[b] var mix
Capital costs	259 732	259 732	259 732	259 732	304 893	274 832	274 832
Startup costs	12 000	48 000	48 000	48 000	46 000	48 000	46 000
Inventory costs	352 048	82 839	82 839	82 839	89 760	89 136	88 696
Total costs[d]	*623 780*	*390 571*	*390 571*	*390 571*	*440 653*	*411 968*	*409 528*
Design decisions (written as size (number)):							
stage 1	1500(1)		1500(1)		2000(1)	1500(1)	
stage 2	1200(1)		1200(1)		1500(1)	1350(1)	
stage 3	1200(1)		1200(1)		1500(1)	1350(1)	
stage 4	1200(1)		1200(1)		1500(1)	1350(1)	
Total amount end-of-period inventory	0	0	0	0	0	7 680	8 117

[c]ex post inventory holding costs
[d]in monetary units

Table F.4: Example 5: 6 products - total capital, startup and ex post inventory holding costs and optimal design decisions, for a plant operating in MPC mode

	S-SPC	M-SPC					
		Equal delivery qty			Variable delivery qty		
Min. objective/ incl. inv. costs[c]	single[a] period	no inv	inv fix mix	inv var mix	no inv	inv fix mix	inv[b] var mix
Capital costs	254 741	254 741	259 732	259 732	304 717	265 982	265 982
Startup costs	40 000	40 000	32 000	32 000	32 000	36 000	36 000
Inventory costs	345 162	86 818	107 352	107 352	79 854	102 667	102 667
Total costs[d]	*639 903*	*381 559*	*399 084*	*399 084*	*416 571*	*404 649*	*404 649*
Design decisions (written as size (number)):							
stage 1	1350(1)	1350(1)	1500(1)		1800(1)	1500(1)	
stage 2	1200(1)	1200(1)	1200(1)		1650(1)	1200(1)	
stage 3	1200(1)	1200(1)	1200(1)		1500(1)	1350(1)	
stage 4	1200(1)	1200(1)	1200(1)		1350(1)	1350(1)	
Total amount end-of-period inventory	0	0	19 309	19 309	0	13 015	13 015

[c]ex post inventory holding costs
[d]in monetary units